

Precalculus · Polynomial and Rational Functions in Depth · Worksheet

Common Core A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This precalculus polynomial rational worksheet gives 24 problems on polynomial and rational functions in depth, ordered from easy to hard, with a separate answer key.

These 24 problems build Precalculus fluency with polynomial and rational functions: the Rational Root Theorem, complex zeros and conjugate pairs (A-APR.B.2, A-APR.B.3, N-CN.C.9), end behavior and asymptotes (F-IF.C.7c, F-IF.C.7d), and inequalities solved with sign charts.

The problems run from listing candidates to multi-step models. The last six use contexts such as box volume, average cost and profit, so show every factorization and every interval test.

Precalculus Common Core: A-APR.B.2 · A-APR.B.3 · F-IF.C.7c · F-IF.C.7d · N-CN.C.9

Precalculus Polynomial Rational Worksheet: problems 1-5

Problem 1: List possible rational zeros

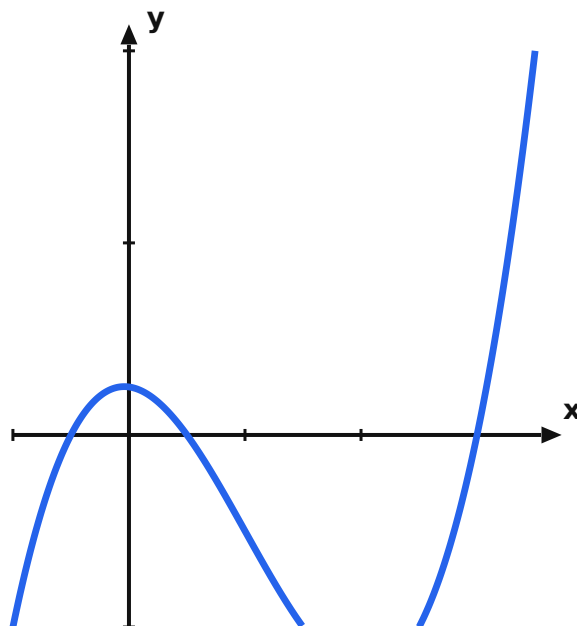
List every possible rational zero of $P(x) = 3x^4 + 2x^3 - 5x + 8$.

Problem 2: Rational zeros of a cubic

Find all zeros of $P(x) = x^3 - 4x^2 + x + 6$.

Problem 3: Factor with the Rational Root Theorem

The graph shows $P(x) = 4x^3 - 12x^2 - x + 3$, which crosses the x-axis three times.



Graph of P for exercise 3.

- List the possible rational zeros.
- Find all zeros and write $P(x)$ in factored form.

Problem 4: Counting zeros

Let $f(x) = (x-1)^2(x+4)^3(x^2+1)$.

- State the degree of f .
- List every complex zero with its multiplicity.
- How many distinct real zeros does f have?

Problem 5: Complex zeros of a cubic

Find all real and complex zeros of $P(x) = x^3 - 2x^2 + 4x - 8$.

Precalculus Polynomial Rational Worksheet: problems 6-10

Problem 6: Build a polynomial from zeros

Write a monic cubic polynomial with real coefficients whose zeros include 3 and $2-i$. Write it in standard form.

Problem 7: Use a given complex zero

One zero of $P(x) = x^4 - 2x^3 + 6x^2 - 2x + 5$ is $1+2i$. Find the other three zeros and plot the nonreal zeros on a complex plane.

Problem 8: End behavior of polynomials

Describe the end behavior of each function by completing: as $x \rightarrow -\infty$, $f(x) \rightarrow ?$ and as $x \rightarrow \infty$, $f(x) \rightarrow ?$.

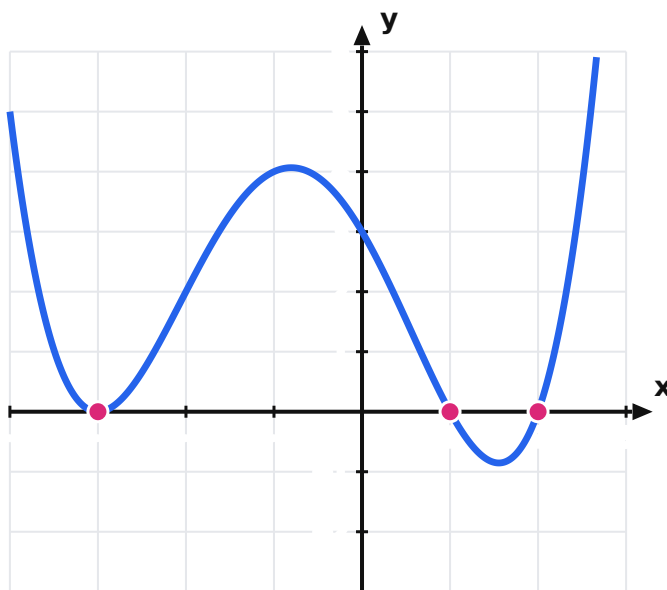
a. $f(x) = -2x^5 + 3x^2 - 1$

b. $f(x) = x^4 - 8x^2 + 3$

c. $f(x) = \frac{1}{2}x^3 - 7x$

Problem 9: Zeros and multiplicity from a graph

The graph shows a polynomial h with zeros at -3 , 1 and 2 .



