

Precalculus · Vectors in the Plane · Worksheet Answer Key

Common Core N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · mathovore.fr

This precalculus vectors worksheet answer key solves all 22 problems of the vectors in the plane worksheet step by step.

Every solution below shows the key steps and ends with the final answer in bold. Compare your work step by step: setting up the component form correctly is usually the hardest part, and the later arithmetic follows from it.

Precalculus Common Core: N-VM.A.1 · N-VM.A.2 · N-VM.A.3 · N-VM.B.4 · N-VM.B.5

Precalculus Vectors Worksheet Answer Key: problems 1-5

Problem 1: Component form from two points

- Subtract coordinates: $\langle 7-2, 13-1 \rangle = \langle 5, 12 \rangle$. Then $\sqrt{25+144} = \sqrt{169} = 13$. $\langle 5, 12 \rangle$, **magnitude 13**.
- $\langle 1-(-3), -2-4 \rangle = \langle 4, -6 \rangle$. Then $\sqrt{16+36} = \sqrt{52} = 2\sqrt{13} \approx 7.21$. $\langle 4, -6 \rangle$, **magnitude $2\sqrt{13} \approx 7.21$** .
- $\langle 0-5, 0-5 \rangle = \langle -5, -5 \rangle$. Then $\sqrt{25+25} = 5\sqrt{2} \approx 7.07$. $\langle -5, -5 \rangle$, **magnitude $5\sqrt{2} \approx 7.07$** .

Problem 2: Magnitude of a vector

- $\sqrt{36+64} = \sqrt{100} = 10$
- $\sqrt{25+144} = \sqrt{169} = 13$
- $\sqrt{81+1600} = \sqrt{1681} = 41$
- $\sqrt{4+9} = \sqrt{13} \approx 3.61$

Problem 3: Direction angles

- $\tan \theta = \frac{3}{3} = 1$, quadrant I: 45° .
- Reference angle $\tan^{-1}(1) = 45^\circ$, quadrant II: $180^\circ - 45^\circ = 135^\circ$.
- Reference angle 45° , quadrant III: $180^\circ + 45^\circ = 225^\circ$.
- $\tan \theta = -\sqrt{3}$, reference angle 60° , quadrant IV: $360^\circ - 60^\circ = 300^\circ$.

Problem 4: Components from magnitude and angle

- $\langle 10\cos 30^\circ, 10\sin 30^\circ \rangle = \langle 5\sqrt{3}, 5 \rangle \approx \langle 8.66, 5 \rangle$.
- $\langle 8\cos 120^\circ, 8\sin 120^\circ \rangle = \langle -4, 4\sqrt{3} \rangle \approx \langle -4, 6.93 \rangle$.
- $\langle 12\cos 270^\circ, 12\sin 270^\circ \rangle = \langle 0, -12 \rangle$.
- $\langle 6\sqrt{2} \cdot \frac{\sqrt{2}}{2}, 6\sqrt{2} \cdot \frac{\sqrt{2}}{2} \rangle = \langle 6, 6 \rangle$.

Problem 5: Add and subtract vectors

- a. $\langle 3+(-1), -2+5 \rangle = \langle 2, 3 \rangle$
- b. $\langle 3-(-1), -2-5 \rangle = \langle 4, -7 \rangle$
- c. $\langle -1-3, 5-(-2) \rangle = \langle -4, 7 \rangle$. Notice $\vec{v}-\vec{u}$ is the opposite of $\vec{u}-\vec{v}$.

Precalculus Vectors Worksheet Answer Key: problems 6-10

Problem 6: Scalar multiplication

- a. $\langle 12, -8 \rangle$
- b. $\langle 3, -15 \rangle$
- c. $\langle 3, -5 \rangle$
- d. $\langle -6, 4 \rangle$

Problem 7: Linear combinations

- a. $\langle 6, -4 \rangle - \langle -3, 15 \rangle = \langle 9, -19 \rangle$
- b. $\langle 9, -6 \rangle + \langle -2, 10 \rangle = \langle 7, 4 \rangle$
- c. $\sqrt{81+361} = \sqrt{442} \approx 21.02$

Problem 8: Unit vectors

- a. $|\vec{v}|=10$, so $\langle \frac{4}{5}, -\frac{3}{5} \rangle$.
- b. $|\vec{v}|=13$, so $\langle \frac{5}{13}, \frac{12}{13} \rangle$.
- c. $|\vec{v}|=\sqrt{49+576}=25$, so $\langle -\frac{7}{25}, \frac{24}{25} \rangle$.
- d. $|\vec{v}|=9$, so $\langle 0, -1 \rangle$.

