

Precalculus · Angles, Radians and the Unit Circle · Worksheet Answer Key

Common Core F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · mathovore.fr

This precalculus unit circle worksheet answer key solves all 23 problems of the angles, radians and the unit circle worksheet step by step.

Complete worked solutions for the radians and unit circle worksheet. Exact answers are in bold. For every trigonometric value, name the quadrant, the reference angle and the sign, because that three-step habit prevents most errors.

Precalculus Common Core: F-TF.A.1 · F-TF.A.2 · F-TF.A.3 · F-TF.A.4

Precalculus Unit Circle Worksheet Answer Key: problems 1-5

Problem 1: Degrees to radians

- a. $30^\circ \cdot \frac{\pi}{180} = \frac{\pi}{6}$
- b. $135^\circ \cdot \frac{\pi}{180} = \frac{3\pi}{4}$
- c. $270^\circ \cdot \frac{\pi}{180} = \frac{3\pi}{2}$
- d. $-210^\circ \cdot \frac{\pi}{180} = -\frac{7\pi}{6}$
- e. $15^\circ \cdot \frac{\pi}{180} = \frac{\pi}{12}$

Problem 2: Radians to degrees

- a. $\frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ$
- b. $\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = 315^\circ$
- c. $\frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = 120^\circ$
- d. $\frac{11\pi}{6} \cdot \frac{180^\circ}{\pi} = 330^\circ$
- e. $3 \cdot \frac{180^\circ}{\pi} \approx 171.89^\circ$

Problem 3: Coterminal angles

- a. $\frac{17\pi}{6} - 2\pi = \frac{5\pi}{6}$
- b. $-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$
- c. $\frac{25\pi}{4} - 6\pi = \frac{\pi}{4}$
- d. $-\frac{11\pi}{4} + 4\pi = \frac{5\pi}{4}$

Problem 4: Reference angles

- a. Quadrant II, reference angle $\frac{\pi}{6}$
- b. Quadrant IV, reference angle $\frac{\pi}{4}$

- c. Quadrant IV, reference angle $\frac{\pi}{6}$
- d. Coterminal angle $\frac{\pi}{3}$; Quadrant I, reference angle $\frac{\pi}{3}$
- e. Coterminal angle $\frac{4\pi}{3}$; Quadrant III, reference angle $\frac{\pi}{3}$

Problem 5: Special angle values

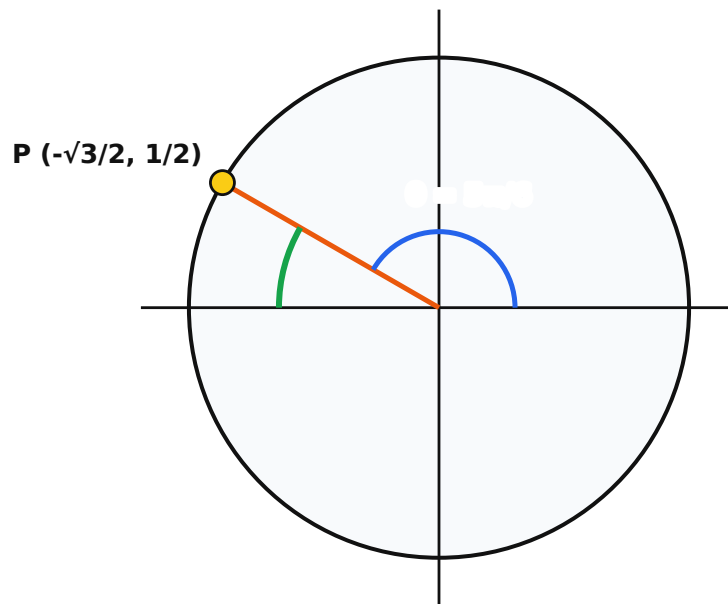
Read the coordinates $(\cos\theta, \sin\theta)$ and divide for the tangent.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Precalculus Unit Circle Worksheet Answer Key: problems 6-10

Problem 6: Point on the unit circle

- a. The reference angle satisfies $\sin\frac{\pi}{6} = \frac{1}{2}$. In Quadrant II, $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$
- b. $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$, $\tan\theta = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$



The angle is $5\pi/6$; the green arc is the reference angle $\pi/6$.

