

Precalculus · Polar Coordinates and Parametric Equations · Worksheet Answer Key

Common Core N-CN.B.4, F-IF.C.7 · mathovore.fr

This precalculus polar parametric worksheet answer key solves all 22 problems of the polar coordinates and parametric equations worksheet step by step.

Full worked solutions for all 22 problems follow, with each final answer in bold. Compare each step with your own work, and check conversions by recomputing $x = r \cos \theta$ and $y = r \sin \theta$.

Precalculus Common Core: N-CN.B.4 · F-IF.C.7

Precalculus Polar Parametric Worksheet Answer Key: problems 1-5

Problem 1: Read points on a polar grid

Count rings for r and read the spoke for θ .

- a. A: $(3, 30^\circ)$
- b. B: $(4, 120^\circ)$
- c. C: $(2, 225^\circ)$
- d. D: $(5, 300^\circ)$

Problem 2: Convert polar to rectangular

- a. $x = 4 \cos \frac{\pi}{3} = 2$, $y = 4 \sin \frac{\pi}{3} = 2\sqrt{3}$: $(2, 2\sqrt{3})$
- b. $x = 6 \cdot \left(-\frac{\sqrt{2}}{2}\right) = -3\sqrt{2}$, $y = 6 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$: $(-3\sqrt{2}, 3\sqrt{2})$
- c. $x = 5 \cos \pi = -5$, $y = 5 \sin \pi = 0$: $(-5, 0)$
- d. $x = 2 \cos\left(-\frac{\pi}{2}\right) = 0$, $y = 2 \sin\left(-\frac{\pi}{2}\right) = -2$: $(0, -2)$

Problem 3: Convert rectangular to polar

- a. $r = \sqrt{9+27} = 6$; $\tan \theta = \sqrt{3}$ in Quadrant I, so $\theta = \frac{\pi}{3}$: $\left(6, \frac{\pi}{3}\right)$
- b. The point lies on the negative x-axis, so $r = 4$ and $\theta = \pi$: $(4, \pi)$
- c. $r = \sqrt{25+25} = 5\sqrt{2}$; $\tan \theta = -1$ in Quadrant II, so $\theta = \frac{3\pi}{4}$: $\left(5\sqrt{2}, \frac{3\pi}{4}\right)$
- d. $r = \sqrt{4+12} = 4$; $\tan \theta = -\sqrt{3}$ in Quadrant IV, so $\theta = \frac{5\pi}{3}$: $\left(4, \frac{5\pi}{3}\right)$

Problem 4: Rewrite equivalent polar pairs

- a. Change the sign of r and add 180° : $(2, 250^\circ)$
- b. Add 360° : $-45^\circ + 360^\circ = 315^\circ$, so $(5, 315^\circ)$

- c. Change the sign of r and add π : $\frac{\pi}{6} + \pi = \frac{7\pi}{6}$, so $(4, \frac{7\pi}{6})$
 d. Subtract 4π (two full turns): $5\pi - 4\pi = \pi$, so $(3, \pi)$

Problem 5: Distance between polar points

- a. The angle between the rays is 60° . $d^2 = 16 + 36 - 2(4)(6)\cos 60^\circ = 52 - 24 = 28$, so $d = 2\sqrt{7} \approx 5.29$
 b. The angle between the rays is 120° . $d^2 = 25 + 25 - 2(5)(5)\cos 120^\circ = 50 + 25 = 75$, so $d = 5\sqrt{3} \approx 8.66$

Precalculus Polar Parametric Worksheet Answer Key: problems 6-10

Problem 6: Rectangular equations to polar form

- a. Replace x with $r\cos\theta$: $r\cos\theta = 4$
 b. $r\sin\theta = r\cos\theta$ gives $\tan\theta = 1$, so $\theta = \frac{\pi}{4}$
 c. $r^2 = 25$, so $r = 5$
 d. $r^2 = 6r\sin\theta$, so $r = 6\sin\theta$ (the pole is included at $\theta = 0$)

Problem 7: Polar equations to rectangular form

- a. $r^2 = 49$, so $x^2 + y^2 = 49$
 b. $\tan\theta = \sqrt{3}$ gives $y = \sqrt{3}x$
 c. Multiply by r : $x^2 + y^2 = 8y$. Complete the square: $x^2 + (y-4)^2 = 16$
 d. Multiply by r : $x^2 + y^2 = 2x + 2y$. Complete both squares: $(x-1)^2 + (y-1)^2 = 2$

