

Precalculus · Polynomial and Rational Functions in Depth · Lesson

Common Core A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9 · mathovore.fr

This precalculus polynomial rational lesson explains polynomial and rational functions in depth in one printable page, with definitions, a method and a worked example.

The Rational Root Theorem lists every possible rational zero p/q of an integer-coefficient polynomial, and the Fundamental Theorem of Algebra says a degree n polynomial has exactly n complex zeros, with nonreal zeros of real polynomials arriving as conjugate pairs. Sign charts then solve polynomial and rational inequalities.

Precalculus Common Core: A-APR.B.2 · A-APR.B.3 · F-IF.C.7c · F-IF.C.7d · N-CN.C.9

Precalculus Polynomial Rational Lesson: key facts

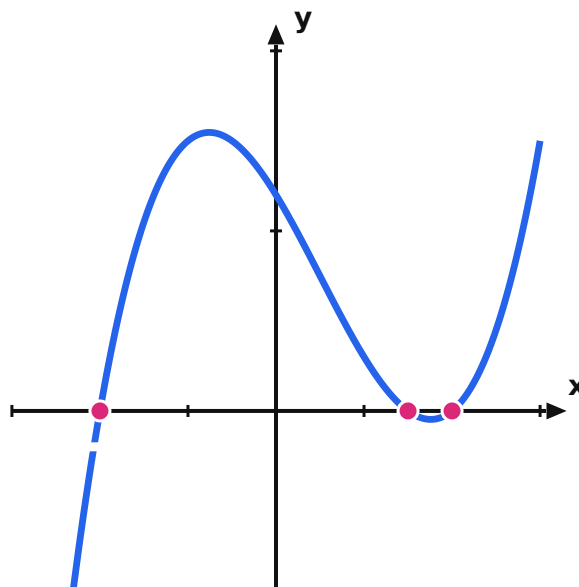
- Standards A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d and N-CN.C.9 covered in depth at the Precalculus level.
- Rational zeros have the form $\frac{p}{q}$ with p dividing the constant term and q dividing the leading coefficient.
- A degree n polynomial has exactly n complex zeros counting multiplicity; real polynomials have conjugate pairs $a \pm bi$.
- End behavior follows the leading term; a slant asymptote exists when the numerator's degree is one more than the denominator's.
- Solve inequalities by finding critical numbers, testing intervals on a sign chart, and checking endpoints.

What does the Rational Root Theorem say?

Suppose a polynomial has integer coefficients. Then every rational zero, written in lowest terms as $\frac{p}{q}$, has p dividing the constant term and q dividing the leading coefficient. The theorem gives a finite list of candidates to test.

Rational Root Theorem

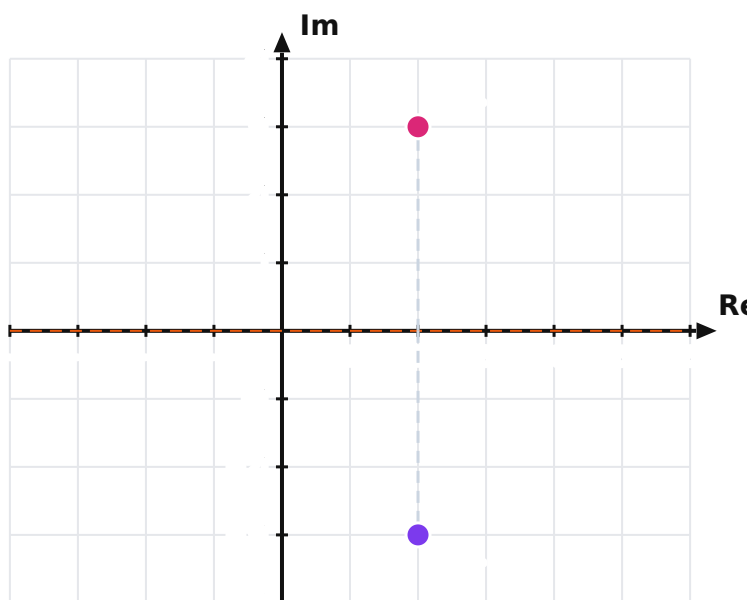
For $P(x) = 2x^3 - 3x^2 - 8x + 12$, p is one of $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ and q is 1 or 2. Testing $x = 2$ gives $16 - 12 - 16 + 12 = 0$, so $x = 2$ is a zero. Dividing leaves $2x^2 + x - 6 = (2x - 3)(x + 2)$, so the zeros are $2, \frac{3}{2}$ and -2 .



