

Precalculus · Series, Binomial Theorem and an Introduction to Limits · Lesson

Common Core A-APR.C.5, A-SSE.B.4, F-BF.A.2 · mathovore.fr

This precalculus sequences series limits lesson explains series, binomial theorem and an introduction to limits in one printable page, with definitions, a method and a worked example.

A series is the sum of the terms of a sequence, written compactly with sigma notation. Arithmetic series add a constant difference, geometric series multiply by a constant ratio, and an infinite geometric series converges to a first term divided by one minus r exactly when the absolute value of r is less than 1.

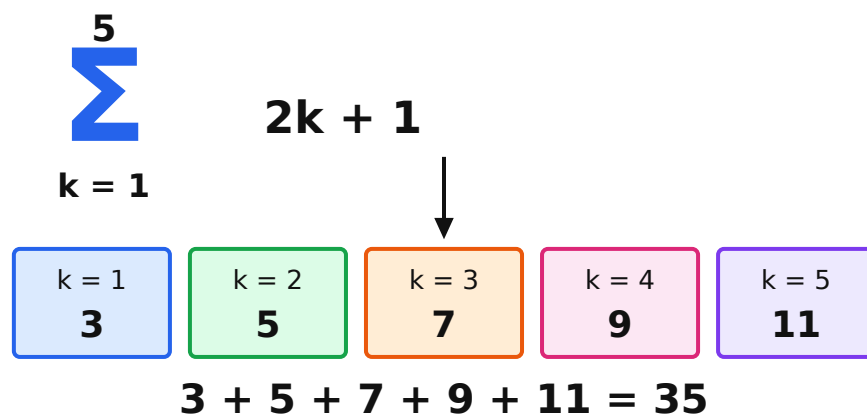
Precalculus Common Core: A-APR.C.5 · A-SSE.B.4 · F-BF.A.2

Precalculus Sequences Series Limits Lesson: key facts

- Standards A-SSE.B.4, A-APR.C.5 and F-BF.A.2: sum finite and infinite geometric series, use the binomial theorem, and write sequences recursively and explicitly.
- Arithmetic series: $S_n = \frac{n(a_1 + a_n)}{2}$. Finite geometric series: $S_n = \frac{a_1(1 - r^n)}{1 - r}$.
- An infinite geometric series with $|r| < 1$ has sum $\frac{a_1}{1 - r}$; otherwise it diverges.
- Binomial theorem: $(a + b)^n = \sum_{k=0}^n C(n, k) a^{n-k} b^k$, with coefficients from Pascal's triangle.
- A limit describes the value approached: $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$ even though the function is undefined at 2.

What is a series and how does sigma notation write it?

A **sequence** is a list of numbers; a **series** is the sum of those numbers. Sigma notation packs a long sum into one line: $\sum_{k=1}^5 (2k+1)$ tells you to substitute $k = 1, 2, 3, 4, 5$ into $2k+1$ and add, giving $3+5+7+9+11=35$. The number under the sigma is where the index starts and the number above it is where it stops.



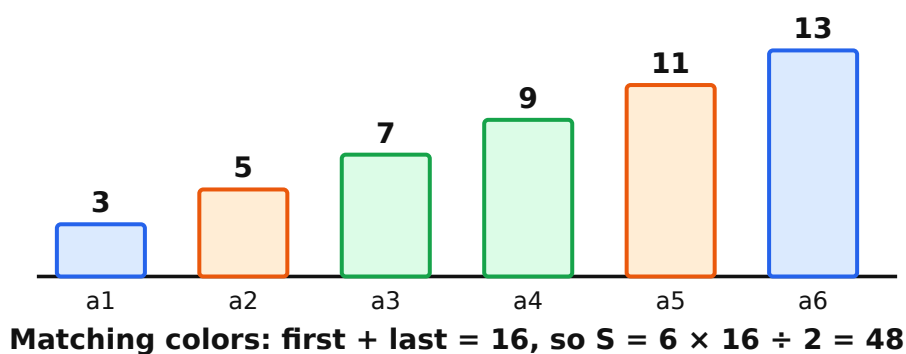
The index k runs from 1 to 5; each value of k gives one term.

How do you add an arithmetic series?

Arithmetic sequence and series

Explicit rule $a_n = a_1 + (n-1)d$; recursive rule $a_n = a_{n-1} + d$ with a_1 given. The sum of the first n terms is $S_n = \frac{n(a_1 + a_n)}{2}$.

The formula works because the first and last terms, the second and second-to-last, and so on, all have the same sum. For 3, 5, 7, 9, 11, 13 every pair adds to 16, and there are 3 pairs, so the total is 48.



