

Precalculus · Series, Binomial Theorem and an Introduction to Limits · Worksheet Answer Key

Common Core A-APR.C.5, A-SSE.B.4, F-BF.A.2 · mathovore.fr

This precalculus sequences series worksheet answer key solves all 25 problems of the series, binomial theorem and an introduction to limits worksheet step by step.

Every solution below shows the formula used, the substitution and the final value in bold. Use it to find the exact step where a mistake crept in, especially for ratios, term counts and signs in binomial expansions.

Precalculus Common Core: A-APR.C.5 · A-SSE.B.4 · F-BF.A.2

Precalculus Sequences Series Worksheet Answer Key: problems 1-5

Problem 1: Evaluate a sum

Substitute $k=1,2,3,4,5$ into $3k-1$: 2, 5, 8, 11, 14.

$$\sum_{k=1}^5 3k - 1$$

$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
2	5	8	11	14

$$2 + 5 + 8 + 11 + 14 = 40$$

The index k runs from 1 to 5; each value of k gives one term.

$$2+5+8+11+14=40$$

Problem 2: Sums of powers

- a. $1+4+9+16=30$
- b. $1+2+4+8=15$

Problem 3: Write in sigma notation

- a. Each term is $3k$ for $k=1$ to 5: $\sum_{k=1}^5 3k$

b. Each term is 2^k for $k=1$ to 5: $\sum_{k=1}^5 2^k$

Problem 4: Explicit and recursive rules

a. Common difference 4. Explicit: $a_n = 5 + 4(n-1) = 4n + 1$. Recursive: $a_1 = 5$, $a_n = a_{n-1} + 4$.
 $a_{20} = 4(20) + 1 = \mathbf{81}$

b. Common ratio 3. Explicit: $a_n = 2 \cdot 3^{n-1}$. Recursive: $a_1 = 2$, $a_n = 3a_{n-1}$. $a_6 = 2 \cdot 3^5 = 2 \cdot 243 = \mathbf{486}$

Problem 5: Find the nth term

a. $a_{20} = 7 + 4(19) = 7 + 76 = \mathbf{83}$

b. $7 + 4(n-1) = 103 \Rightarrow 4(n-1) = 96 \Rightarrow n-1 = 24$, so **term 25**

Precalculus Sequences Series Worksheet Answer Key: problems 6-10

Problem 6: Arithmetic series

$d = 7$, so $a_{15} = 12 + 7(14) = 110$.

$$S_{15} = \frac{15(12+110)}{2} = 15 \cdot 61 = \mathbf{915}$$

Problem 7: Decreasing series

$d = -3$. Solve $100 - 3(n-1) = 1$: $n-1 = 33$, so $n = 34$ terms.

$$S_{34} = \frac{34(100+1)}{2} = 17 \cdot 101 = \mathbf{1,717}$$

Problem 8: Finite geometric series

a. $r = 2$: $S_8 = \frac{3(1-2^8)}{1-2} = \frac{3(-255)}{-1} = \mathbf{765}$

b. $r = \frac{1}{3}$, $n = 5$: $S_5 = \frac{81(1-\frac{1}{243})}{1-\frac{1}{3}} = 81 \cdot \frac{242}{243} \cdot \frac{3}{2} = \mathbf{121}$

Problem 9: Does it converge?

a. $r = \frac{1}{2}$, $|r| < 1$: converges to $\frac{8}{1-1/2} = \mathbf{16}$

b. $r = 2$: **diverges**

c. $r = -\frac{1}{3}$: converges to $\frac{9}{1+1/3} = \frac{9}{4/3} = \mathbf{27/4}$