Graph of h for exercise 9.

- State the multiplicity of each zero as odd or even and explain how you know.
- State the least possible degree.
- Is the leading coefficient positive or negative?

Problem 10: Asymptotes of a rational function

Find all vertical and horizontal asymptotes of $f(x) = \frac{3x^2+1}{x^2-4}$.

Precalculus Polynomial Rational Worksheet: problems 11-15

Problem 11: Slant asymptote by division

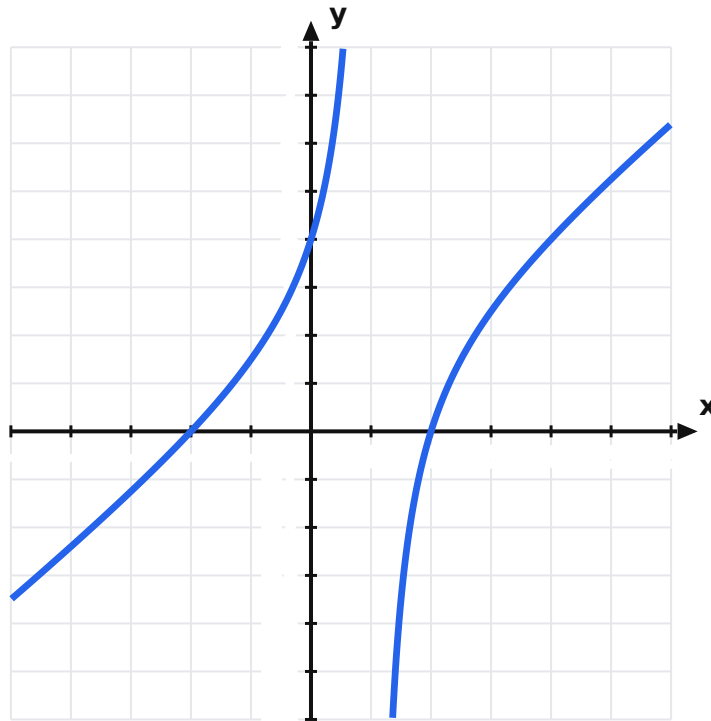
Find the slant asymptote of $f(x) = \frac{x^2+3x-1}{x-1}$.

Problem 12: Slant asymptote with a negative remainder term

Find the slant asymptote and the vertical asymptote of $f(x) = \frac{2x^2 - 5x + 3}{x + 2}$.

Problem 13: Analyze a rational graph

The graph shows $f(x) = \frac{x^2 - 4}{x - 1}$.



Graph of f for exercise 13.

- Write the domain in interval notation.
- Find the equations of the vertical and slant asymptotes.
- Find the x -intercepts and the y -intercept.

Problem 14: Quadratic inequality

Solve $x^2 - x - 6 > 0$ and write the answer in interval notation.

Problem 15: Cubic inequality

Solve $(x + 1)(x - 2)(x - 4) \leq 0$.

Precalculus Polynomial Rational Worksheet: problems 16-20

Problem 16: Inequality with a repeated factor

Solve $(x-2)^2(x+1) \leq 0$.

Problem 17: Rational inequality with zero on one side

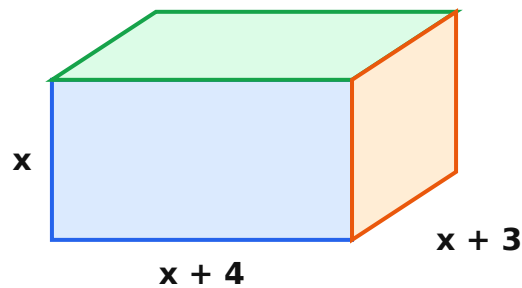
Solve $\frac{x-1}{x+3} \geq 0$.

Problem 18: Rational inequality with a constant on the right

Solve $\frac{2x}{x-1} < 3$.

Problem 19: Gift box volume

Priya designs a gift box with height x feet, width $x+3$ feet and length $x+4$ feet. The volume must be 60 cubic feet.

