

Precalculus · Solving Systems with Matrices · Worksheet Answer Key

Common Core A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12 · mathovore.fr

This precalculus matrix systems worksheet answer key solves all 24 problems of the solving systems with matrices worksheet step by step.

Every solution shows the determinant, the inverse and the multiplication $A^{-1}B$, followed by a check in the original equations. Compare each line of your work, not only the final answer, and note which step an error began.

Precalculus Common Core: A-REI.C.8 · A-REI.C.9 · N-VM.C.11 · N-VM.C.12

Precalculus Matrix Systems Worksheet Answer Key: problems 1-5

Problem 1: Write a system as $AX = B$

Rows of A hold the coefficients of each equation, so A has rows 3, 1 and 2, -1. The unknown column is $X = (x, y)$ and the constant column is $B = (7, 3)$.

$$3x + y = 7 \text{ and } 2x - y = 3$$

$$\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

A coefficients X unknowns B constants

$$AX = B$$

Matrix form of the system.

Check by multiplying: row 1 gives $3x + y = 7$ and row 2 gives $2x - y = 3$. $AX = B$ with the matrices shown

Problem 2: Read a matrix equation

Row 1 times X gives $4x - y = 9$. Row 2 gives $2x + 5y = -1$.

Test $(2, -1)$: $4(2) - (-1) = 8 + 1 = 9$ and $2(2) + 5(-1) = 4 - 5 = -1$. Both hold. **The system is $4x - y = 9$, $2x + 5y = -1$, and $(2, -1)$ is a solution.**

Problem 3: Determinants of 2 by 2 matrices

Use $ad-bc$.

- a. $3(4)-5(2)=12-10=2$
- b. $(-2)(-3)-6(1)=6-6=0$
- c. $4(2)-(-1)(3)=8+3=11$
- d. $\frac{1}{2}(8)-3(2)=4-6=-2$

(a) 2 (b) 0 (c) 11 (d) -2

Problem 4: Does the inverse exist?

- a. $6(3)-9(2)=0$: no inverse.
- b. $5(1)-2(3)=-1$: inverse exists.
- c. $(-4)(-2)-8(1)=0$: no inverse.
- d. $7(3)-1(0)=21$: inverse exists.

Inverses exist for (b) and (d) only.

Problem 5: Inverse with determinant 1

$\det = 3(2)-5(1)=1$. Swap the diagonal entries and negate the others:

2	-5
-1	3

The inverse has rows 2, -5 and -1, 3.

Precalculus Matrix Systems Worksheet Answer Key: problems 6-10

Problem 6: Inverses and the identity

- a. $\det = 8-6=2$, so $A^{-1} = \frac{1}{2}$ times the matrix with rows 2, -3 and -2, 4, which is:

1	$-\frac{3}{2}$
-1	2

Check: first row of A times first column of A^{-1} is $4(1)+3(-1)=1$; first row times second column is $4(-\frac{3}{2})+3(2)=0$; second row gives $2(1)+2(-1)=0$ and $2(-\frac{3}{2})+2(2)=1$.

- b. $\det = 20+6=26$, so $A^{-1} = \frac{1}{26}$ times rows 4, -2 and 3, 5:

$\frac{2}{13}$	$-\frac{1}{13}$
$\frac{3}{26}$	$\frac{5}{26}$

- c. $\det = 5-6=-1$, so A^{-1} has rows -5, 2 and 3, -1:

-5	2
3	-1

See the three inverses above.

Problem 7: Are these inverses?

AC : row 1 gives $2(2)+3(-1)=1$ and $2(-3)+3(2)=0$; row 2 gives $1(2)+2(-1)=0$ and $1(-3)+2(2)=1$. So $AC=I$.

CA : row 1 gives $2(2)-3(1)=1$ and $2(3)-3(2)=0$; row 2 gives $-1(2)+2(1)=0$ and $-1(3)+2(2)=1$. So $CA=I$.

Yes, $C=A^{-1}$.

