

Precalculus · Vectors in the Plane · Lesson

Common Core N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · mathovore.fr

This precalculus vectors lesson explains vectors in the plane in one printable page, with definitions, a method and a worked example.

A vector is a quantity with magnitude and direction, written in component form as $\langle a, b \rangle$. Its magnitude is the square root of a squared plus b squared. Vectors add, subtract, and scale component by component, and the dot product $u \cdot v$ equals $|u||v| \cos \theta$, which gives angles and tests for perpendicular vectors.

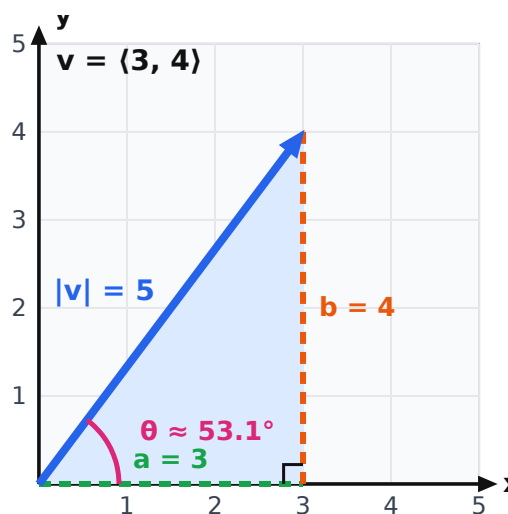
Precalculus Common Core: N-VM.A.1 · N-VM.A.2 · N-VM.A.3 · N-VM.B.4 · N-VM.B.5

Precalculus Vectors Lesson: key facts

- Magnitude of $\langle a, b \rangle$ is $\sqrt{a^2 + b^2}$; direction angle satisfies $\tan \theta = b/a$ in the correct quadrant.
- Add and subtract component by component; the parallelogram rule gives the same sum geometrically.
- Scalar multiplication $k\langle a, b \rangle = \langle ka, kb \rangle$ changes the length by $|k|$ and reverses direction when k is negative.
- A unit vector in the direction of v is $v/|v|$; every vector is $a \cdot i + b \cdot j$.
- $u \cdot v = u_1v_1 + u_2v_2 = |u||v| \cos \theta$, and $u \cdot v = 0$ exactly when u and v are perpendicular.

What is a vector, and how do you find its magnitude and direction?

A **vector** is a quantity with both a size and a direction, drawn as an arrow. In component form $\vec{v} = \langle a, b \rangle$ the number a is the horizontal change and b is the vertical change. The **magnitude** is $|\vec{v}| = \sqrt{a^2 + b^2}$. The **direction angle** θ is measured counterclockwise from the positive x-axis, and $\tan \theta = \frac{b}{a}$ once you place the angle in the correct quadrant. For $\vec{v} = \langle 3, 4 \rangle$, the magnitude is $\sqrt{9+16} = 5$ and $\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.1^\circ$.



The components 3 and 4 form a right triangle; the magnitude is its hypotenuse.

Vocabulary

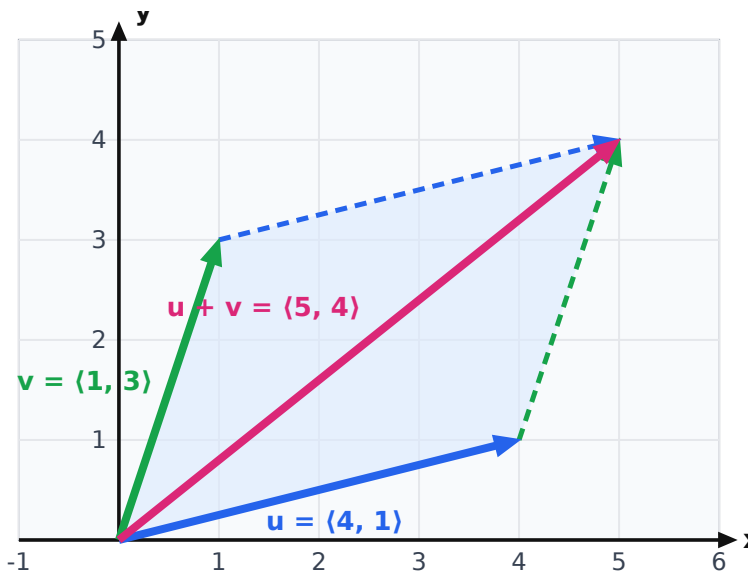
Unit vector: a vector of magnitude 1. The unit vector in the direction of $\vec{v}$ is $\frac{\vec{v}}{|\vec{v}|}$.

Standard unit vectors: $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$, so $\langle a, b \rangle = a\vec{i} + b\vec{j}$.

Components from magnitude and angle: $\langle r \cos \theta, r \sin \theta \rangle$.

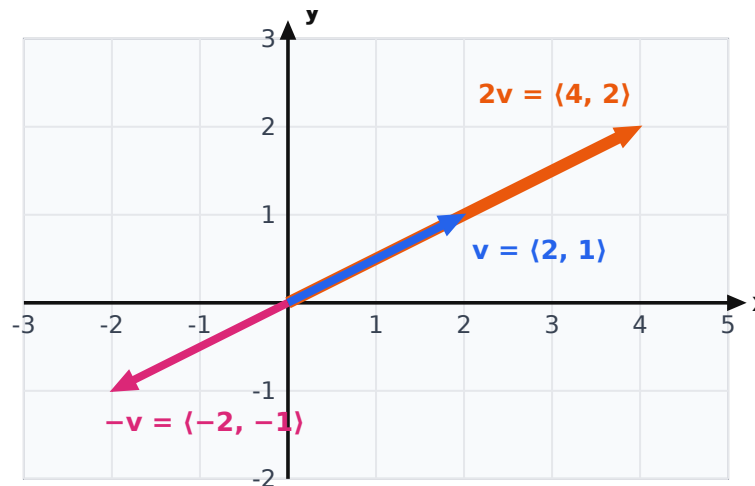
How do you add, subtract, and scale vectors?