Problem 9: Unit vector notation i and j

- a. $4\vec{i}-3\vec{j}$
- b. $\langle 7, 2 \rangle$, magnitude $\sqrt{49+4}=\sqrt{53} \approx 7.28$.
- c. $\langle -1, 6 \rangle$
- d. $6\vec{i}-3\vec{j}+\vec{i}+4\vec{j}=7\vec{i}+\vec{j}$

Problem 10: Vectors with a given length

- a. $|\langle 3, 4 \rangle|=5$, so multiply the unit vector by 20: $20\langle \frac{3}{5}, \frac{4}{5} \rangle = \langle 12, 16 \rangle$.
- b. Reverse the sign: $\langle -12, -16 \rangle$.
- c. $|\langle -5, 12 \rangle|=13$, so scale by $\frac{26}{13}=2$: $\langle -10, 24 \rangle$. Check: $\sqrt{100+576}=26$.

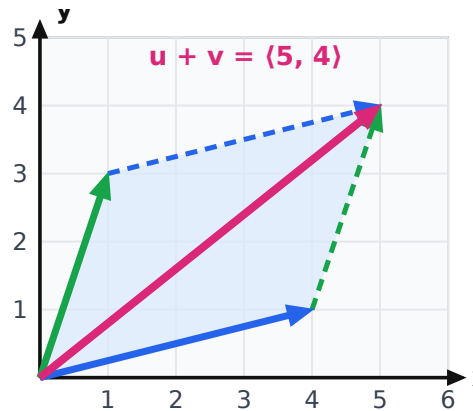
Precalculus Vectors Worksheet Answer Key: problems 11-15

Problem 11: Parallelogram rule

Slide each arrow to the tip of the other. The fourth corner is $(5, 4)$, so $\vec{u}+\vec{v}=\langle 4+1, 1+3 \rangle=\langle 5, 4 \rangle$.

$$\vec{u} - \vec{v} = \langle 4-1, 1-3 \rangle = \langle 3, -2 \rangle.$$

Length of the diagonal: $\sqrt{25+16} = \sqrt{41} \approx 6.40$.



The parallelogram completed: the diagonal is $\vec{u} + \vec{v}$.

Problem 12: Tip-to-tail path

$\vec{a} + \vec{b} + \vec{c} = \langle 2+4-5, 3-1+2 \rangle = \langle 1, 4 \rangle$, which matches the end point $(1, 4)$ in the figure.

Magnitude $\sqrt{1+16} = \sqrt{17} \approx 4.12$ blocks. Direction $\tan^{-1}(4) \approx 76.0^\circ$.

Problem 13: Dot products

- a. $8-3=5$
- b. $12-12=0$
- c. $-2+12=10$
- d. $-6+20=14$

Only pair (b) has a dot product of 0, so **pair (b) is perpendicular**.

Problem 14: Angle between two vectors

- a. $\cos \theta = \frac{1}{1\sqrt{2}}$, so $\theta = 45.0^\circ$.
- b. $\cos \theta = \frac{6}{\sqrt{10}\sqrt{10}} = 0.6$, so $\theta \approx 53.1^\circ$.
- c. $\cos \theta = \frac{-10}{\sqrt{5}\sqrt{20}} = \frac{-10}{10} = -1$, so $\theta = 180.0^\circ$ (opposite directions).
- d. $\vec{u} \cdot \vec{v} = -3+2 = -1$, $\cos \theta = \frac{-1}{\sqrt{5}\sqrt{10}} \approx -0.1414$, so $\theta \approx 98.1^\circ$.