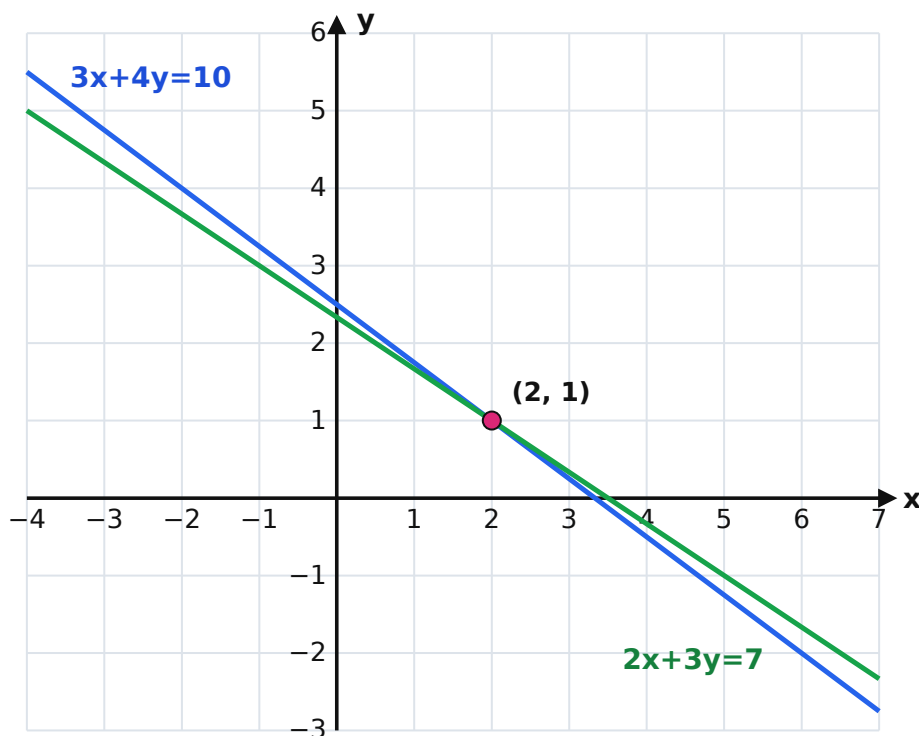
Problem 8: Solve with a given inverse

$X=A^{-1}B$ with $B=(7, 4)$. Then $x=3(7)-5(4)=1$ and $y=-1(7)+2(4)=1$.

Check: $2+5=7$ and $1+3=4$. $(1, 1)$

Problem 9: Solve a system with the inverse

$\det A=9-8=1$, so A^{-1} has rows 3, -4 and -2, 3. Then $x=3(10)-4(7)=2$ and $y=-2(10)+3(7)=1$.



The lines cross at $(2, 1)$.

(2, 1). Check: $6+4=10$ and $4+3=7$.

Problem 10: Determinant 11

$\det A = 5(1) - (-2)(3) = 11$. $A^{-1} = \frac{1}{11}$ times rows 1, 2 and -3 , 5.

$$x = \frac{1}{11}(1 \cdot 4 + 2 \cdot 9) = \frac{22}{11} = 2 \text{ and } y = \frac{1}{11}(-3 \cdot 4 + 5 \cdot 9) = \frac{33}{11} = 3.$$

(2, 3). Check: $10-6=4$ and $6+3=9$.

Precalculus Matrix Systems Worksheet Answer Key: problems 11-15

Problem 11: A fractional solution

$\det A = 8 - 6 = 2$. $A^{-1} = \frac{1}{2}$ times rows 2, -1 and -6 , 4.

