

Precalculus · Polynomial and Rational Functions in Depth · Worksheet Answer Key

Common Core A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9 · mathovore.fr

This precalculus polynomial rational worksheet answer key solves all 24 problems of the polynomial and rational functions in depth worksheet step by step.

Complete solutions for all 24 problems appear below. For each polynomial, confirm that the number of zeros matches the degree. For each inequality, verify the answer by testing a point from every interval, including the excluded endpoints.

Precalculus Common Core: A-APR.B.2 · A-APR.B.3 · F-IF.C.7c · F-IF.C.7d · N-CN.C.9

Precalculus Polynomial Rational Worksheet Answer Key: problems 1-5

Problem 1: List possible rational zeros

The constant term is 8, with divisors 1, 2, 4, 8. The leading coefficient is 3, with divisors 1, 3.

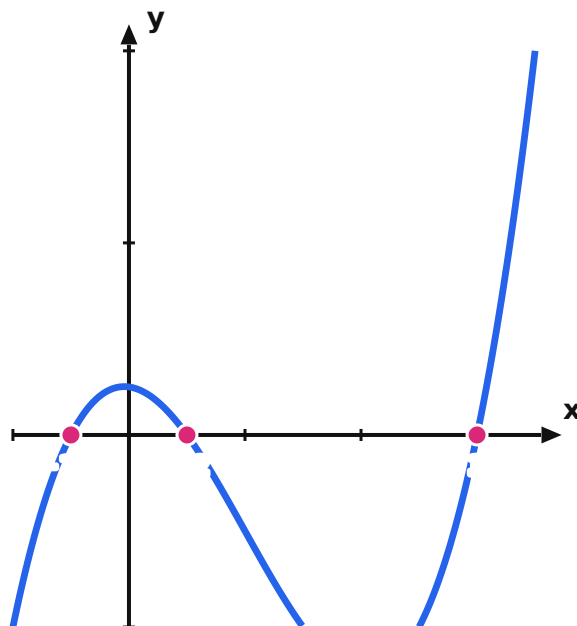
Form $\pm \frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$. That is 16 candidates.

Problem 2: Rational zeros of a cubic

Candidates: $\pm 1, \pm 2, \pm 3, \pm 6$. Test $x = -1$: $-1 - 4 - 1 + 6 = 0$, so $x + 1$ is a factor. Division gives $x^2 - 5x + 6 = (x - 2)(x - 3)$.

Answer: $x = -1, 2, 3$.

Problem 3: Factor with the Rational Root Theorem



The zeros are $-1/2$, $1/2$ and 3 .

- The constant term 3 has divisors $1, 3$; the leading coefficient 4 has divisors $1, 2, 4$.
Candidates: $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$
- Group: $4x^2(x-3)-(x-3)=(x-3)(4x^2-1)=(x-3)(2x-1)(2x+1)$. Zeros: $3, \frac{1}{2}, -\frac{1}{2}$. Factored form: $P(x)=(x-3)(2x-1)(2x+1)$

Problem 4: Counting zeros

- Add the exponents: $2+3+2=7$. Degree: 7
- $x=1$ (multiplicity 2), $x=-4$ (multiplicity 3), $x=i$ and $x=-i$ (multiplicity 1 each). That is $2+3+1+1=7$ zeros, as the Fundamental Theorem of Algebra predicts.
- The real zeros are 1 and -4 . Answer: **2**

Problem 5: Complex zeros of a cubic

Factor by grouping: $x^2(x-2)+4(x-2)=(x-2)(x^2+4)$. Then $x-2=0$ gives $x=2$, and $x^2+4=0$ gives $x=\pm 2i$.

Answer: $2, 2i, -2i$.

Precalculus Polynomial Rational Worksheet Answer Key: problems 6-10

Problem 6: Build a polynomial from zeros

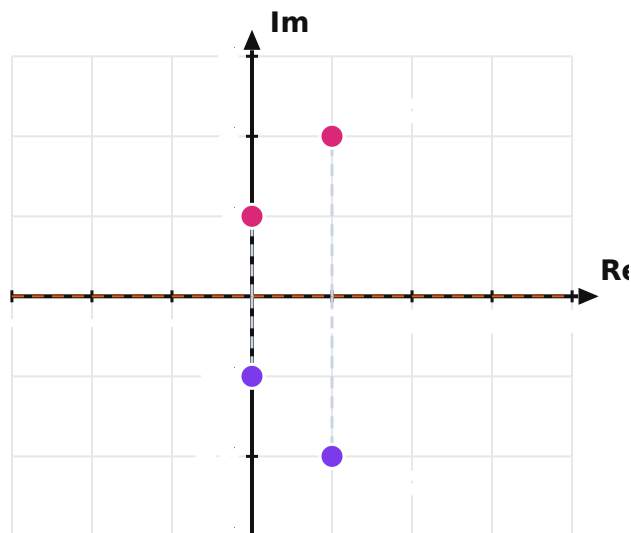
Complex zeros of a real polynomial come in conjugate pairs, so $2+i$ is also a zero. The pair gives $(x-(2-i))(x-(2+i))=x^2-4x+5$.

