

Precalculus · Trigonometric Identities · Worksheet Answer Key

Common Core F-TF.C.8, F-TF.C.9 · mathovore.fr

This precalculus trig identities worksheet answer key solves all 28 problems of the trigonometric identities worksheet step by step.

Full worked solutions for every problem, with exact values and decimal checks. Compare each step, especially the sign choices from the quadrant, and use the final bold answers to score your work.

Precalculus Common Core: F-TF.C.8 · F-TF.C.9

Precalculus Trig Identities Worksheet Answer Key: problems 1-5

Problem 1: Reciprocal and quotient forms

Apply the reciprocal and quotient identities.

- a. $\frac{1}{\cos \theta}$
- b. $\frac{1}{\sin \theta}$
- c. $\frac{\sin \theta}{\cos \theta}$
- d. $\frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{1}{\cos \theta}$

Problem 2: Reciprocal values

- a. $\csc \theta = \frac{1}{3/5} = \frac{5}{3}$
- b. $\sec \theta = -\frac{13}{5}$
- c. $\cot \theta = \frac{24}{7}$
- d. $\cot \theta = \frac{1}{-2} = -\frac{1}{2}$

Problem 3: Pythagorean forms

- a. $\sin^2 \theta = 1 - 0.6^2 = 1 - 0.36 = 0.64$
- b. $\cos^2 \theta = 1 - \frac{1}{4} = \frac{3}{4}$
- c. $\sec^2 \theta = 1 + 3^2 = 10$
- d. $\csc^2 \theta = 1 + 2^2 = 5$

Problem 4: Cosine from sine, Quadrant I

$\cos^2 \theta = 1 - \frac{16}{25} = \frac{9}{25}$. Cosine is positive in Quadrant I, so $\cos \theta = \frac{3}{5}$. Then $\tan \theta = \frac{4/5}{3/5} = \frac{4}{3}$.

Problem 5: Cosine from sine, Quadrant II

$\cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$. Cosine is negative in Quadrant II, so $\cos\theta = -\frac{12}{13}$. Then $\tan\theta = \frac{5/13}{-12/13} = -\frac{5}{12}$.

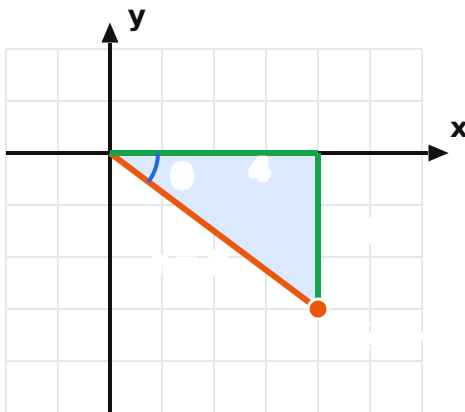
Precalculus Trig Identities Worksheet Answer Key: problems 6-10

Problem 6: Sine from cosine, Quadrant III

$\sin^2\theta = 1 - \frac{64}{289} = \frac{225}{289}$. Sine is negative in Quadrant III, so $\sin\theta = -\frac{15}{17}$. Then $\tan\theta = \frac{-15/17}{-8/17} = \frac{15}{8}$.

Problem 7: Sine and cosine from tangent

$\sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$. Cosine is positive in Quadrant IV, so $\sec\theta = \frac{5}{4}$ and $\cos\theta = \frac{4}{5}$. Then $\sin\theta = \tan\theta\cos\theta = -\frac{3}{4} \cdot \frac{4}{5} = -\frac{3}{5}$.



The reference triangle confirms the result.

Problem 8: All six functions

$\cos^2\theta = 1 - \frac{576}{625} = \frac{49}{625}$, so $\cos\theta = -\frac{7}{25}$. Then $\tan\theta = \frac{24}{7}$, $\csc\theta = -\frac{25}{24}$, $\sec\theta = -\frac{25}{7}$, $\cot\theta = \frac{7}{24}$.

Problem 9: Simplify products

- a. $\sin\theta \cdot \frac{\cos\theta}{\sin\theta} = \cos\theta$
- b. $\cos\theta \cdot \frac{\sin\theta}{\cos\theta} = \sin\theta$
- c. $\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$
- d. $1 - \cos^2\theta = \sin^2\theta$

Problem 10: Simplify with Pythagorean identities

- a. $1 + \tan^2\theta = \sec^2\theta$
- b. $1 - \sin^2\theta = \cos^2\theta$
- c. $\frac{\tan^2\theta}{\tan\theta} = \tan\theta$

d. $\cos^2\theta \sec^2\theta = 1$

Precalculus Trig Identities Worksheet Answer Key: problems 11-15

Problem 11: Identity or not?