d. $r = 1.1$: **diverges**

Problem 10: Infinite geometric sum

a. $r = \frac{1}{3}$: $\frac{12}{1-1/3} = 12 \cdot \frac{3}{2} = \mathbf{18}$

b. $\frac{5}{1-0.2} = \frac{5}{0.8} = \mathbf{6.25}$

Precalculus Sequences Series Worksheet Answer Key: problems 11-15

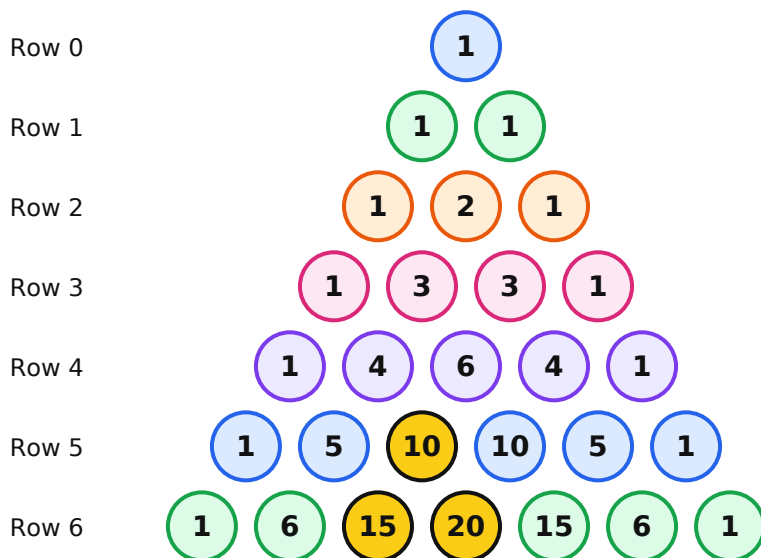
Problem 11: Repeating decimals

a. $0.27 + 0.0027 + \dots$ has $a_1 = 0.27$, $r = 0.01$: $\frac{0.27}{0.99} = \frac{27}{99} = \mathbf{3/11}$

b. $a_1 = 0.6$, $r = 0.1$: $\frac{0.6}{0.9} = \mathbf{2/3}$

Problem 12: Pascal's triangle

Each entry is the sum of the two above it: row 5 is 1, 5, 10, 10, 5, 1 and row 6 is 1, 6, 15, 20, 15, 6, 1.



Each entry is the sum of the two entries above it.

a. The missing entries are **10, 15** and **20**.

b. Row 7: **1, 7, 21, 35, 35, 21, 7, 1**

Problem 13: Expand a binomial

a. Row 5 is 1, 5, 10, 10, 5, 1: $x^5 + 5 \cdot 2x^4 + 10 \cdot 4x^3 + 10 \cdot 8x^2 + 5 \cdot 16x + 32 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

b. Row 3 is 1, 3, 3, 1 with $b = -3$: $x^3 + 3(-3)x^2 + 3(9)x + (-27) = x^3 - 9x^2 + 27x - 27$

Problem 14: Binomial with a coefficient

Row 4 is 1, 4, 6, 4, 1 with $a = 2x$ and $b = -1$.

$$(2x)^4 + 4(2x)^3(-1) + 6(2x)^2(1) + 4(2x)(-1) + 1 = 16x^4 - 32x^3 + 24x^2 - 8x + 1$$

Problem 15: Find a specific term

- a. The x^3 term is $C(7,3) x^3 3^4 = 35 \cdot 81 x^3$. The coefficient is **2,835**.
 b. $C(6,2)(2x)^2(1)^4 = 15 \cdot 4x^2 = 60x^2$

Precalculus Sequences Series Worksheet Answer Key: problems 16-20

Problem 16: Limit of a sequence

- a. $a_{10} = \frac{31}{12} \approx 2.5833$; $a_{100} = \frac{301}{102} \approx 2.9510$; $a_{1000} = \frac{3001}{1002} \approx 2.9950$
 b. Divide top and bottom by n : $\frac{3+1/n}{1+2/n} \rightarrow \frac{3}{1}$. The limit is **3**.

Problem 17: Limit from a graph

- a. No. Substituting $x=3$ gives $\frac{0}{0}$, which is why the graph has an open circle.
 b. Values near 3 approach the height of the hole: **6**.

Problem 18: Limit by factoring

- a. $\frac{(x+4)(x-3)}{x-3} = x+4$ for $x \neq 3$, so the limit is **7**.
 b. $\frac{(x+5)(x-5)}{x-5} = x+5$ for $x \neq 5$, so the limit is **10**.