Multiply: $(x-3)(x^2-4x+5) = x^3-4x^2+5x-3x^2+12x-15$. Answer: $x^3-7x^2+17x-15$.

Problem 7: Use a given complex zero

The conjugate $1-2i$ is also a zero. Its quadratic factor is $(x-1)^2+4 = x^2-2x+5$.

Divide: $(x^4-2x^3+6x^2-2x+5) \div (x^2-2x+5) = x^2+1$, so the remaining zeros are $\pm i$.



The four zeros form two conjugate pairs.

Answer: $1-2i$, i , $-i$.

Problem 8: End behavior of polynomials

- Odd degree, negative leading coefficient: $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$
- Even degree, positive leading coefficient: $f(x) \rightarrow \infty$ at both ends
- Odd degree, positive leading coefficient: $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow \infty$ as $x \rightarrow \infty$

Problem 9: Zeros and multiplicity from a graph

- The graph touches the axis and turns at $x = -3$, so its multiplicity is **even** (at least 2). It crosses at $x = 1$ and $x = 2$, so those multiplicities are **odd** (at least 1).
- Least degree: $2+1+1=4$. Answer: **4**
- Both ends rise on an even-degree graph, so the leading coefficient is **positive**. (The graph is $h(x) = \frac{(x+3)^2(x-1)(x-2)}{6}$, and $h(0)=3$.)

Problem 10: Asymptotes of a rational function

The denominator $x^2-4 = (x-2)(x+2)$ is zero at $x = \pm 2$ and the numerator is not zero there, so the vertical asymptotes are $x = 2$ and $x = -2$.

The degrees are equal, so the horizontal asymptote is the ratio of leading coefficients: $y = 3$.

Precalculus Polynomial Rational Worksheet Answer Key: problems 11-15

Problem 11: Slant asymptote by division

Divide: $x^2 + 3x - 1 = (x - 1)(x + 4) + 3$, so $f(x) = x + 4 + \frac{3}{x - 1}$.

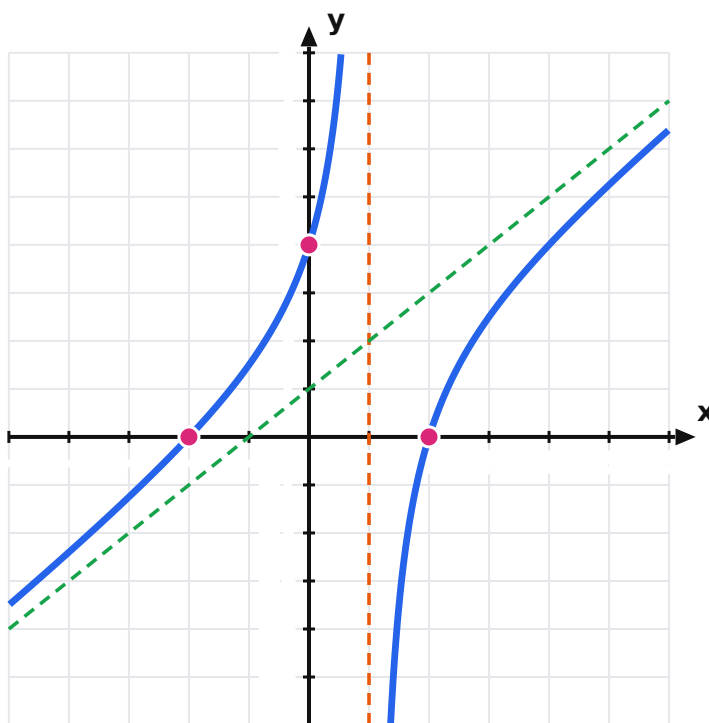
As $|x|$ grows, $\frac{3}{x - 1} \rightarrow 0$. The slant asymptote is $y = x + 4$.