Problem 8: Identify a polar circle

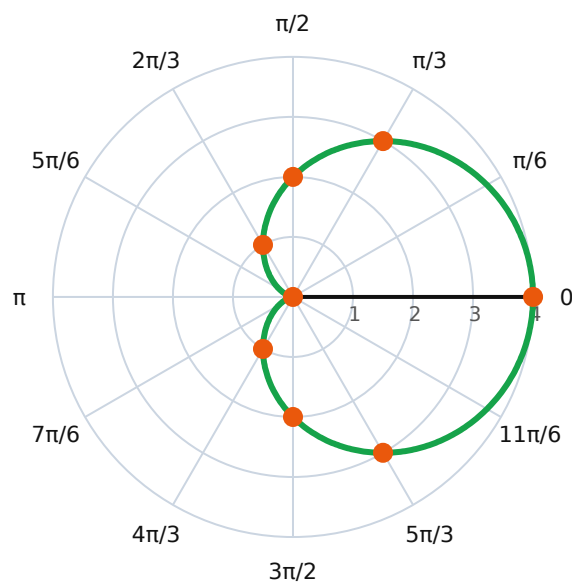
The circle passes through the pole and reaches $r = 10$ at $\theta = 0$, so its diameter is 10.

Center $(5, 0)$, radius **5**. Algebraically, $r^2 = 10r\cos\theta$ gives $x^2 + y^2 = 10x$, so $(x-5)^2 + y^2 = 25$.

Problem 9: Complete a cardioid table

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
r	4	3	2	1	0	1	2	3

Because $a = b = 2$, the graph is a **cardioid**. The greatest value is $r = 4$ at $\theta = 0$, and the curve passes through the pole at $\theta = \pi$.



The cardioid $r = 2 + 2 \cos \theta$ with the table points marked.

Problem 10: Classify polar graphs

- $a = b = 3$: **cardioid**
- $a = 2 < b = 4$: **limaçon with an inner loop**
- $\frac{a}{b} = 1.5$, between 1 and 2: **dimpled limaçon**
- $\frac{a}{b} = 2.5 \geq 2$: **convex limaçon** (no loop, no dimple)
- $n = 3$ is odd: **rose with 3 petals**
- $n = 2$ is even: **rose with 4 petals**

Precalculus Polar Parametric Worksheet Answer Key: problems 11-15

Problem 11: Find the tips of a rose

Since $n = 3$ is odd there are **3 petals**, each of length **5**.

Solve $\cos 3\theta = 1$: $3\theta = 0, 2\pi, 4\pi$, so $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$. These angles point to the three petal tips.

Problem 12: Pole crossings of a rose

Solve $4 \cos 2\theta = 0$: $2\theta = \frac{\pi}{2} + \pi k$, so $\theta = \frac{\pi}{4} + \frac{\pi k}{2}$. In $[0, 2\pi)$: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

$n = 2$ is even, so there are **4 petals**. $r = 4$ when $\cos 2\theta = 1$: $2\theta = 0, 2\pi$, so $\theta = 0, \pi$.

Problem 13: Limaçon with an inner loop

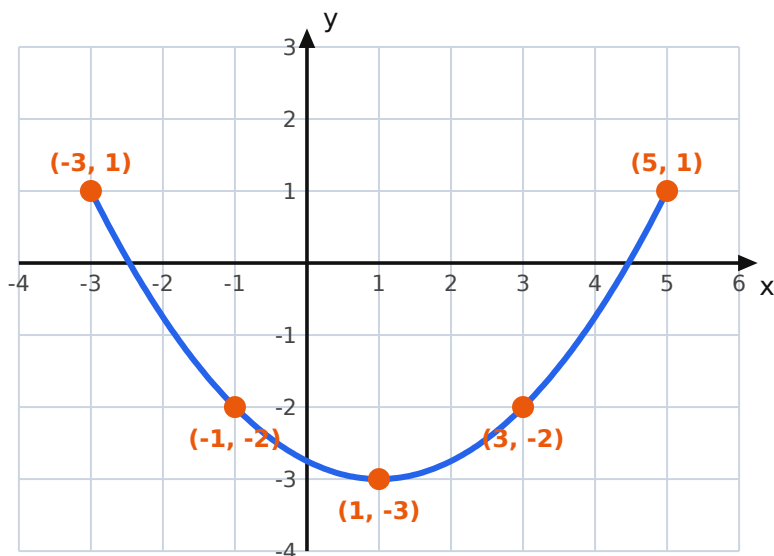
The greatest value is $2+4=6$ at $\theta=0$.

$2+4\cos\theta=0$ gives $\cos\theta=-\frac{1}{2}$, so $\theta=\frac{2\pi}{3}, \frac{4\pi}{3}$.

At $\theta=\pi$, $r=2-4=-2$. The point $(-2,\pi)$ is $(x,y)=(-2\cos\pi,-2\sin\pi)=(2,0)$, on the inner loop.

