

Precalculus · Trigonometric Identities · Lesson

Common Core F-TF.C.8, F-TF.C.9 · mathovore.fr

This precalculus trig identities lesson explains trigonometric identities in one printable page, with definitions, a method and a worked example.

A trigonometric identity is an equation that holds for every angle where both sides are defined. The key identities are the reciprocal and quotient identities, the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, the sum and difference formulas for sine, cosine and tangent, and the double-angle formulas that follow from them.

Precalculus Common Core: F-TF.C.8 · F-TF.C.9

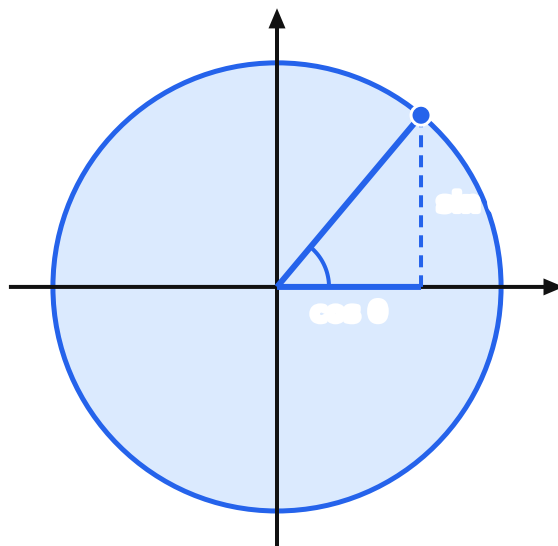
Precalculus Trig Identities Lesson: key facts

- Standards F-TF.C.8 and F-TF.C.9: prove and use the Pythagorean identity, and the addition formulas for sine, cosine and tangent.
- Pythagorean identities: $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \csc^2\theta$.
- Sum formulas: $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\cos(A+B) = \cos A \cos B - \sin A \sin B$.
- Double angle: $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$.
- To verify an identity, work on one side only, rewrite in sine and cosine, and simplify until it matches the other side.

What are the basic trigonometric identities?

An **identity** is true for every angle where both sides are defined. The point at angle θ on the unit circle is $(\cos\theta, \sin\theta)$ and lies 1 unit from the origin, so $\cos^2\theta + \sin^2\theta = 1$.

Dividing both sides of the Pythagorean identity by $\cos^2\theta$ or by $\sin^2\theta$ produces the other two Pythagorean identities. Memorize one and derive the rest when you need them. The sign of each function depends on the quadrant, so always check which quadrant the angle is in before choosing a square root.



The Pythagorean identity from the unit circle.

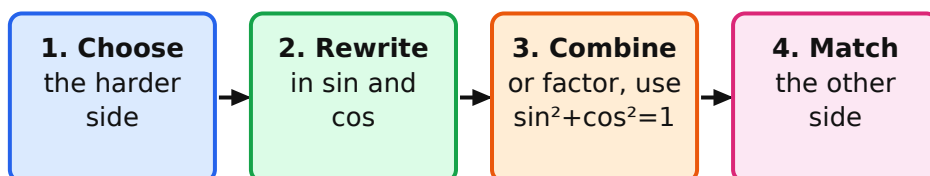
Core identities

Type	Identities
Reciprocal	$\csc \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{1}{\tan \theta}$
Quotient	$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$
Pythagorean	$\sin^2 \theta + \cos^2 \theta = 1, 1 + \tan^2 \theta = \sec^2 \theta,$ $1 + \cot^2 \theta = \csc^2 \theta$

How do you verify an identity?

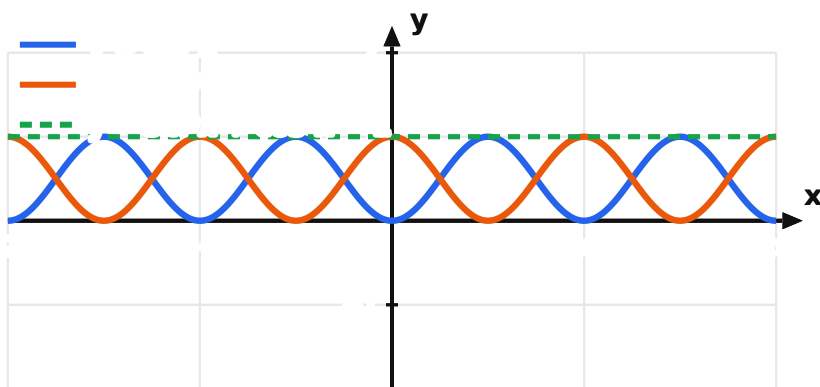
1. Pick the more complicated side and work only on that side.
2. Rewrite every function in terms of sine and cosine.
3. Combine fractions, factor, or substitute a Pythagorean identity.
4. Stop when the side matches the other one.

Never move terms across the equal sign, because that assumes the identity is true. If both sides look complicated, simplify each side separately until they reach the same expression.



The four steps of a verification.