Problem 15: Perpendicular and parallel vectors

- a. $4k-18=0$, so $k = \frac{9}{2}$.
- b. Parallel means a scalar multiple. Since $6=2 \cdot 3$, also $k = 2 \cdot 2 = 4$.
- c. Swap the components and change one sign: $\langle 2, 5 \rangle$. Check: $10-10=0$ and $\sqrt{4+25} = \sqrt{29}$. $\langle 2, 5 \rangle$ (or $\langle -2, -5 \rangle$).

Precalculus Vectors Worksheet Answer Key: problems 16-20

Problem 16: Angles of a triangle

At A: $\overrightarrow{AB} = \langle 4, 0 \rangle$, $\overrightarrow{AC} = \langle 1, 3 \rangle$, $\cos A = \frac{4}{4\sqrt{10}} \approx 0.3162$, so $A \approx 71.57^\circ$.

At B: $\overrightarrow{BA} = \langle -4, 0 \rangle$, $\overrightarrow{BC} = \langle -3, 3 \rangle$, $\cos B = \frac{12}{4 \cdot 3\sqrt{2}} \approx 0.7071$, so $B = 45.00^\circ$.

At C: $\overrightarrow{CA} = \langle -1, -3 \rangle$, $\overrightarrow{CB} = \langle 3, -3 \rangle$, $\cos C = \frac{6}{\sqrt{10} \cdot 3\sqrt{2}} \approx 0.4472$, so $C \approx 63.43^\circ$.

Sum: $71.57 + 45.00 + 63.43 = 180.00$. The angles check out.

Problem 17: Hiking displacement

$\langle 3, 0 \rangle + \langle 0, 4 \rangle + \langle -6, 0 \rangle = \langle -3, 4 \rangle$.

Distance: $\sqrt{9+16} = \mathbf{5 \text{ miles}}$.

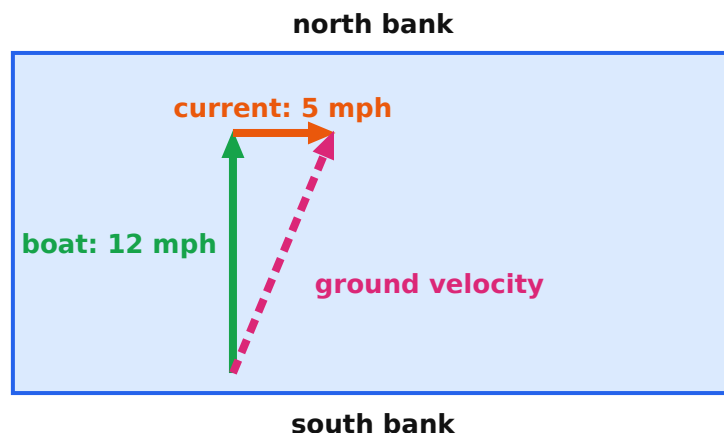
Direction: reference angle $\tan^{-1}\left(\frac{4}{3}\right) \approx 53.13^\circ$ in quadrant II, so $180^\circ - 53.13^\circ = 126.87^\circ$ (about 36.9° west of north).

Problem 18: Boat crossing a river

Ground velocity $\langle 5, 12 \rangle$, speed $\sqrt{25+144} = \mathbf{13 \text{ mph}}$.

Time to cross: only the across component counts, $\frac{0.6}{12} = 0.05$ hour, which is **3 minutes**.

Drift: $5 \times 0.05 = 0.25$ mile, which is $0.25 \times 5280 = \mathbf{1,320 \text{ feet}}$ downstream.



Speed 13 mph: the 5-12-13 triangle.

Problem 19: Airplane and crosswind

Ground velocity (east, north): $\langle 50, 300 \rangle$. Speed $\sqrt{2500 + 90000} = \sqrt{92500} \approx \mathbf{304.1 \text{ mph}}$.