The cubic crosses the x-axis at the rational numbers -2 , $3/2$ and 2 , all on the list of candidates.

What does the Fundamental Theorem of Algebra guarantee?

A polynomial of degree $n \geq 1$ has exactly n complex zeros when each zero is counted as many times as its multiplicity. If the coefficients are real, the nonreal zeros come in **conjugate pairs** $a+bi$ and $a-bi$. For $x^2-4x+13$ the discriminant is -36 , so the zeros are $2+3i$ and $2-3i$.



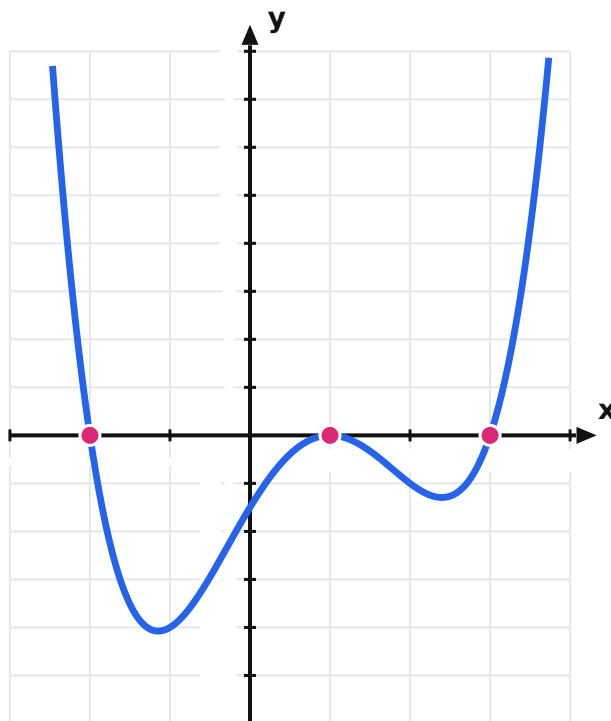
Conjugate zeros are mirror images across the real axis.

How do you describe end behavior?

For large $|x|$ a polynomial behaves like its leading term $a_n x^n$. Use the degree and the sign of the leading coefficient.

Degree	Leading coefficient	As $x \rightarrow -\infty$	As $x \rightarrow \infty$
even	positive	rises	rises
even	negative	falls	falls
odd	positive	falls	rises
odd	negative	rises	falls

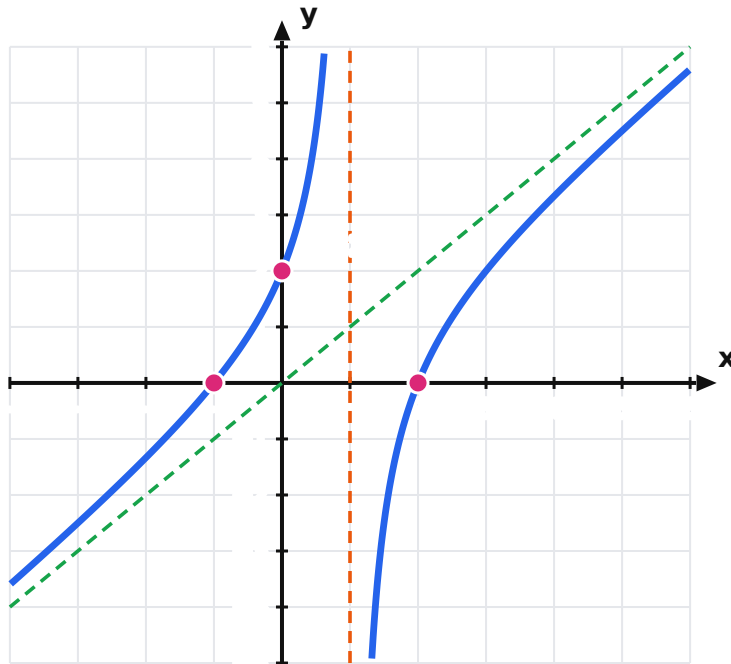
A zero of even multiplicity, such as $x=1$ below, touches the axis and turns; an odd multiplicity crosses.



