

Precalculus · Angles, Radians and the Unit Circle · Lesson

Common Core F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · mathovore.fr

This precalculus unit circle lesson explains angles, radians and the unit circle in one printable page, with definitions, a method and a worked example.

A radian is the angle whose arc equals the radius, so π radians equals 180 degrees. On the unit circle the point at angle θ is $(\cos \theta, \sin \theta)$. Reference angles and symmetry turn the values at $\pi/6$, $\pi/4$ and $\pi/3$ into every other angle, and arc length is $s = r$ times θ .

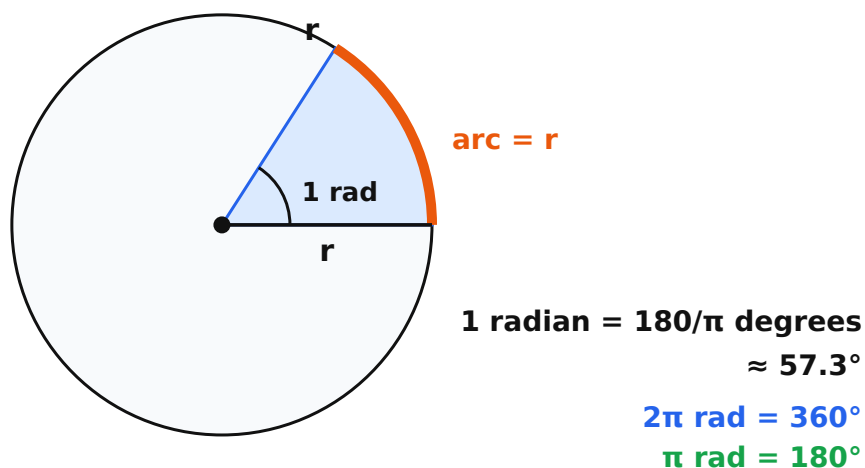
Precalculus Common Core: F-TF.A.1 · F-TF.A.2 · F-TF.A.3 · F-TF.A.4

Precalculus Unit Circle Lesson: key facts

- 180 degrees = π radians; multiply degrees by $\pi/180$ to get radians.
- The point at angle θ on the unit circle is $(\cos \theta, \sin \theta)$, and $\tan \theta = \sin \theta / \cos \theta$.
- Special values: $\sin$ and $\cos$ of $\pi/6$, $\pi/4$, $\pi/3$ are $1/2$, $\sqrt{2}/2$, $\sqrt{3}/2$ in some order.
- Arc length is $s = r \cdot \theta$ and linear speed is $v = r \cdot \omega$, with θ in radians.
- Sine and cosine have period 2π , tangent has period π ; cosine is even, sine and tangent are odd.

What is a radian?

A **radian** is the measure of a central angle whose arc is exactly as long as the radius. A full circle has circumference $2\pi r$, so one full turn is 2π radians and π radians equals 180° . To convert, multiply degrees by $\frac{\pi}{180}$, or radians by $\frac{180}{\pi}$. For example, $135^\circ = \frac{3\pi}{4}$.



One radian is the angle that cuts off an arc equal to the radius.