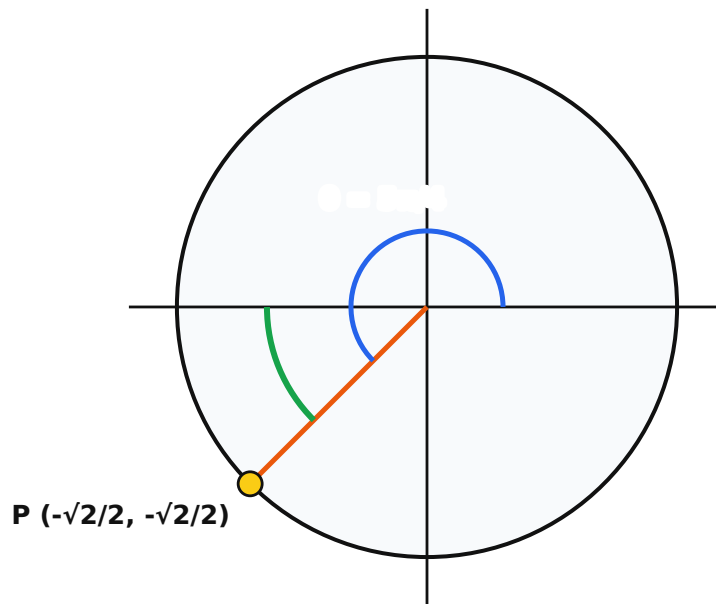
Problem 7: Exact values by symmetry

- a. $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$. Reference angle $\frac{\pi}{6}$, Quadrant II, sine is positive there.
- b. $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$. Reference angle $\frac{\pi}{4}$, Quadrant IV, cosine is positive there.
- c. $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$. Reference angle $\frac{\pi}{3}$, Quadrant II, tangent is negative there.

- d. $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$. Reference angle $\frac{\pi}{6}$, Quadrant IV, sine is negative there.
- e. $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$. Reference angle $\frac{\pi}{3}$, Quadrant III, cosine is negative there.

Problem 8: Reading a point from an angle

- a. The terminal side is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$ past the negative x-axis.
- b. Both coordinates have magnitude $\frac{\sqrt{2}}{2}$ and are negative in Quadrant III: $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
- c. $\tan\frac{5\pi}{4} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$



P is at $(-\sqrt{2}/2, -\sqrt{2}/2)$.

Problem 9: Even and odd identities

- a. $\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.
- b. $\cos\left(-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.
- c. $\tan\left(-\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.
- d. $\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right) = -\frac{1}{2}$.
- e. $\cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$.

Problem 10: Coordinates on the unit circle

- a. $\sin^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$, and sine is negative in Quadrant IV, so $\sin\theta = -\frac{4}{5}$. Then $\tan\theta = \frac{-4/5}{3/5} = -\frac{4}{3}$
- b. $\frac{25}{169} + y^2 = 1$ gives $y^2 = \frac{144}{169}$. In Quadrant III, $y = -\frac{12}{13}$

Precalculus Unit Circle Worksheet Answer Key: problems 11-15

Problem 11: Periodicity

- a. $\sin\left(\frac{25\pi}{6}\right) = \sin\left(\frac{25\pi}{6} - 4\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- b. $\cos\left(-\frac{13\pi}{3}\right) = \cos\left(-\frac{13\pi}{3} + 6\pi\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$.
- c. $\tan\left(\frac{9\pi}{4}\right) = \tan\left(\frac{9\pi}{4} - 2\pi\right) = \tan\left(\frac{\pi}{4}\right) = 1$.
- d. $\sin\left(-\frac{17\pi}{4}\right) = \sin\left(-\frac{17\pi}{4} + 6\pi\right) = \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.
- e. $\tan\left(-\frac{11\pi}{6}\right) = \tan\left(-\frac{11\pi}{6} + 2\pi\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$.