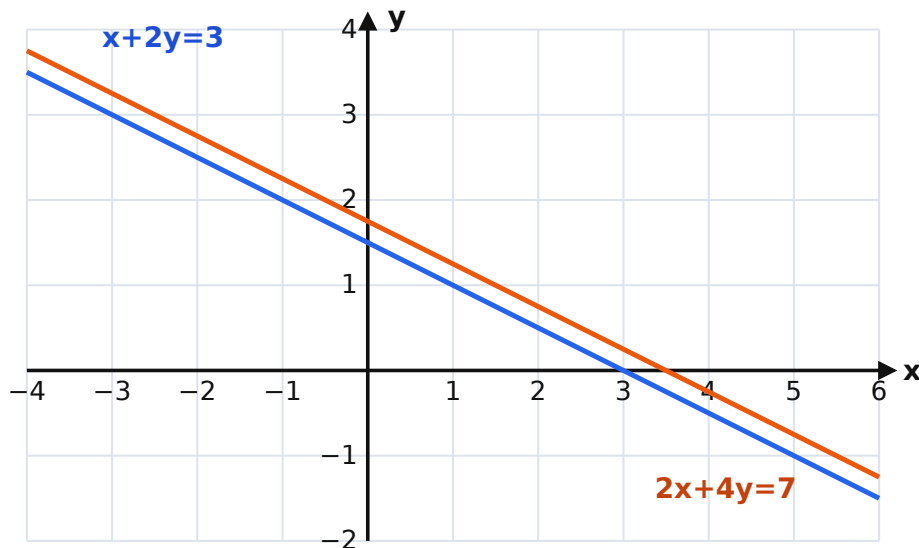
$$x = \frac{1}{2}(2 \cdot 3 - 1 \cdot 5) = \frac{1}{2} \text{ and } y = \frac{1}{2}(-6 \cdot 3 + 4 \cdot 5) = 1.$$

$(\frac{1}{2}, 1)$. Check: $2+1=3$ and $3+2=5$.

Problem 12: When the determinant is zero

Both systems have $\det A = 1(4) - 2(2) = 0$, so A^{-1} does not exist.

- The second equation is twice the first, so the lines coincide: infinitely many solutions.
- The left sides match but $2 \cdot 3 = 6 \neq 7$, so the lines are parallel: no solution.



Part (b): parallel lines.

(a) infinitely many solutions; (b) no solution

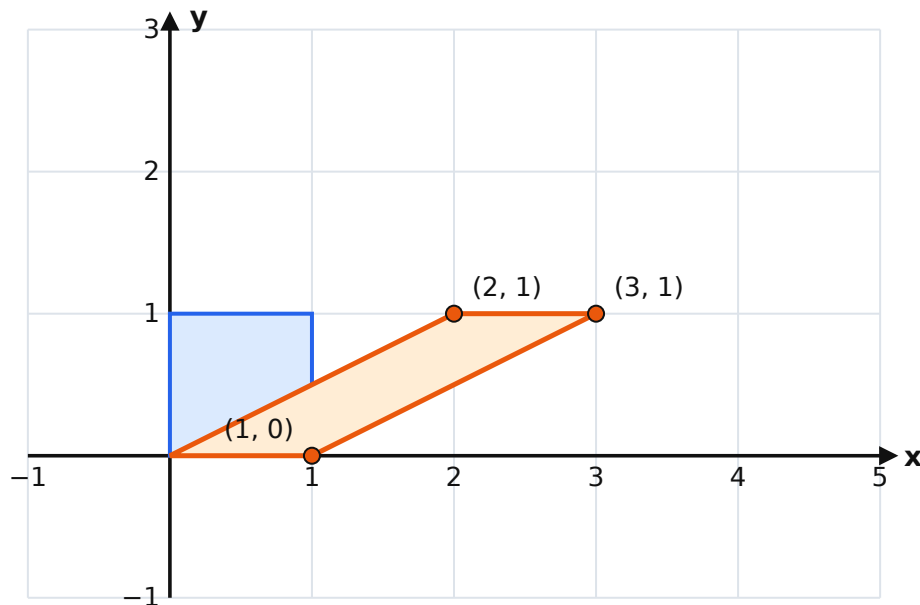
Problem 13: Apply a matrix to a point

- a. $(2 \cdot 4 + 0, 0 + 3(-1)) = (8, -3)$
- b. $(0 \cdot 2 - 1 \cdot 5, 1 \cdot 2 + 0) = (-5, 2)$
- c. $(0 - 1 \cdot 1, 1(-3) + 0) = (-1, -3)$