Degree 4 with positive leading coefficient: both ends rise. The graph touches at $x = 1$.

How do you find a slant asymptote?

When the numerator's degree is exactly one more than the denominator's, divide. The quotient is the slant asymptote and the remainder term fades as $|x|$ grows. For $f(x) = \frac{x^2 - x - 2}{x - 1}$, division gives $f(x) = x - \frac{2}{x - 1}$, so the slant asymptote is $y = x$ and the vertical asymptote is $x = 1$.

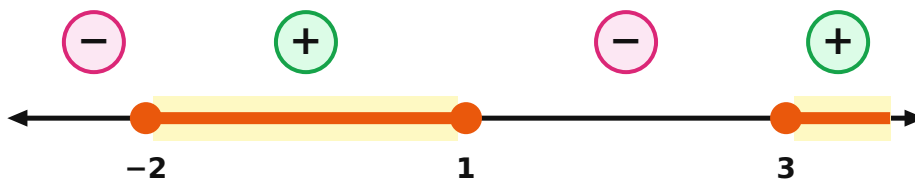


Dashed lines mark the vertical asymptote $x = 1$ and the slant asymptote $y = x$.

How do you solve polynomial and rational inequalities?

1. Move every term to one side so the other side is 0.
2. Factor the expression.
3. Mark the critical numbers: zeros of the numerator and values that make the denominator zero.
4. Test one value in each interval to find its sign.
5. Write the intervals that match; include an endpoint only for $\leq$ or $\geq$ and only if the expression is defined there.

Example: $(x+2)(x-1)(x-3) \geq 0$. At $x=0$ the product is 6, which is positive, and the signs alternate across single zeros. The solution is $[-2, 1] \cup [3, \infty)$.



Signs alternate across each single zero; shaded intervals satisfy ≥ 0 .

Polynomial and Rational Functions in Depth: frequently asked

questions

What are the possible rational zeros of a polynomial?

Take every divisor p of the constant term and every divisor q of the leading coefficient, then form $\pm \frac{p}{q}$. For $3x^3 + x - 8$ the candidates have p in 1,2,4,8 and q in 1,3.

Why do complex zeros come in pairs?

For a polynomial with real coefficients, the quadratic formula produces $\pm \sqrt{b^2 - 4ac}$ when the discriminant is negative, so $a + bi$ and $a - bi$ always appear together.

How can you tell if a graph has a slant asymptote?

Its equation is a rational function whose numerator has degree exactly one more than its denominator. Long division gives the line, and the graph approaches that line as x grows large in either direction.

When do you include endpoints in an inequality answer?

Include a critical number only when the inequality has an equals sign and the expression is defined there. Zeros of a denominator are never included.

How does multiplicity change a graph?

An odd multiplicity makes the graph cross the x-axis, while an even multiplicity makes it touch and turn. In $(x-2)^2(x+1)$, the zero 2 touches and -1 crosses.

How to use this precalculus polynomial rational lesson

Use this precalculus polynomial rational lesson before the worksheet, then check what you learned with the quiz.