Problem 19: When does it converge?

- a. The ratio is $\frac{x}{3}$. Convergence needs $|\frac{x}{3}| < 1$, so $-3 < x < 3$.
 b. $r = \frac{1}{3}$: $\frac{1}{1-1/3} = \mathbf{3/2}$
 c. $r = -\frac{1}{2}$: $\frac{1}{1+1/2} = \mathbf{2/3}$

Problem 20: Theater seats

The seats rise by 3 each row, so $a_n = 4 + 3(n-1)$.

Rows 1 to 6: $S_6 = \frac{6(4+19)}{2} = \mathbf{69 \text{ seats}}$.

Row 20 has $a_{20} = 4 + 57 = 61$ seats, so $S_{20} = \frac{20(4+61)}{2} = \mathbf{650 \text{ seats}}$.

Precalculus Sequences Series Worksheet Answer Key: problems 21-25

Problem 21: Doubling savings

Geometric with $a_1 = 0.01$, $r = 2$, $n = 12$.

$$S_{12} = \frac{0.01(1-2^{12})}{1-2} = 0.01(4095) = \mathbf{\$40.95}$$

Problem 22: Salary raises

Recursive: $a_1 = 40000$, $a_n = a_{n-1} + 2500$.

Year 10: $a_{10} = 40000 + 9(2500) = \mathbf{\$62,500}$.

Total: $S_{10} = \frac{10(40000 + 62500)}{2} = \mathbf{\$512,500}$.

Problem 23: Bouncing ball

Down 10 ft first. After that each bounce goes up and down: heights 6, 3.6, 2.16, ... form a geometric series with $a_1 = 6$, $r = 0.6$.

Up-and-down total: $2 \cdot \frac{6}{1-0.6} = 2 \cdot 15 = 30$.

Distance: $10 + 30 = \mathbf{40 \text{ feet}}$.

Problem 24: Medicine in the body

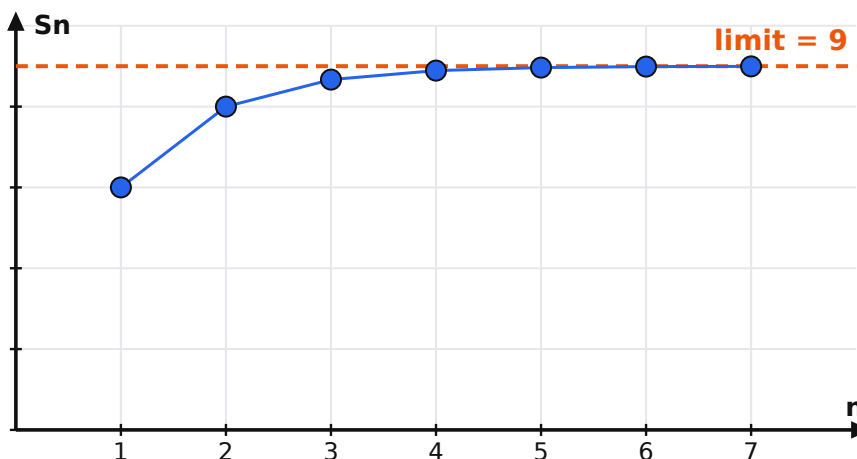
Day 1: 100 mg. Day 2: $100 + 0.2(100) = 120$ mg. Day 3: $100 + 0.2(120) = 124$ mg, so **124 mg**.

Day n is $100(1 + 0.2 + 0.2^2 + \dots)$. The long-run total is $\frac{100}{1-0.2} = \mathbf{125 \text{ mg}}$.

Problem 25: Cleaning a pond

The points level off near 9 pounds.

Exactly: $a_1 = 6$, $r = \frac{1}{3}$, so the total is $\frac{6}{1-\frac{1}{3}} = 6 \cdot \frac{3}{2} = \mathbf{9 \text{ pounds}}$.



The partial sums level off at the limit of the series.

How to use this precalculus sequences series worksheet answer key

Read this precalculus sequences series worksheet answer key after trying each problem yourself, and redo any problem you missed.