(a) $(8, -3)$ **(b)** $(-5, 2)$ **(c)** $(-1, -3)$

Problem 14: Shear the unit square

Columns of S are the images of $(1, 0)$ and $(0, 1)$, so $(1, 0) \rightarrow (1, 0)$ and $(0, 1) \rightarrow (2, 1)$. Also $(1, 1) \rightarrow (1+2, 0+1) = (3, 1)$.



Blue: original. Orange: image.

Images: $(0, 0)$, $(1, 0)$, $(3, 1)$, $(2, 1)$. The base and height are both 1, so the area is still $1 = \det S$.

Problem 15: Build the matrix from two images

The columns of the matrix are the images: first column $(0, 1)$, second column $(-1, 0)$. So the matrix has rows $0, -1$ and $1, 0$, a 90° counterclockwise rotation.

Image of $(3, 2)$: $(0 \cdot 3 - 1 \cdot 2, 1 \cdot 3 + 0 \cdot 2) = (-2, 3)$.

Rows $0, -1$ **and** $1, 0$; **image** $(-2, 3)$

Precalculus Matrix Systems Worksheet Answer Key: problems 16-20

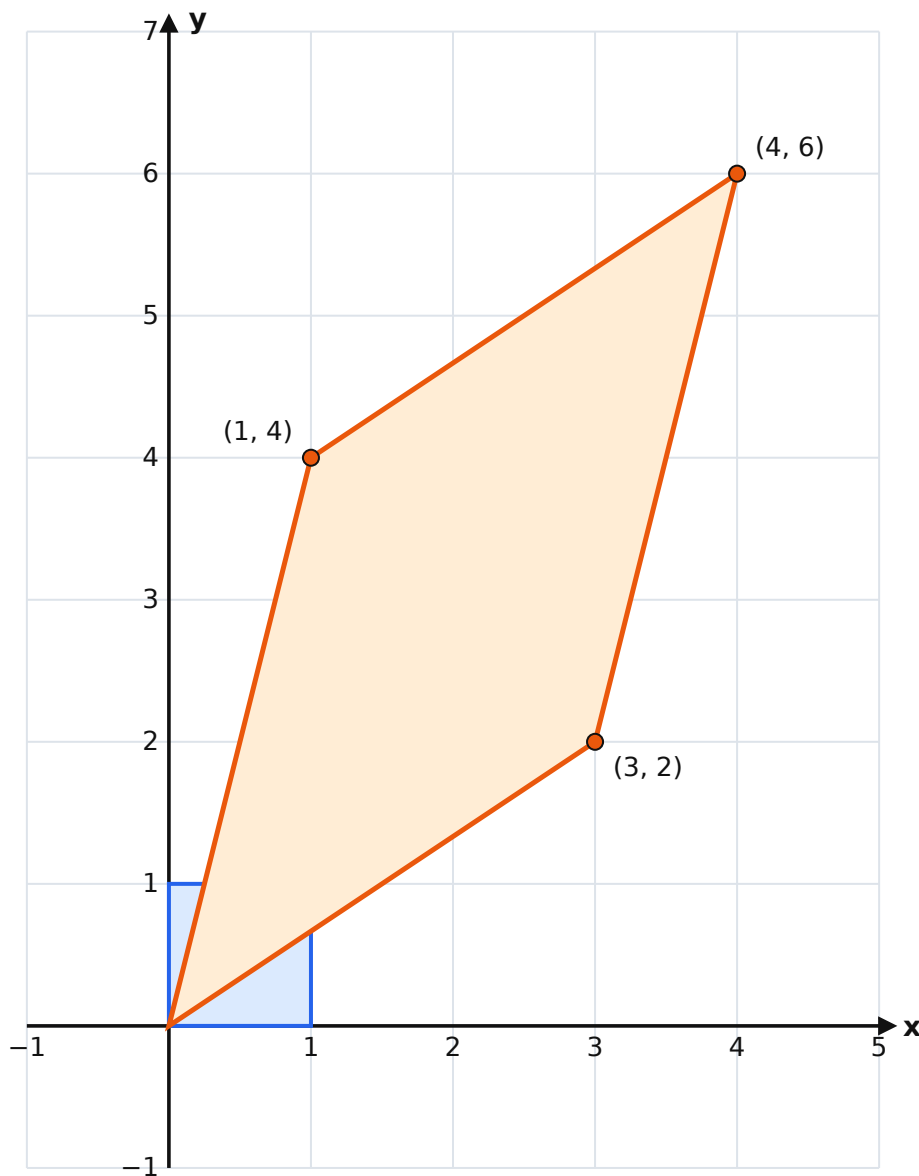
Problem 16: Reflections

- a. $(1 \cdot 5 + 0, 0 - 1 \cdot 4) = (5, -4)$.
- b. The x -coordinate stays and the y -coordinate changes sign: a reflection across the x -axis.
- c. $\det T = 1(-1) - 0 = -1$. The area is unchanged ($|-1| = 1$); the negative sign shows the orientation is flipped.

(a) $(5, -4)$ **(b)** reflection across the x -axis **(c)** -1

Problem 17: Area scale factor

(a) $\det A = 12 - 2 = 10$, so areas are multiplied by $|10| = 10$. The picture confirms it: the image of the unit square has area 10.



Orange: image of the unit square, area 10.

(b) $10 \times 6 = 60$. (c) $35 \div 10 = 3.5$.

(a) 10 (b) 60 (c) 3.5