Problem 14: Make a table of parametric points

Substitute each t : $(-3,1)$, $(-1,-2)$, $(1,-3)$, $(3,-2)$, $(5,1)$. They lie on a parabola opening upward with its lowest point at $(1,-3)$.



The five points joined in order of increasing t .

Problem 15: Eliminate the parameter, linear

- From $y=t+4$, $t=y-4$. Then $x=3(y-4)-2=3y-14$, so $y=\frac{x+14}{3}$.
- From $x=5-2t$, $t=\frac{5-x}{2}$. Then $y=4\cdot\frac{5-x}{2}+1=10-2x+1$, so $y=11-2x$.

Precalculus Polar Parametric Worksheet Answer Key: problems 16-20

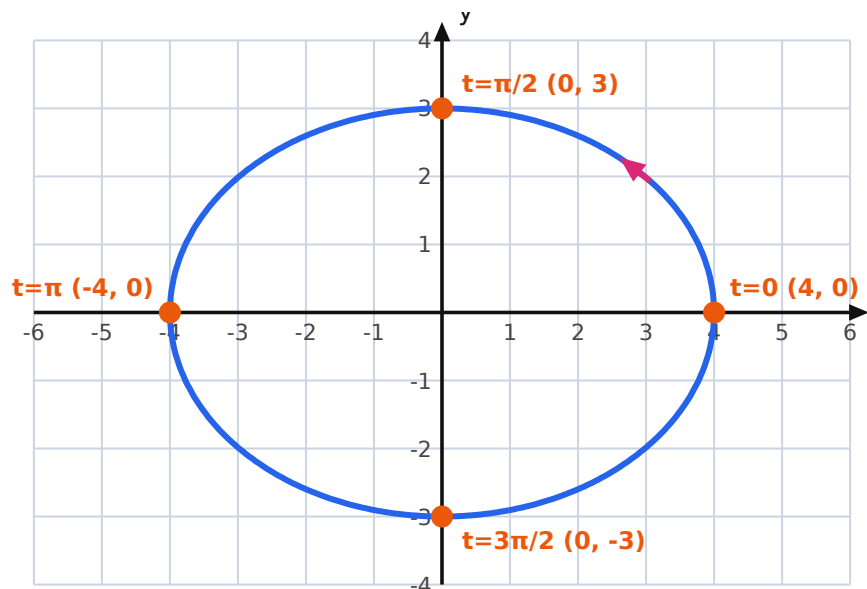
Problem 16: Eliminate the parameter, quadratic

- From $y=2t-1$, $t=\frac{y+1}{2}$. Then $x=\frac{(y+1)^2}{4}$, a parabola opening to the right.
- From $x=t+3$, $t=x-3$. Then $y=(x-3)^2-6(x-3)+5=x^2-6x+9-6x+18+5$, so $y=x^2-12x+32$, a parabola opening upward. Check: $t=1$ gives $(4,0)$, and $16-48+32=0$.

Problem 17: Eliminate the parameter, ellipse

Write $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{3}$. Since $\cos^2 t + \sin^2 t = 1 : \frac{x^2}{16} + \frac{y^2}{9} = 1$, an ellipse.

The points are $(4,0)$, $(0,3)$, $(-4,0)$, $(0,-3)$, so the curve is traced counterclockwise.

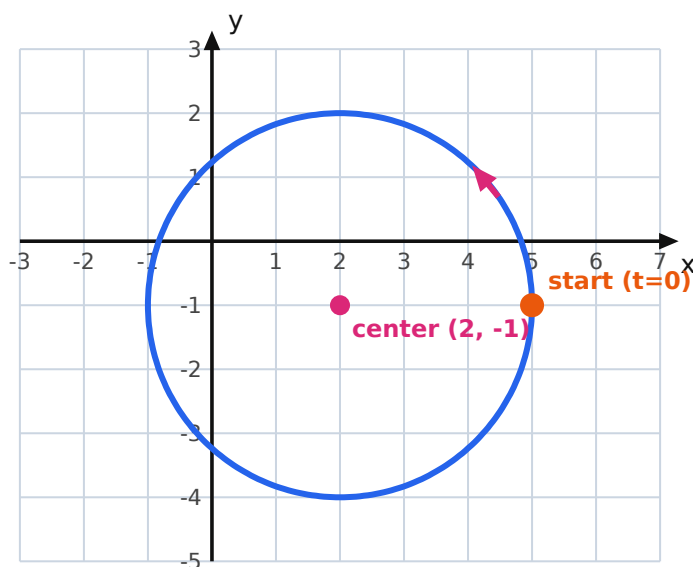


The ellipse traced from $t = 0$ to $t = 2\pi$.