Problem 12: Solving trigonometric equations

- a. The reference angle is $\frac{\pi}{6}$. The value is positive, so the terminal side is in Quadrant I or II: $\theta = \frac{\pi}{6}$ and $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.
- b. The reference angle is $\frac{\pi}{4}$. The value is negative, so the terminal side is in Quadrant II or III: $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ and $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$.
- c. The reference angle is $\frac{\pi}{4}$. The value is negative, so the terminal side is in Quadrant II or IV: $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ and $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.
- d. The reference angle is $\frac{\pi}{3}$. The value is negative, so the terminal side is in Quadrant III or IV: $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ and $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

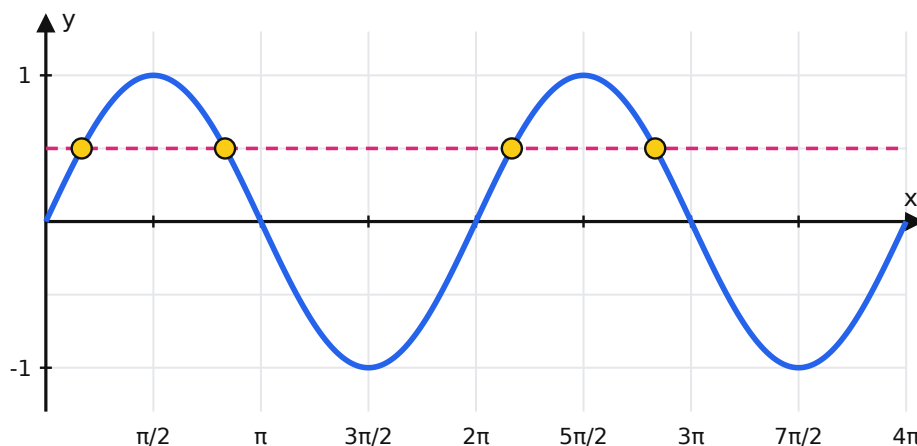
Problem 13: Even or odd functions

- a. $f(-x) = (-x)\sin(-x) = (-x)(-\sin x) = x\sin x = f(x)$, so **even**
- b. $g(-x) = (-x)\cos(-x) = -x\cos x = -g(x)$, so **odd**
- c. $h\left(\frac{\pi}{4}\right) = \sqrt{2}$ but $h\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$, which equals neither $h\left(\frac{\pi}{4}\right)$ nor $-h\left(\frac{\pi}{4}\right)$, so **neither**

Problem 14: Solutions from a graph

On $[0, 2\pi)$, $\sin x = \frac{1}{2}$ at $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$. Sine has period 2π , so add 2π : $\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$ and $\frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$.

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$



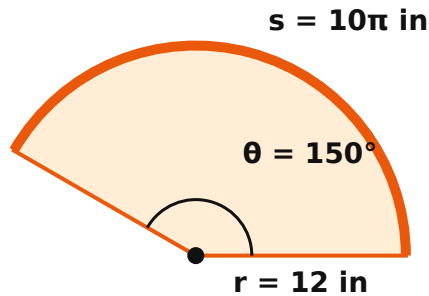
The four marked points match the four solutions.

Problem 15: Arc length

a. $150^\circ = \frac{5\pi}{6}$

b. $s = r\theta = 12 \cdot \frac{5\pi}{6} = 10\pi \approx 31.42$ in

c. $s = 8 \cdot 2.5 = 20$ ft



The arc length is 10π inches.

