

Precalculus · Polar Coordinates and Parametric Equations · Lesson

Common Core N-CN.B.4, F-IF.C.7 · mathovore.fr

This precalculus polar parametric lesson explains polar coordinates and parametric equations in one printable page, with definitions, a method and a worked example.

Polar coordinates locate a point by its distance r from the pole and its angle θ from the polar axis, written (r, θ) . They convert with $x = r \cos \theta$ and $y = r \sin \theta$. Parametric equations give x and y as functions of a parameter t , which models motion and can be eliminated to find the curve.

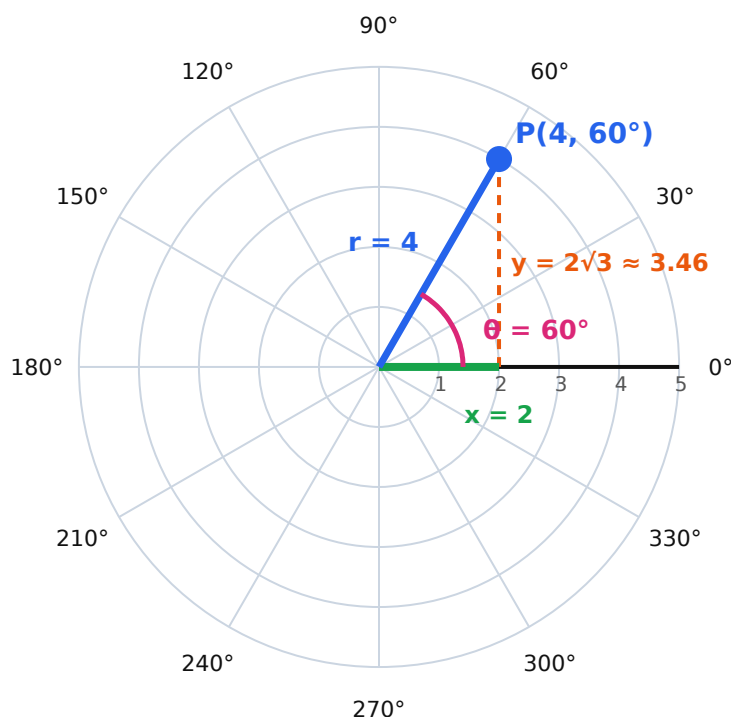
Precalculus Common Core: N-CN.B.4 · F-IF.C.7

Precalculus Polar Parametric Lesson: key facts

- Polar to rectangular: $x = r \cos \theta$, $y = r \sin \theta$; rectangular to polar: $r^2 = x^2 + y^2$, $\tan \theta = \frac{y}{x}$.
- Every point has infinitely many polar names: $(r, \theta) = (r, \theta + 2\pi k) = (-r, \theta + \pi)$.
- A rose $r = a \cos n\theta$ has n petals for odd n and $2n$ petals for even n .
- A limaçon $r = a + b \cos \theta$ has an inner loop when $a < b$ and is a cardioid when $a = b$.
- To eliminate the parameter, solve one equation for t and substitute into the other.

What are polar coordinates?

A point can be located by a distance and a turn instead of by horizontal and vertical steps. The fixed point is the **pole** and the fixed ray, along the positive x-axis, is the **polar axis**. The pair (r, θ) means: turn through angle θ from the polar axis, then move r units in that direction. A negative r means move the opposite way.



Point $P(4, 60^\circ)$: 4 units from the pole along the 60° spoke.

Many names for one point

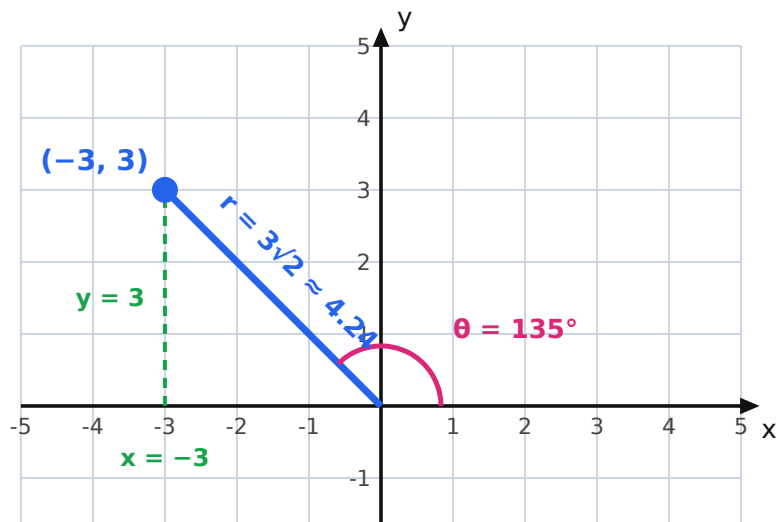
(r, θ) , $(r, \theta + 2\pi k)$ and $(-r, \theta + \pi)$ all name the same point, for any integer k . For example, $(4, 60^\circ)$ and $(-4, 240^\circ)$ are the same point.