Problem 18: Shifted circle from parametric equations

Rewrite as $x-2=3\cos t$ and $y+1=3\sin t$. Squaring and adding gives $(x-2)^2 + (y+1)^2 = 9$.

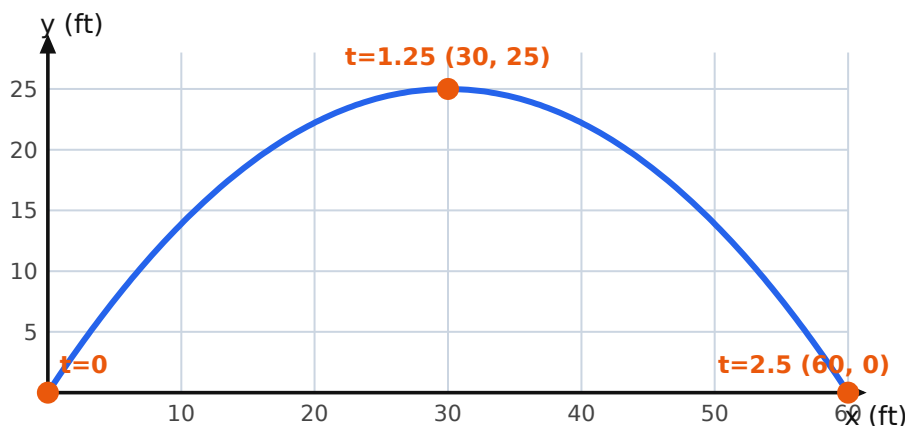
Center $(2,-1)$, radius **3**. At $t=0$ the point is $(5,-1)$; at $t=\pi$ it is $(-1,-1)$. The motion is counterclockwise.



The circle $(x - 2)^2 + (y + 1)^2 = 9$ traced counterclockwise.

Problem 19: Soccer kick

- $y = 40 - 16 = \mathbf{24 \text{ ft}}$
- $40t - 16t^2 = 0$ gives $8t(5 - 2t) = 0$, so $t = 2.5 \text{ s}$
- $x = 24(2.5) = \mathbf{60 \text{ ft}}$
- The vertex is at $t = \frac{40}{32} = 1.25$, where $y = 50 - 25 = \mathbf{25 \text{ ft}}$
- $t = \frac{x}{24}$, so $y = \frac{40x}{24} - \frac{16x^2}{576} = \frac{5x}{3} - \frac{x^2}{36}$. Check: $x = 60$ gives $100 - 100 = 0$.



The ball lands 60 ft away after 2.5 seconds.

Problem 20: Ferris wheel

- One turn needs $\frac{\pi t}{10} = 2\pi$, so $t = 20 \text{ s}$
- $\cos \frac{\pi}{2} = 0$, $\sin \frac{\pi}{2} = 1$: $(0, 65)$, the top
- The lowest point is $35 - 30 = \mathbf{5 \text{ ft}}$, at $t = 15$
- $x^2 + (y - 35)^2 = 900 \cos^2 + 900 \sin^2 = x^2 + (y - 35)^2 = 900$
- $35 + 30 \sin \frac{\pi t}{10} = 50$ gives $\sin \frac{\pi t}{10} = \frac{1}{2}$, so $\frac{\pi t}{10} = \frac{\pi}{6}$ and $t = \frac{5}{3} \approx 1.67 \text{ s}$

Precalculus Polar Parametric Worksheet Answer Key: problems 21-22

Problem 21: Radar stations

For boat 1: $x = 12 \cos 40^\circ \approx \mathbf{9.19}$ and $y = 12 \sin 40^\circ \approx \mathbf{7.71}$.

The angle between the rays is 60° . $d^2 = 144 + 81 - 2(12)(9) \cos 60^\circ = 225 - 108 = 117$, so $d = 3\sqrt{13} \approx \mathbf{10.82 \text{ mi}}$.

Problem 22: Rose flower bed

- $n = 3$ is odd: $\mathbf{3 \text{ petals}}$

b. $\sin 3\theta = 1$ gives $3\theta = \frac{\pi}{2} + 2\pi k$, so $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

c. Tips are 8 ft from the pole and 120° apart: $d^2 = 64 + 64 - 128\cos 120^\circ = 192$, so $d = 8\sqrt{3} \approx 13.86$ ft

d. The three tips form an equilateral triangle: $3 \cdot 8\sqrt{3} = 24\sqrt{3} \approx 41.57$ ft

How to use this precalculus polar parametric worksheet answer key

Read this precalculus polar parametric worksheet answer key after trying each problem yourself, and redo any problem you missed.