Vocabulary

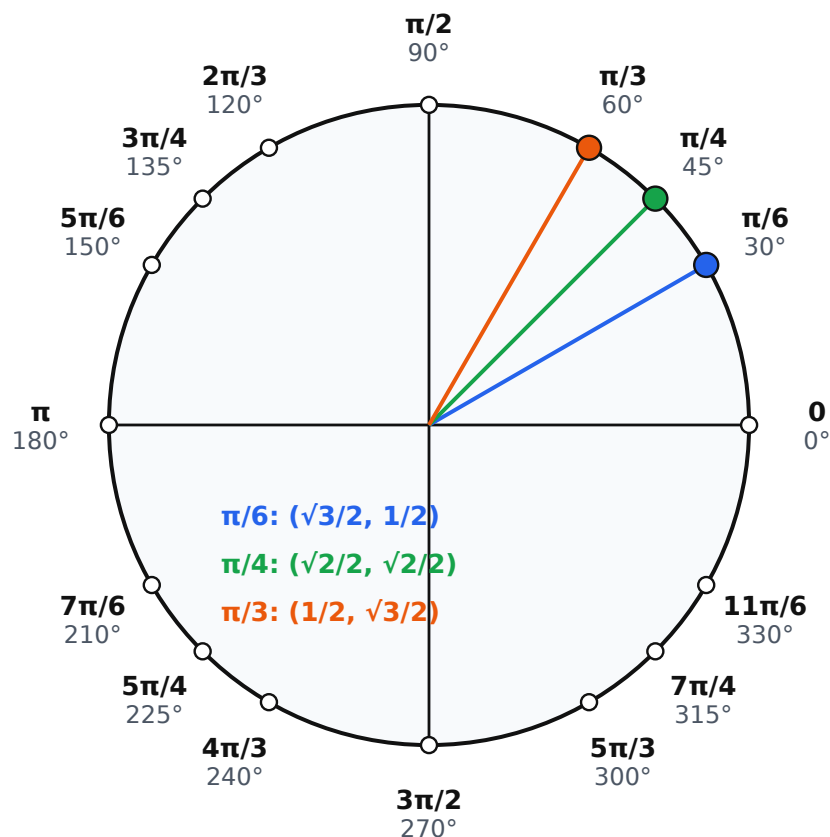
Unit circle: the circle $x^2+y^2=1$ centered at the origin. The point at angle θ is $(\cos\theta, \sin\theta)$.

Coterminal angles: angles that share a terminal side, such as θ and $\theta+2\pi$. **Reference angle:** the acute angle between the terminal side and the x-axis.

What are the special angle values on the unit circle?

Because the point at angle θ has coordinates $(\cos\theta, \sin\theta)$, every value comes from reading a coordinate, and $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Angle	$\cos\theta$	$\sin\theta$	$\tan\theta$
$\frac{\pi}{6}$ (30°)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$ (45°)	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$ (60°)	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$

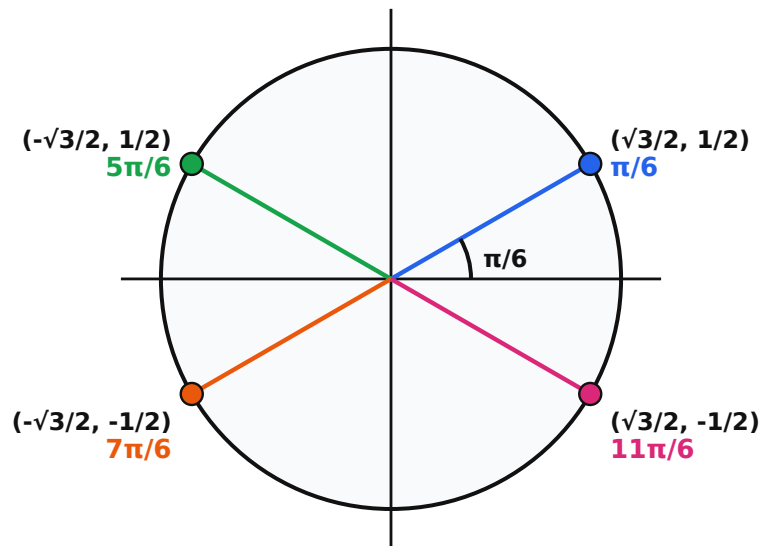


The unit circle with special angles in radians and degrees.

How do reference angles and symmetry give every other value?

1. Rewrite the angle between 0 and 2π by adding or subtracting full turns.
2. Find the reference angle: $\pi - \theta$ in Quadrant II, $\theta - \pi$ in Quadrant III, $2\pi - \theta$ in Quadrant IV.
3. Read the value for the reference angle from the table.
4. Attach the sign: sine is positive in Quadrants I and II, cosine in I and IV, tangent in I and III.