- a. The curves meet only at isolated points such as $x=0$ and $x=\frac{\pi}{2}$; an identity needs the graphs to match everywhere.
- b. $\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \approx 1.414 \neq 1$, so **the statement is false**
- c. $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$

Problem 12: Simplify rational expressions

- a. $\frac{1-\cos^2\theta}{1-\cos\theta} = \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} = 1+\cos\theta$
- b. $\frac{1-\sin^2\theta}{1+\sin\theta} = 1+\sin\theta$
- c. $\frac{(1+\sin\theta)+(1-\sin\theta)}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = 2 \sec^2\theta$

Problem 13: Verify an identity: sum of fractions

Combine the left side over $\sin\theta(1+\cos\theta)$: $\frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$. The numerator is $\sin^2\theta+1+2\cos\theta+\cos^2\theta=2+2\cos\theta=2(1+\cos\theta)$. Cancel $1+\cos\theta$ to get $\frac{2}{\sin\theta}=2\csc\theta$. **Verified.**

Problem 14: Verify an identity: tangent plus cotangent

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \sec\theta\csc\theta. \text{ Verified.}$$

Problem 15: Verify an identity: secant minus cosine

$$\sec\theta - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}. \text{ Dividing by } \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ gives } \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \sin\theta. \text{ Verified.}$$

Precalculus Trig Identities Worksheet Answer Key: problems 16-20

Problem 16: Exact values with the sum formula

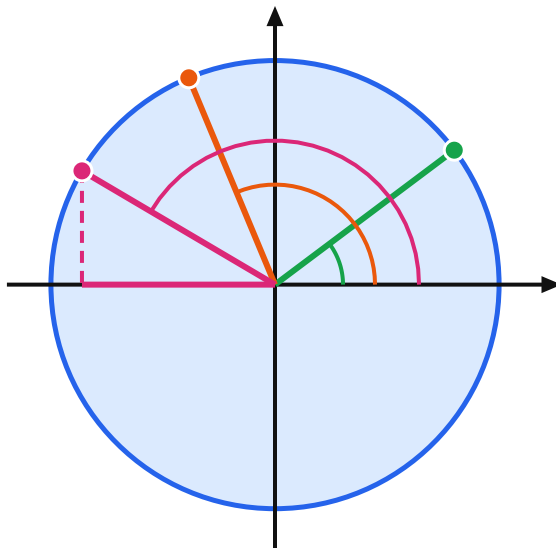
- a. $\sin(45^\circ+30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$
- b. $\cos(60^\circ+45^\circ) = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$
- c. $\tan(45^\circ-30^\circ) = \frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{(3-\sqrt{3})^2}{6} = \frac{12-6\sqrt{3}}{6} = 2-\sqrt{3}$

Problem 17: Exact values in radians

- a. $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}: \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$
- b. $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}: \frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \frac{(3+\sqrt{3})^2}{6} = \frac{12+6\sqrt{3}}{6} = 2+\sqrt{3}$
- c. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}: \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

Problem 18: Sum formulas from two triangles

$\cos A = \frac{4}{5}$ and $\sin B = \frac{12}{13}$. Then $\sin(A+B) = \frac{3}{5} \cdot \left(-\frac{5}{13}\right) + \frac{4}{5} \cdot \frac{12}{13} = \frac{-15+48}{65} = \frac{33}{65}$ and $\cos(A+B) = \frac{4}{5} \cdot \left(-\frac{5}{13}\right) - \frac{3}{5} \cdot \frac{12}{13} = \frac{-20-36}{65} = -\frac{56}{65}$. Sine is positive and cosine negative, so **A + B is in Quadrant II**.



Check: 33, 56 and 65 form a right triangle.

Problem 19: Recognize the formula

- a. $\sin(50^\circ - 20^\circ) = \sin 30^\circ = \frac{1}{2}$
- b. $\cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$
- c. $\tan(40^\circ + 5^\circ) = \tan 45^\circ = 1$
- d. $\sin\left(\frac{2\pi}{9} + \frac{\pi}{9}\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Problem 20: Prove shift identities

- a. $\sin \theta \cos \frac{\pi}{2} + \cos \theta \sin \frac{\pi}{2} = \sin \theta \cdot 0 + \cos \theta \cdot 1 = \cos \theta$
- b. $\cos \theta \cos \pi - \sin \theta \sin \pi = \cos \theta (-1) - \sin \theta (0) = -\cos \theta$
- c. $\frac{\tan \theta + \tan \pi}{1 - \tan \theta \tan \pi} = \frac{\tan \theta + 0}{1 - 0} = \tan \theta$

Precalculus Trig Identities Worksheet Answer Key: problems 21-25

Problem 21: Double-angle values

$\cos \theta = \frac{12}{13}$ in Quadrant IV. Then $\sin 2\theta = 2 \left(-\frac{5}{13}\right) \left(\frac{12}{13}\right) = -\frac{120}{169}$, $\cos 2\theta = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$ and $\tan 2\theta = \frac{-120/169}{119/169} = -\frac{120}{119}$.