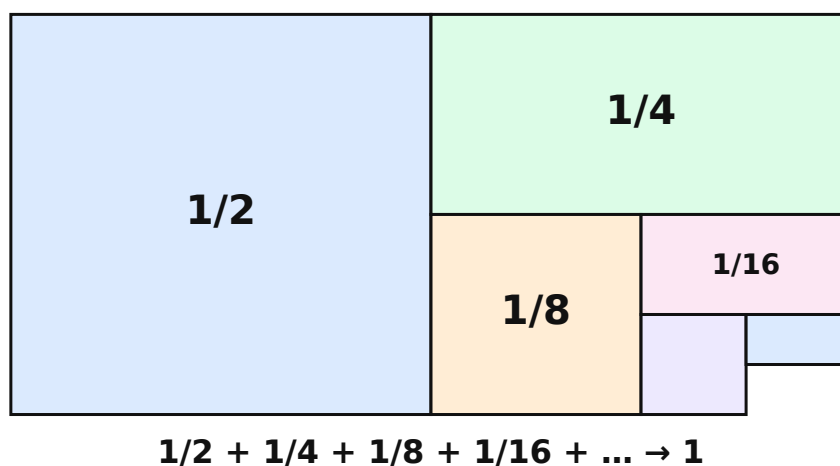
Pairing terms from both ends gives equal sums.

When does an infinite geometric series have a sum?

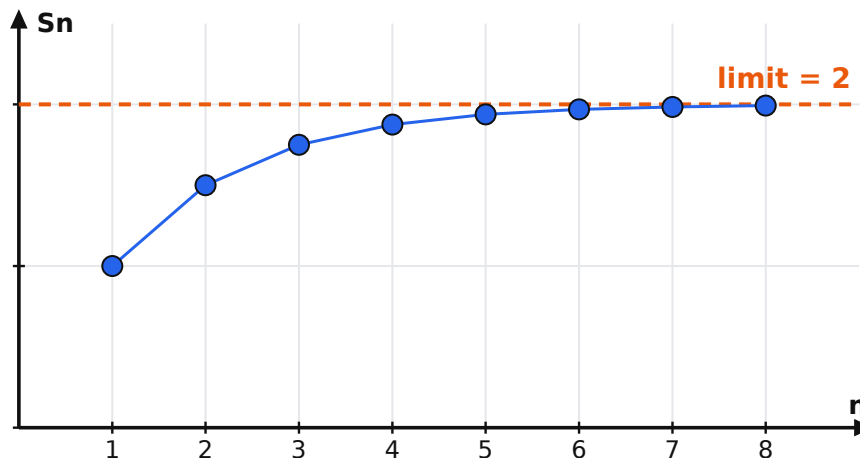
Geometric series

For a geometric sequence with ratio r , $S_n = \frac{a_1(1-r^n)}{1-r}$ when $r \neq 1$. If $|r| < 1$, the partial sums settle down and $\sum_{k=0}^{\infty} a_1 r^k = \frac{a_1}{1-r}$. If $|r| \geq 1$, the series diverges.

Cutting a square in half again and again shows why $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$ approaches 1 (here $a_1 = \frac{1}{2}$ and $r = \frac{1}{2}$, so the sum is $\frac{1/2}{1-1/2} = 1$). The graph of partial sums for $1 + 1/2 + 1/4 + \dots$ climbs toward 2 and never passes it.



Each piece is half of what remains, so the total area approaches 1.

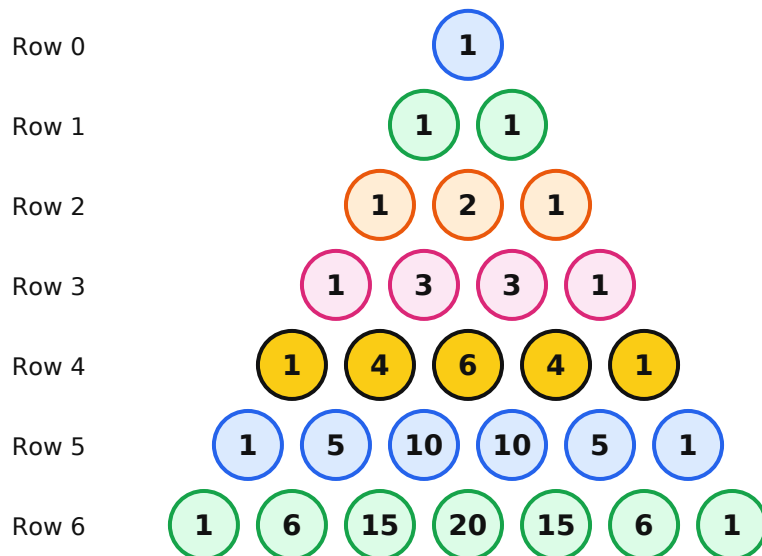


The partial sums level off at the limit of the series.