Problem 12: Slant asymptote with a negative remainder term

Divide: $(x + 2)(2x - 9) = 2x^2 - 5x - 18$, so $2x^2 - 5x + 3 = (x + 2)(2x - 9) + 21$ and $f(x) = 2x - 9 + \frac{21}{x + 2}$.

Slant asymptote: $y = 2x - 9$. Vertical asymptote: $x = -2$.

Problem 13: Analyze a rational graph



Asymptotes $x = 1$ and $y = x + 1$, with intercepts $(-2, 0)$, $(2, 0)$ and $(0, 4)$.

- The denominator is zero at $x = 1$. Domain: $(-\infty, 1) \cup (1, \infty)$
- Vertical asymptote: $x = 1$. Divide: $x^2 - 4 = (x - 1)(x + 1) - 3$, so $f(x) = x + 1 - \frac{3}{x - 1}$ and the slant asymptote is $y = x + 1$
- Set the numerator to zero: $x^2 = 4$, so x-intercepts $(-2, 0)$ **and** $(2, 0)$. Also $f(0) = \frac{-4}{-1} = 4$, so y-intercept $(0, 4)$

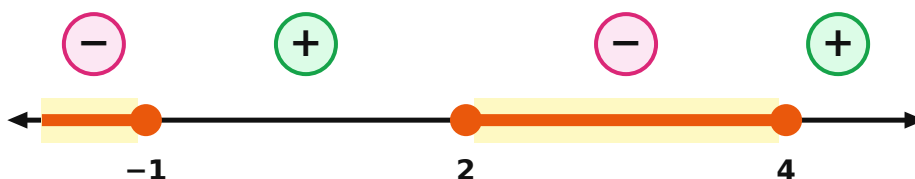
Problem 14: Quadratic inequality

Factor: $(x-3)(x+2) > 0$. Critical numbers: -2 and 3 . Test $x=0$: -6 is negative, so the middle interval fails; the outer intervals work.

Answer: $(-\infty, -2) \cup (3, \infty)$.

Problem 15: Cubic inequality

Critical numbers: $-1, 2, 4$. At $x=0$ the product is $(1)(-2)(-4)=8$, positive, so the signs run $-, +, -, +$ from left to right.



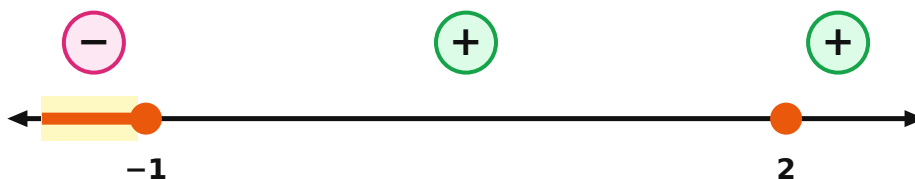
Shaded intervals are where the product is negative or zero.

The product is ≤ 0 where it is negative or zero. Answer: $(-\infty, -1] \cup [2, 4]$.

Precalculus Polynomial Rational Worksheet Answer Key: problems 16-20

Problem 16: Inequality with a repeated factor

The factor $(x-2)^2$ is never negative and equals 0 at $x=2$. So the sign depends on $x+1$, except that $x=2$ gives exactly 0 .

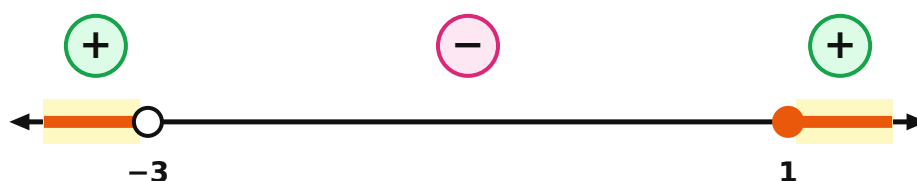


The sign does not change at $x = 2$, but the value is 0 there, so 2 is an isolated solution.

Answer: $x \leq -1$ or $x = 2$.

Problem 17: Rational inequality with zero on one side

Critical numbers: 1 (numerator zero, included) and -3 (denominator zero, excluded).

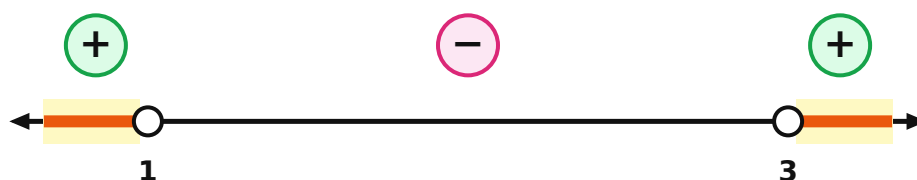


Open circle at -3 because the fraction is undefined there.