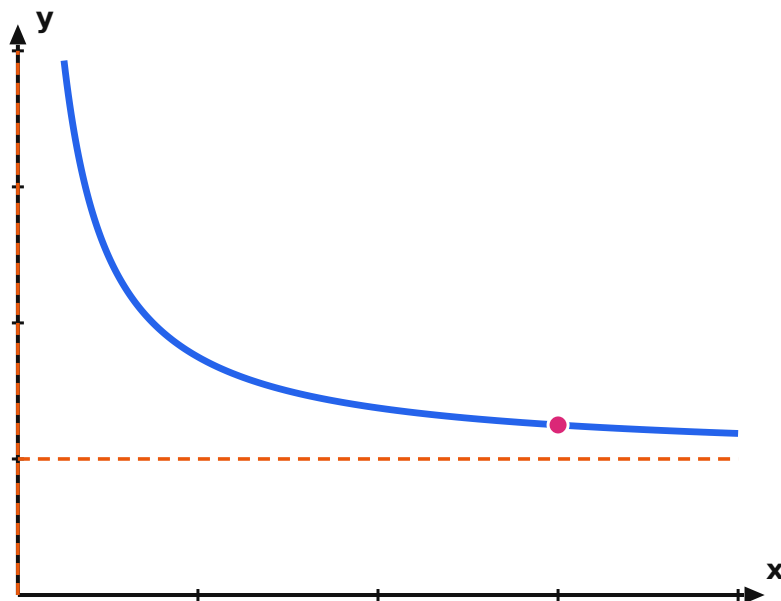


The box in exercise 19.

- Write an equation of the form $P(x) = 0$.
- Use the Rational Root Theorem to find the real solution.
- Find the other two zeros and the box dimensions.

Problem 20: Average cost

A bakery's average cost per cookie box is $A(x) = \frac{20x+1500}{x}$ dollars when it makes x boxes.



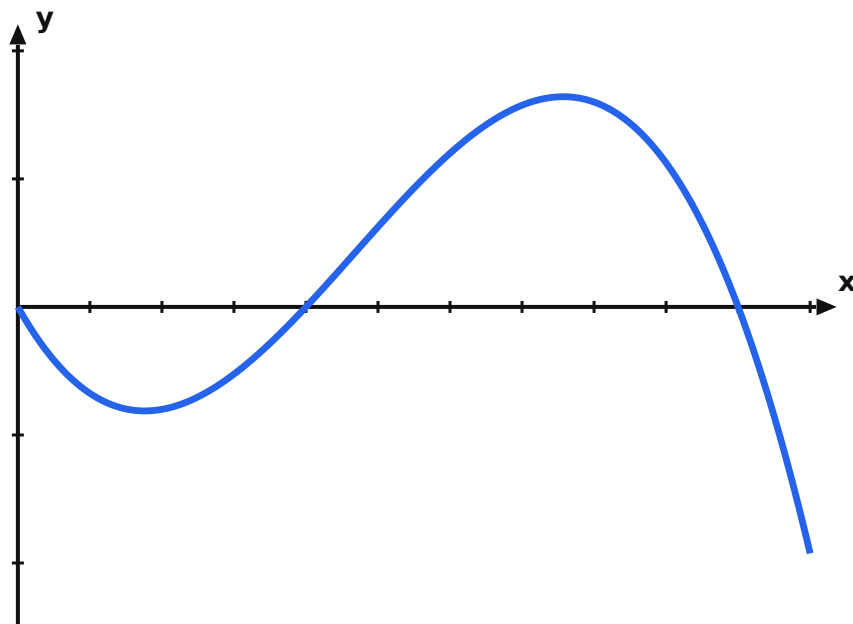
Average cost per box falls toward \$20 as production grows.

- Find the horizontal asymptote and explain what it means.
- Find $A(100)$.
- How many boxes must be made so that the average cost is at most \$25?

Precalculus Polynomial Rational Worksheet: problems 21-24

Problem 21: Profit model

The profit of a school store is $P(x) = -x^3 + 14x^2 - 40x$ thousand dollars when x hundred items are sold, for $0 < x < 11$.



Profit model for exercise 21.

- Factor $P(x)$ completely.
- Solve $P(x) > 0$ for $0 < x < 11$.
- Find the profit when 600 items are sold.

Problem 22: Polynomial with an imaginary zero

Write a monic polynomial of degree 4 with real coefficients that has zeros 2, -1 and $3i$. Write it in standard form and describe its end behavior.

Problem 23: Rational inequality with a quadratic

Solve $\frac{x^2 - x - 6}{x - 1} \geq 0$.

Problem 24: Roller coaster height

The height of a roller coaster track above ground level, in tens of feet, is $h(x) = x^3 - 6x^2 + 11x - 6$ at horizontal distance x hundred feet, for $0 \leq x \leq 4$. Find where the track is below ground level.

How to use this precalculus polynomial rational worksheet

Print this precalculus polynomial rational worksheet as a PDF, solve every problem on paper, then compare with the answer key.