How do you convert between polar and rectangular coordinates?

Direction	Formulas
Polar to rectangular	$x = r \cos \theta$, $y = r \sin \theta$
Rectangular to polar	$r^2 = x^2 + y^2$, $\tan \theta = \frac{y}{x}$

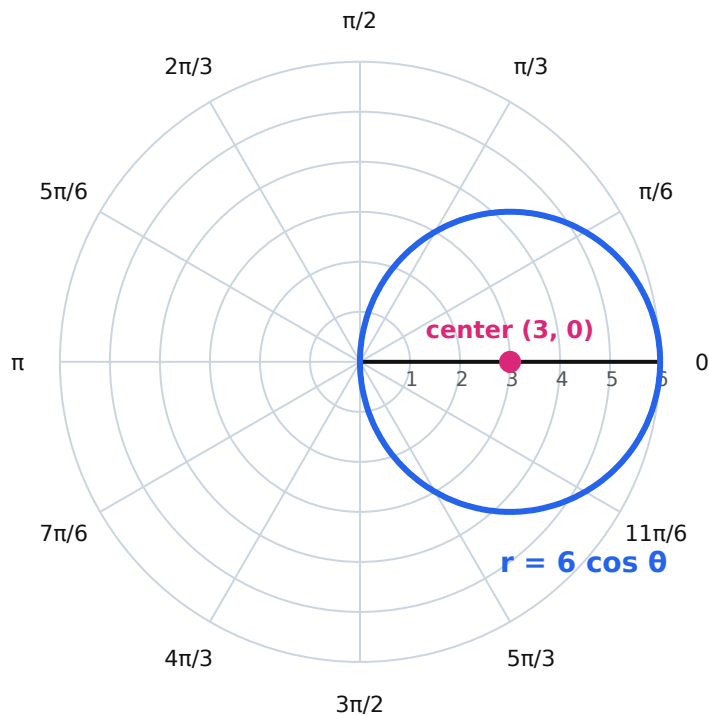
1. Find $r = \sqrt{x^2 + y^2}$.
2. Find a reference angle from $\tan \theta = \frac{y}{x}$, then place θ in the quadrant that holds the point.
3. Check by recomputing $x = r \cos \theta$ and $y = r \sin \theta$.

Example: for $(-3, 3)$, $r = \sqrt{18} = 3\sqrt{2}$. Since $\tan \theta = -1$ and the point is in Quadrant II, $\theta = \frac{3\pi}{4}$. The polar pair is $(3\sqrt{2}, \frac{3\pi}{4})$.



The point $(-3, 3)$ and its distance and angle from the origin.

Equations convert the same way. Take $r = 6 \cos \theta$. Multiply both sides by r to get $r^2 = 6r \cos \theta$, so $x^2 + y^2 = 6x$. Completing the square gives $(x-3)^2 + y^2 = 9$, a circle of radius 3 centered at $(3, 0)$.

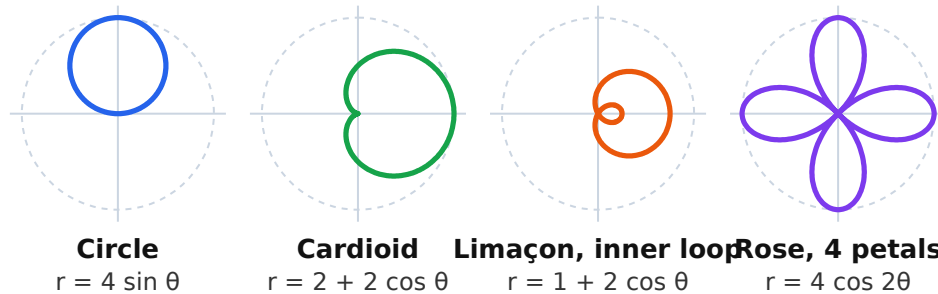


The graph of $r = 6 \cos \theta$ is a circle through the pole.