Answer: $(-\infty, -3) \cup [1, \infty)$.

Problem 18: Rational inequality with a constant on the right

Do not multiply by $x-1$ because its sign changes. Subtract: $\frac{2x}{x-1} - 3 = \frac{2x-3x+3}{x-1} = \frac{3-x}{x-1} < 0$, which is the same as $\frac{x-3}{x-1} > 0$.



Both critical numbers are open circles because the inequality is strict.

Check $x=0$: $0 < 3$ is true, and $x=2$: $4 < 3$ is false. Answer: $(-\infty, 1) \cup (3, \infty)$.

Problem 19: Gift box volume

- $x(x+3)(x+4)=60$ expands to $x^3+7x^2+12x=60$. Answer: $x^3+7x^2+12x-60=0$
- Test $x=2$: $8+28+24-60=0$. So $x=2$ is a real solution.
- Divide by $x-2$ to get $x^2+9x+30$. Its discriminant is $81-120=-39$, so the other zeros are $\frac{-9 \pm i\sqrt{39}}{2}$, which are not real lengths. The dimensions are **2 ft by 5 ft by 6 ft**.

Problem 20: Average cost

- Divide: $A(x)=20+\frac{1500}{x}$, so the horizontal asymptote is $y=20$: with huge production the average cost per box approaches \$20.
- $A(100)=\frac{2000+1500}{100}=35$. Answer: **\$35**
- $\frac{20x+1500}{x} \leq 25$ with $x>0$ gives $20x+1500 \leq 25x$, so $1500 \leq 5x$ and $x \geq 300$. Answer: **at least 300 boxes**

Precalculus Polynomial Rational Worksheet Answer Key: problems 21-24

Problem 21: Profit model

- a. $P(x) = -x(x^2 - 14x + 40) = -x(x-4)(x-10)$. Answer: $-x(x-4)(x-10)$
b. Critical numbers 0, 4, 10. On $0 < x < 4$ the factors give $-(+)(-)(-)$ which is negative; on $4 < x < 10$ it is positive; on $x > 10$ it is negative. Answer: $4 < x < 10$
c. $P(6) = -6(2)(-4) = 48$. Answer: **\$48 thousand**

Problem 22: Polynomial with an imaginary zero

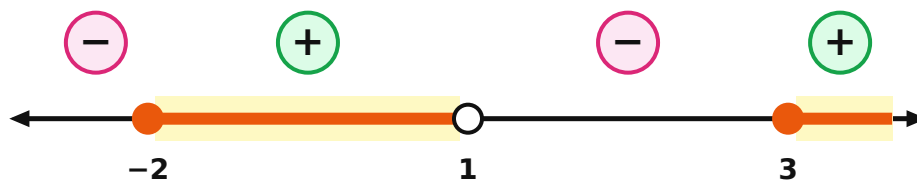
The conjugate $-3i$ is also a zero, giving $(x-3i)(x+3i) = x^2 + 9$. Also $(x-2)(x+1) = x^2 - x - 2$.

Multiply: $(x^2 - x - 2)(x^2 + 9) = x^4 - x^3 + 7x^2 - 9x - 18$. Answer: $x^4 - x^3 + 7x^2 - 9x - 18$.

Even degree and positive leading coefficient, so the graph rises at both ends.

Problem 23: Rational inequality with a quadratic

Factor: $\frac{(x-3)(x+2)}{x-1} \geq 0$. Critical numbers: -2 (included), 1 (excluded) and 3 (included).



The closed circles at -2 and 3 are zeros; 1 is excluded.

Answer: $[-2, 1) \cup [3, \infty)$.

Problem 24: Roller coaster height

Test the divisors of 6: $h(1) = 1 - 6 + 11 - 6 = 0$, so $(x-1)$ is a factor. Division gives $x^2 - 5x + 6 = (x-2)(x-3)$, so $h(x) = (x-1)(x-2)(x-3)$.

Signs: negative on $[0, 1)$, positive on $(1, 2)$, negative on $(2, 3)$, positive on $(3, 4]$. Check: $h(0) = -6$ and $h(2.5) = -0.375$.

Answer: $[0, 1) \cup (2, 3)$, that is 0 up to 100 feet and 200 to 300 feet.

How to use this precalculus polynomial rational worksheet answer key

Read this precalculus polynomial rational worksheet answer key after trying each problem yourself, and redo any problem you missed.