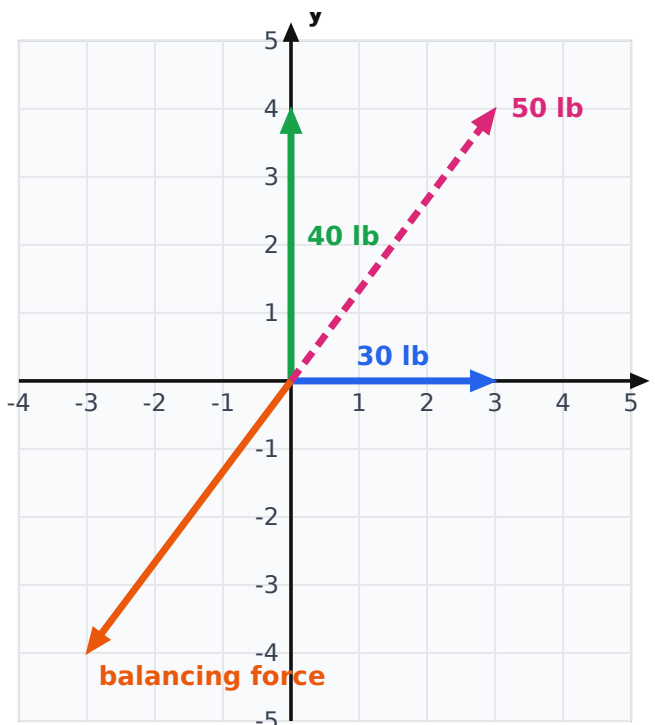
Angle east of north: $\tan^{-1}\left(\frac{50}{300}\right) \approx 9.5^\circ$.

Distance in 2 hours: $2 \times 304.14 \approx \mathbf{608.3 \text{ miles}}$.

Problem 20: Forces in equilibrium

Resultant: $\langle 30, 0 \rangle + \langle 0, 40 \rangle = \langle 30, 40 \rangle$, magnitude $\sqrt{900 + 1600} = \mathbf{50 \text{ pounds}}$, direction $\tan^{-1}\left(\frac{4}{3}\right) \approx 53.1^\circ$.

The balancing force is the opposite vector: $\langle -30, -40 \rangle$, which is 50 pounds at $180^\circ + 53.1^\circ = 233.1^\circ$.



The balancing force points opposite the resultant.

Precalculus Vectors Worksheet Answer Key: problems 21-22

Problem 21: Work done by a force

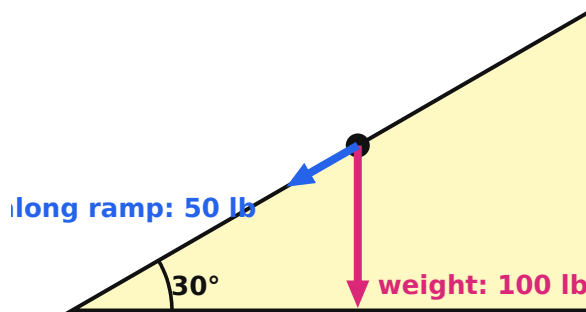
Force $\vec{F} = \langle 40 \cos 60^\circ, 40 \sin 60^\circ \rangle = \langle 20, 20\sqrt{3} \rangle$ pounds; displacement $\vec{d} = \langle 25, 0 \rangle$ feet.

Work $= \vec{F} \cdot \vec{d} = 20 \cdot 25 + 20\sqrt{3} \cdot 0 = \mathbf{500 \text{ foot-pounds}}$.

Problem 22: Crate on a ramp

$$\vec{W} \cdot \vec{u} = 0 \cdot \cos 30^\circ + (-100)(-\sin 30^\circ) = 100 \cdot \frac{1}{2} = \mathbf{50 \text{ pounds}} \text{ down the ramp.}$$

To hold the crate still, push up the ramp with a force of **50 pounds**. The remaining $100 \cos 30^\circ \approx 86.6$ pounds presses into the ramp.



Weight and its component along the ramp.

How to use this precalculus vectors worksheet answer key

Read this precalculus vectors worksheet answer key after trying each problem yourself, and redo any problem you missed.