How does the binomial theorem expand a power?

The coefficients of $(a+b)^n$ are row n of **Pascal's triangle**. The general rule is

$$(a+b)^n = \sum_{k=0}^n C(n,k) a^{n-k} b^k, \quad C(n,k) = \frac{n!}{k! (n-k)!}$$



Each entry is the sum of the two entries above it.

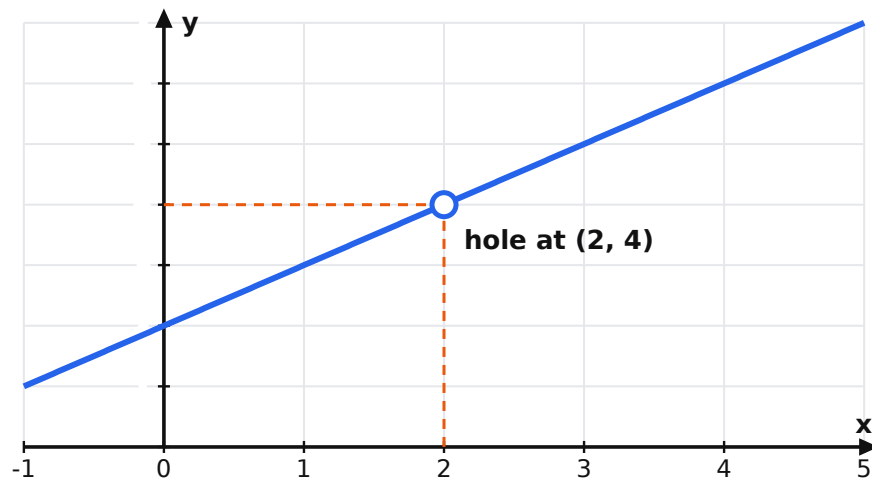
Worked example, expand $(x+2)^4$.

1. Read row 4: 1, 4, 6, 4, 1.
2. Powers of x fall from 4 to 0 while powers of 2 rise from 0 to 4.
3. Multiply: $1 \cdot x^4 + 4 \cdot 2x^3 + 6 \cdot 4x^2 + 4 \cdot 8x + 1 \cdot 16$.
4. Result: $x^4 + 8x^3 + 24x^2 + 32x + 16$.

What is a limit?

A **limit** describes the value a sequence or function approaches, even when it never reaches it. For a sequence, $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$. For a function, $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$, because the quotient equals $x+2$ for every $x \neq 2$, so the graph has a hole at $(2,4)$. The limit ignores what happens exactly at $x=2$.

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	3.9	3.99	3.999	4.001	4.01	4.1



The open circle marks a point the graph skips.

Series, Binomial Theorem and an Introduction to Limits: frequently asked questions

What is the difference between a sequence and a series?

A sequence is an ordered list of numbers such as 3, 5, 7, 9. A series is the sum of the terms, such as $3 + 5 + 7 + 9 = 24$. A partial sum adds only the first n terms.

When does an infinite geometric series have a sum?

It has a sum only when the common ratio satisfies $|r| < 1$. Then the terms shrink fast enough for the total to settle at $\frac{a_1}{1-r}$. If $|r| \geq 1$, the partial sums grow without bound or keep swinging.

How do I find one specific term in a binomial expansion?

Use the general term $C(n, k) a^{n-k} b^k$. For the coefficient of x^3 in $(x+3)^7$, set $k=4$:
 $C(7, 3) \cdot 3^4 = 35 \cdot 81 = 2835$.

Why can a limit exist when the function is undefined at that point?

A limit looks only at values of the function near the point, not at the point itself. The function $(x^2 - 4)/(x - 2)$ is undefined at $x = 2$, yet its values get as close to 4 as you like as x approaches 2.

How do I turn a repeating decimal into a fraction?

Write it as an infinite geometric series. For $0.\overline{27}$, use $a_1 = 0.27$ and $r = 0.01$: $\frac{0.27}{0.99} = \frac{3}{11}$.

How to use this precalculus sequences series limits lesson

Use this precalculus sequences series limits lesson before the worksheet, then check what you learned with the quiz.