Precalculus Unit Circle Worksheet Answer Key: problems 16-20

Problem 16: Central angle from arc length

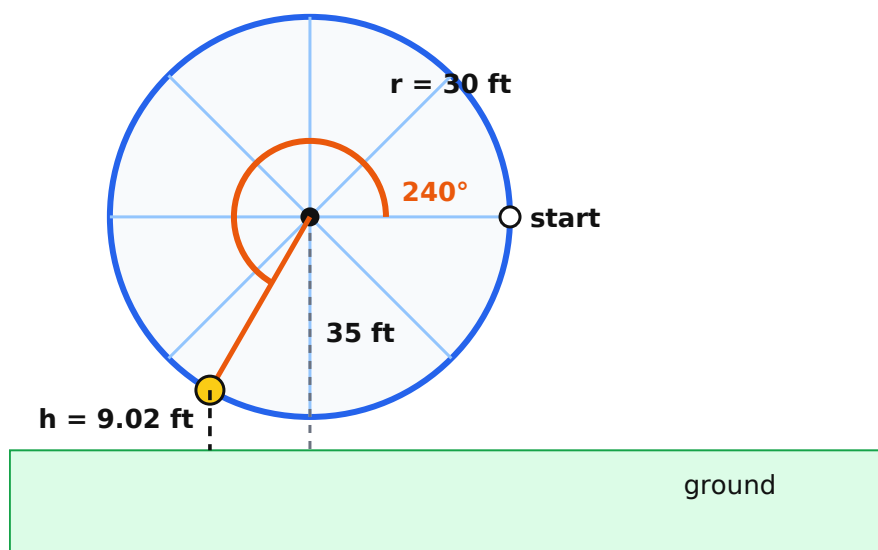
$$\theta = \frac{s}{r} = \frac{9}{6} = 1.5 \text{ radians.}$$

$$1.5 \cdot \frac{180^\circ}{\pi} \approx 85.94^\circ.$$

Problem 17: Ferris wheel height

$$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \text{ (reference angle } \frac{\pi}{3}, \text{ Quadrant III).}$$

$$h = 35 + 30\left(-\frac{\sqrt{3}}{2}\right) = 35 - 15\sqrt{3} \approx 9.02 \text{ ft.}$$



The rider is about 9.02 ft above the ground.

Problem 18: Bicycle wheel speed

$$\omega = 3 \cdot 2\pi = 6\pi \approx 18.85 \text{ rad/s.}$$

The radius is 13 in, so $v = r\omega = 13 \cdot 6\pi = 78\pi \approx 245.04$ in/s.

$$245.04 \cdot \frac{3600}{12 \cdot 5280} \approx 13.92 \text{ mph.}$$

Problem 19: Turntable speed

$$\omega = \frac{100}{3} \cdot 2\pi = \frac{200\pi}{3} \approx 209.44 \text{ rad/min.}$$

$$v = 5 \cdot \frac{200\pi}{3} = \frac{1000\pi}{3} \approx 1047.20 \text{ in/min for a point 5 in from the center.}$$

Problem 20: Windshield wiper

$$140^\circ = 140 \cdot \frac{\pi}{180} = \frac{7\pi}{9}.$$

$$s = r\theta = 18 \cdot \frac{7\pi}{9} = 14\pi \approx 43.98 \text{ in.}$$

Precalculus Unit Circle Worksheet Answer Key: problems 21-23

Problem 21: Lighthouse beam

$$\omega = 2 \cdot 2\pi = 4\pi \approx 12.57 \text{ rad/min.}$$

$$v = 5 \cdot 4\pi = 20\pi \approx 62.83 \text{ mi/min, and } 62.83 \cdot 60 \approx 3769.91 \text{ mph.}$$

Problem 22: Running track curve

$$\theta = \frac{s}{r} = \frac{300}{100} = 3 \text{ radians.}$$

$$3 \cdot \frac{180^\circ}{\pi} \approx 171.89^\circ.$$

Problem 23: Minute hand travel

Twenty minutes is one third of a turn, so $\theta = \frac{2\pi}{3}$.

$$s = 6 \cdot \frac{2\pi}{3} = 4\pi \approx 12.57 \text{ in.}$$

$$\text{Average speed: } \frac{4\pi}{20} = \frac{\pi}{5} \approx 0.63 \text{ in/min.}$$

How to use this precalculus unit circle worksheet answer key

Read this precalculus unit circle worksheet answer key after trying each problem yourself, and redo any problem you missed.