Add or subtract vectors one component at a time: $\langle a, b \rangle + \langle c, d \rangle = \langle a + c, b + d \rangle$. Geometrically, place the tail of the second arrow at the tip of the first, or draw both from one point and complete a parallelogram. The diagonal from the shared tail is the sum. To subtract, add the opposite: $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$.



The parallelogram rule: the diagonal is the sum $\vec{u} + \vec{v} = \langle 5, 4 \rangle$.

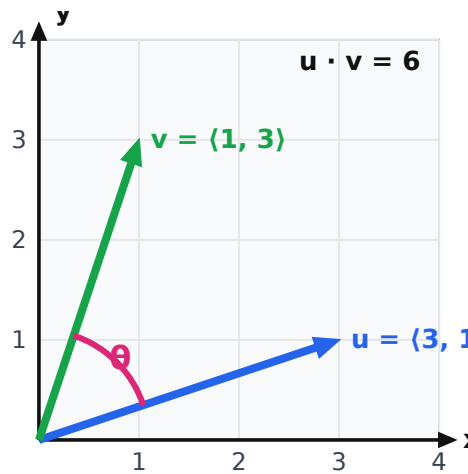
A **scalar** is an ordinary number. Multiplying by a scalar k scales every component: $k\langle a, b \rangle = \langle ka, kb \rangle$. The length changes by the factor $|k|$, and a negative k reverses the direction.



Doubling stretches the arrow; a negative scalar flips it.

What do the dot product and the angle between vectors tell you?

The **dot product** of $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ is the number $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2$. It also equals $|\vec{u}||\vec{v}|\cos\theta$, so $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$. A dot product of 0 means the vectors are perpendicular. For $\langle 3, 1 \rangle$ and $\langle 1, 3 \rangle$: $\cos\theta = \frac{6}{\sqrt{10}\sqrt{10}} = 0.6$, so $\theta \approx 53.1^\circ$.

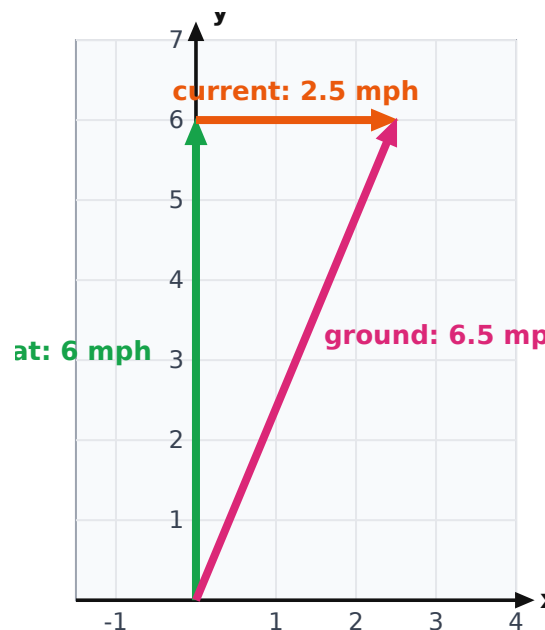


The angle between two vectors that start at the same point.

How do you solve velocity and force problems?

1. Choose axes and write each quantity as a vector in component form.
2. Add the vectors to get the resultant.
3. Find its magnitude and direction angle.
4. State the answer with units.

Example. A boat heads north across a river at 6 mph while the current flows east at 2.5 mph. The ground velocity is $\langle 0, 6 \rangle + \langle 2.5, 0 \rangle = \langle 2.5, 6 \rangle$. Its speed is $\sqrt{2.5^2 + 6^2} = 6.5$ mph, and its direction is $\tan^{-1}\left(\frac{6}{2.5}\right) \approx 67.4^\circ$ above east, which is about 22.6° east of north.



Tip-to-tail addition of the boat velocity and the current gives the ground velocity.

Vectors in the Plane: frequently asked questions

What is the difference between a vector and a scalar?

A scalar is a single number such as 12 mph or 5 lb. A vector also has a direction, such as 12 mph heading northeast. Speed is a scalar; velocity is a vector.

How do you find the direction angle of a vector in the second or third quadrant?

Compute the reference angle with the inverse tangent of the absolute value of b/a , then adjust for the quadrant. In quadrant II the angle is 180° minus the reference angle; in quadrant III it is 180° plus the reference angle. Many calculators handle this with a two-argument arctangent.

Why is the dot product a number and not a vector?

The dot product multiplies matching components and adds the results, so everything collapses to a single scalar. It measures how much two vectors point in the same direction.

How do you know two vectors are parallel?

Two nonzero vectors are parallel when one is a scalar multiple of the other. For example, $\langle 2, 3 \rangle$ and $\langle 6, 9 \rangle$ are parallel because the second is 3 times the first.

How do you add two vectors given by magnitude and angle?

Convert each vector to components with $r \cos \theta$ and $r \sin \theta$, add the x-components and the y-components, and then convert the resultant back to a magnitude and direction angle.

How to use this precalculus vectors lesson

Use this precalculus vectors lesson before the worksheet, then check what you learned with the quiz.