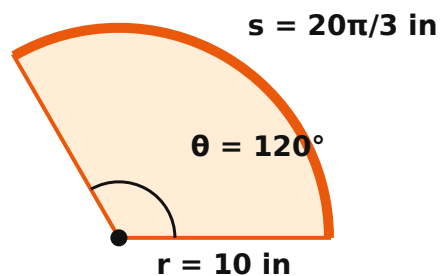
Example. For $\sin \frac{5\pi}{6}$, the reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$ and sine is positive in Quadrant II, so $\sin \frac{5\pi}{6} = \frac{1}{2}$.



The angles $\pi/6$, $5\pi/6$, $7\pi/6$ and $11\pi/6$ all use the same reference angle.

How do arc length and angular speed work?

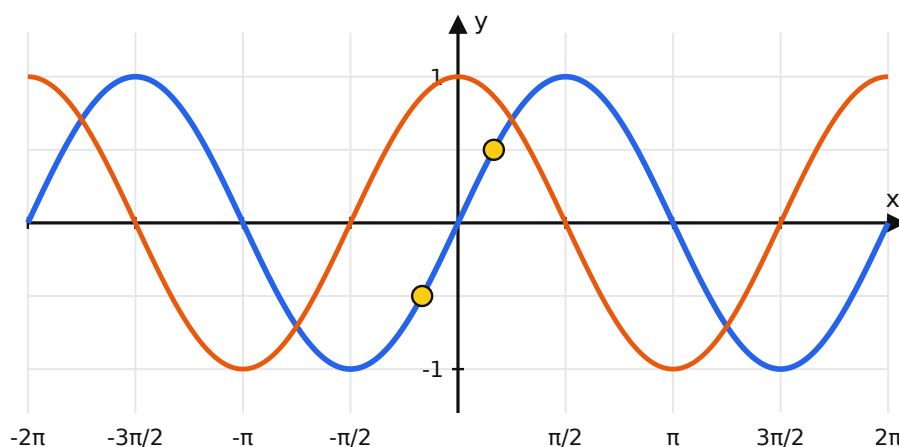
On a circle of radius r , a central angle of θ radians cuts off an arc of length $s = r\theta$. If the angle changes at $\omega = \frac{\theta}{t}$ radians per unit time, the linear speed is $v = r\omega$. With $r = 10$ in and $\theta = \frac{2\pi}{3}$, the arc length is $s = \frac{20\pi}{3} \approx 20.94$ in.



A 120 degree sector of radius 10 inches has arc length $20\pi/3$ inches.

Why are the trigonometric functions periodic, even or odd?

Adding a full turn returns to the same point, so $\sin(\theta + 2\pi) = \sin\theta$ and $\cos(\theta + 2\pi) = \cos\theta$; tangent repeats every π . Reflecting across the x-axis gives $\cos(-\theta) = \cos\theta$ (even) and $\sin(-\theta) = -\sin\theta$, $\tan(-\theta) = -\tan\theta$ (odd).



Sine and cosine repeat every 2π ; sine is odd and cosine is even.

Angles, Radians and the Unit Circle: frequently asked questions

Why do mathematicians use radians instead of degrees?

Radians connect angle directly to length: an angle of θ radians on a circle of radius r cuts off an arc of length r times θ . This makes formulas such as arc length and angular speed simple, with no conversion factor.

How do you remember the unit circle values?

Memorize the first quadrant: the angles $\pi/6$, $\pi/4$ and $\pi/3$ have sine values $1/2$, $\sqrt{2}/2$, $\sqrt{3}/2$ and cosine values in the reverse order. Reference angles and signs then give every other angle.

What is a reference angle?

It is the acute angle between the terminal side of an angle and the x-axis. The trigonometric values of an angle equal those of its reference angle, up to a sign that depends on the quadrant.

What does coterminal mean?

Two angles are coterminal if their terminal sides coincide, so they differ by a whole number of full turns. For instance, $7\pi/6$ and $-5\pi/6$ are coterminal, and they have the same sine, cosine and tangent.

How do you convert radians per minute into linear speed?

Multiply the angular speed in radians per unit time by the radius: $v = r$ times ω . A wheel of radius 13 inches turning at 6π radians per second moves a point on its rim at 78π inches

per second.

How to use this precalculus unit circle lesson

Use this precalculus unit circle lesson before the worksheet, then check what you learned with the quiz.