Example. Verify $\tan\theta + \frac{\cos\theta}{1+\sin\theta} = \sec\theta$. Rewrite the left side as $\frac{\sin\theta(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)}$. The numerator is $\sin\theta+\sin^2\theta+\cos^2\theta=\sin\theta+1$, so the fraction reduces to $\frac{1}{\cos\theta}=\sec\theta$.



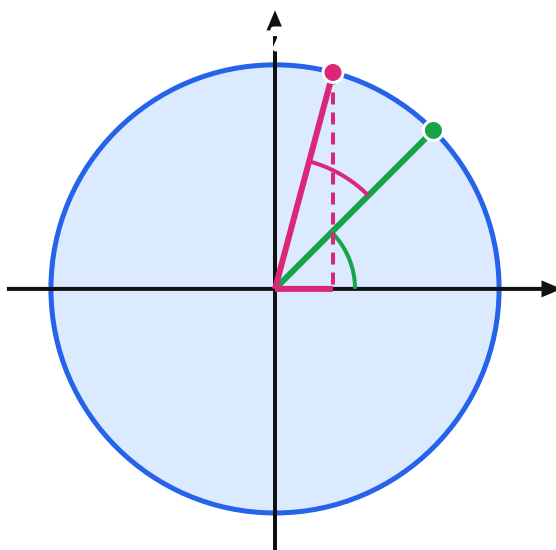
The graphs add up to the constant 1.

What are the sum and difference formulas?

Function	Sum	Difference
Sine	$\sin A \cos B + \cos A \sin B$	$\sin A \cos B - \cos A \sin B$
Cosine	$\cos A \cos B - \sin A \sin B$	$\cos A \cos B + \sin A \sin B$
Tangent	$\frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\frac{\tan A - \tan B}{1 + \tan A \tan B}$

Because $75^\circ = 45^\circ + 30^\circ$, $\sin 75^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \approx 0.966$. Note that $\sin(A+B) \neq \sin A + \sin B$.

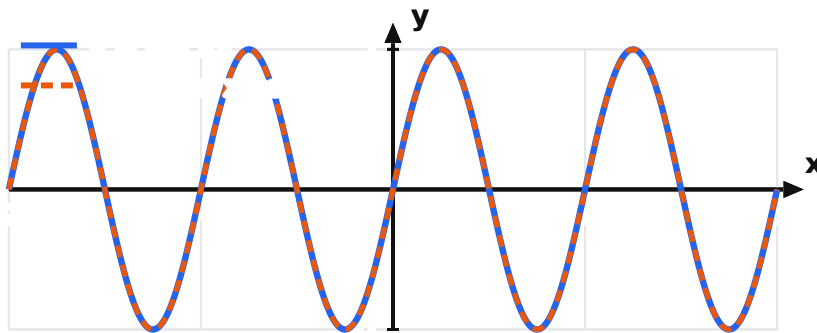
The cosine formulas swap the sign between the two products, and the tangent formulas divide by $1 - \tan A \tan B$ in the sum case. Special angles such as 15° , 75° and 105° get exact values by splitting them into 30° , 45° and 60° .



What are the double-angle formulas?

Setting $B = A$ in the sum formulas gives $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$, and $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$. If $\sin\theta = \frac{3}{5}$ in Quadrant I, then $\cos\theta = \frac{4}{5}$, $\sin 2\theta = \frac{24}{25}$ and $\cos 2\theta = \frac{7}{25}$.

The three cosine forms are interchangeable. Choose $2\cos^2\theta - 1$ when only the cosine is known and $1 - 2\sin^2\theta$ when only the sine is known. The graph shows that $2\sin x \cos x$ and $\sin 2x$ coincide.



Two ways to write the same wave.

Trigonometric Identities: frequently asked questions

What is the difference between an identity and an equation?

An identity is true for every angle where both sides are defined, such as $\sin^2\theta + \cos^2\theta = 1$. An ordinary equation like $\sin\theta = \frac{1}{2}$ is true only for certain angles.

Why can I not move terms across the equal sign when verifying an identity?

Moving terms assumes the identity is already true, which is the very thing to be shown. Work on one side until it becomes the other, or simplify both sides separately to the same expression.

Is $\sin(A + B)$ equal to $\sin A + \sin B$?

No. For $A = B = 60^\circ$, $\sin 120^\circ \approx 0.866$ but $\sin 60^\circ + \sin 60^\circ \approx 1.732$. The correct formula is $\sin A \cos B + \cos A \sin B$.

Which form of the cosine double-angle formula should I use?

Use $2\cos^2\theta - 1$ when you know the cosine, $1 - 2\sin^2\theta$ when you know the sine, and $\cos 2\theta - \sin^2\theta$ when you know both.

How do I choose the sign when I take a square root in an identity problem?

Use the quadrant of the angle. Sine is positive in Quadrants I and II, cosine is positive in Quadrants I and IV, and tangent is positive in Quadrants I and III.

How to use this precalculus trig identities lesson

Use this precalculus trig identities lesson before the worksheet, then check what you learned with the quiz.