What do polar graphs look like?

Equation	Graph
$r = a \cos \theta$ or $r = a \sin \theta$	Circle through the pole, diameter $ a $

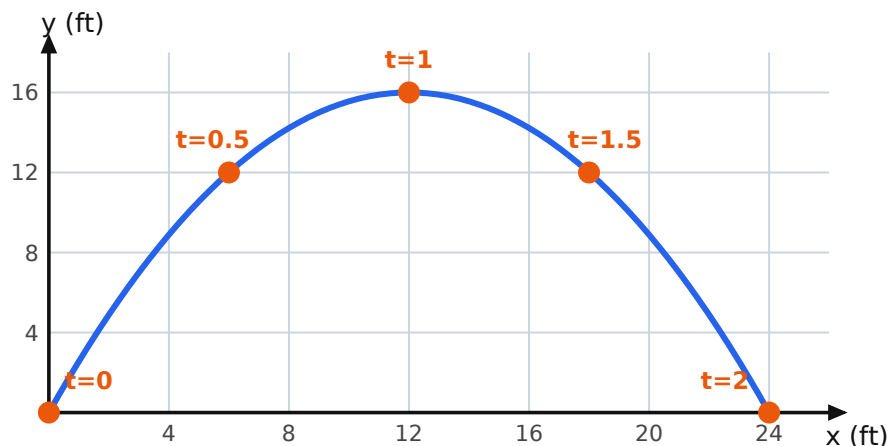
Equation	Graph
$r = a + b \cos \theta$ or $r = a + b \sin \theta$	Limaçon. Cardioid if $a = b$; inner loop if $a < b$; dimple if $b < a < 2b$; convex if $a \geq 2b$
$r = a \cos n\theta$ or $r = a \sin n\theta$	Rose with petals of length $ a $: n petals if n is odd, $2n$ if n is even



A circle, a cardioid, a limaçon with an inner loop and a four-petal rose.

What are parametric equations?

Parametric equations write x and y as separate functions of a third variable t , the **parameter**, often time. A ball kicked at a steady 12 ft/s sideways follows $x = 12t$, $y = 32t - 16t^2$ for $0 \leq t \leq 2$. To **eliminate the parameter**, solve one equation for t and substitute: $t = \frac{x}{12}$, so $y = -\frac{x^2}{9} + \frac{8x}{3}$. The ball peaks at $t = 1$ at 16 ft and lands at $x = 24$ ft. The parametric form also records timing and direction, which $y = f(x)$ loses.



The path of the ball, with points marked every half second.

Polar Coordinates and Parametric Equations: frequently asked questions

What is the difference between polar and rectangular coordinates?

Rectangular coordinates give a point by horizontal and vertical distances (x, y) . Polar

coordinates give it by distance from the pole and angle from the polar axis, (r, θ) . Both describe the same plane, and each pair converts to the other.

Can r be negative in polar coordinates?

Yes. A negative r means you move in the direction opposite to angle θ . So $(-3, \frac{\pi}{4})$ is the same point as $(3, \frac{5\pi}{4})$.

How do you find the number of petals of a rose curve?

Look at the number n in $r = a \cos n\theta$ or $r = a \sin n\theta$. If n is odd the rose has n petals. If n is even it has $2n$ petals. Each petal has length $|a|$.

What does it mean to eliminate the parameter?

It means rewriting parametric equations as one equation in x and y . Solve one equation for t and substitute, or use an identity such as $\cos^2 t + \sin^2 t = 1$.

Why use parametric equations instead of $y = f(x)$?

Parametric equations describe curves that fail the vertical line test, such as circles, and they show where an object is at each time and which way it moves. A single function $y = f(x)$ cannot show either.

How to use this precalculus polar parametric lesson

Use this precalculus polar parametric lesson before the worksheet, then check what you learned with the quiz.