Problem 22: Double-angle simplifications

- a. $\sin 30^\circ = \frac{1}{2}$

- b. $\cos 45^\circ = \frac{\sqrt{2}}{2}$
- c. $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
- d. $\tan 30^\circ = \frac{\sqrt{3}}{3}$

Problem 23: Work backward from $\cos 2\theta$

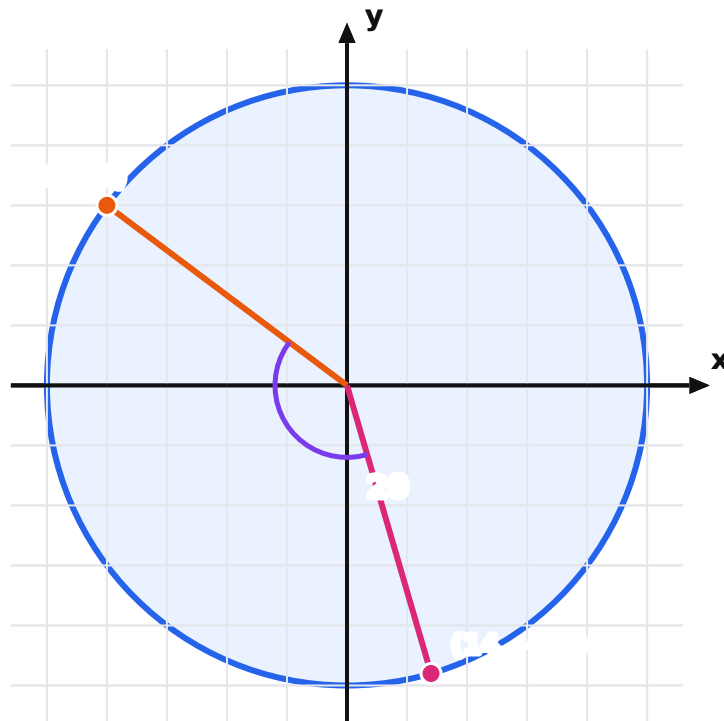
$2\cos 2\theta - 1 = \frac{7}{25}$ gives $\cos 2\theta = \frac{16}{25}$, so $\cos \theta = \frac{4}{5}$. $1 - 2\sin^2 \theta = \frac{7}{25}$ gives $\sin^2 \theta = \frac{9}{25}$, so $\sin \theta = \frac{3}{5}$. Then $\sin 2\theta = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$ and $\tan 2\theta = \frac{24/25}{7/25} = \frac{24}{7}$.

Problem 24: Soccer kick range

- a. $\sin 2\theta = 2(0.6)(0.8) = 0.96$, so $R = \frac{1600(0.96)}{32} = 48$ ft
- b. $\sin 90^\circ = 1$, so $R = \frac{1600}{32} = 50$ ft
- c. 2 ft farther.

Problem 25: Ferris wheel doubling

a) $\sin \theta = \frac{30}{50} = \frac{3}{5}$ and $\cos \theta = -\frac{4}{5}$ (left side). **b)** $\cos 2\theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$ and $\sin 2\theta = 2 \cdot \frac{3}{5} \cdot \left(-\frac{4}{5}\right) = -\frac{24}{25}$. Multiply by 50: (14, -48). **c)** $60 - 48 = 12$ ft above the ground.



One grid square is 10 feet.

Precalculus Trig Identities Worksheet Answer Key: problems 26-28

Problem 26: Rotating a point

- a. $\cos A = \frac{4}{5}$ and $\sin A = \frac{3}{5}$

- b. $x' = 5\cos(A+30^\circ) = 5\left(\frac{4}{5}\cdot\frac{\sqrt{3}}{2} - \frac{3}{5}\cdot\frac{1}{2}\right) = 2\sqrt{3} - \frac{3}{2} \approx 1.96$ and $y' = 5\left(\frac{3}{5}\cdot\frac{\sqrt{3}}{2} + \frac{4}{5}\cdot\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} + 2 \approx 4.60$
 c. $1.964^2 + 4.598^2 \approx 3.857 + 21.143 = 25.000$, so the distance is **5**

Problem 27: Camera tilt

- a. $\tan(A+B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{5/6}{5/6} = 1$
 b. Since $\tan 45^\circ = 1$ and $A+B$ is acute, $A+B = 45^\circ$
 c. $A \approx 26.57^\circ$ and $B \approx 18.43^\circ$; $26.57 + 18.43 = 45.00$. **The check works.**

Problem 28: Polarizing filter

- a. Add 1 to both sides: $1 + \cos 2\theta = 2\cos^2 \theta$, then divide by 2.
 b. $I = 80 \cdot \frac{1 + \frac{1}{2}}{2} = 80 \cdot \frac{3}{4} = 60$ lumens
 c. $I = 80 \cdot \frac{1 + \cos 30^\circ}{2} = 40\left(1 + \frac{\sqrt{3}}{2}\right) = 40 + 20\sqrt{3} \approx 74.64$ lumens
 d. $I = 40(1 + \cos 45^\circ) = 40\left(1 + \frac{\sqrt{2}}{2}\right) = 40 + 20\sqrt{2} \approx 68.28$ lumens

How to use this precalculus trig identities worksheet answer key

Read this precalculus trig identities worksheet answer key after trying each problem yourself, and redo any problem you missed.