Problem 18: Negative determinant

$\det B = 1(1) - 2(3) = -5$. The area scale factor is $|-5| = 5$, so the image has area $5 \times 4 = 20$. The negative sign means the triangle is flipped (reversed orientation).

Area 20; the image is a mirror-like flip.

Problem 19: School store prices

Let p be the pen price and n the notebook price. Then $3p + 2n = 7.25$ and $2p + 5n = 11.25$.
 $\det A = 15 - 4 = 11$; $A^{-1} = \frac{1}{11}$ times rows 5, -2 and -2 , 3 .

$$p = \frac{1}{11}(5 \cdot 7.25 - 2 \cdot 11.25) = \frac{13.75}{11} = 1.25 \text{ and } n = \frac{1}{11}(-2 \cdot 7.25 + 3 \cdot 11.25) = \frac{19.25}{11} = 1.75.$$

Check: $3 \cdot 1.25 + 2 \cdot 1.75 = 7.25$ and $2 \cdot 1.25 + 5 \cdot 1.75 = 11.25$. **Pen \$1.25, notebook \$1.75**

Problem 20: Farmers market

With a and b the prices: $4a + 3b = 19$, $3a + 2b = 13.5$. $\det A = 8 - 9 = -1$, so A^{-1} has rows -2 , 3 and 3 , -4 .

$$a = -2(19) + 3(13.5) = -38 + 40.5 = 2.5 \text{ and } b = 3(19) - 4(13.5) = 57 - 54 = 3.$$

Check: $10 + 9 = 19$ and $7.5 + 6 = 13.5$. **Apples \$2.50 per pound, pears \$3.00 per pound**

Precalculus Matrix Systems Worksheet Answer Key: problems 21-24

Problem 21: Bake sale with three items

With prices c , b , m : A has rows 4, 3, 2, 2, 5, 1 and 1, 2, 6; $B = (17, 15.5, 20.5)$.

Technology gives $\det A = 77 \neq 0$, so A^{-1} exists, and $X = A^{-1}B = (1.5, 2, 2.5)$.

Check day 1: $6 + 6 + 5 = 17$. Day 2: $3 + 10 + 2.5 = 15.5$. Day 3: $1.5 + 4 + 15 = 20.5$. **Cookie \$1.50, brownie \$2.00, muffin \$2.50**

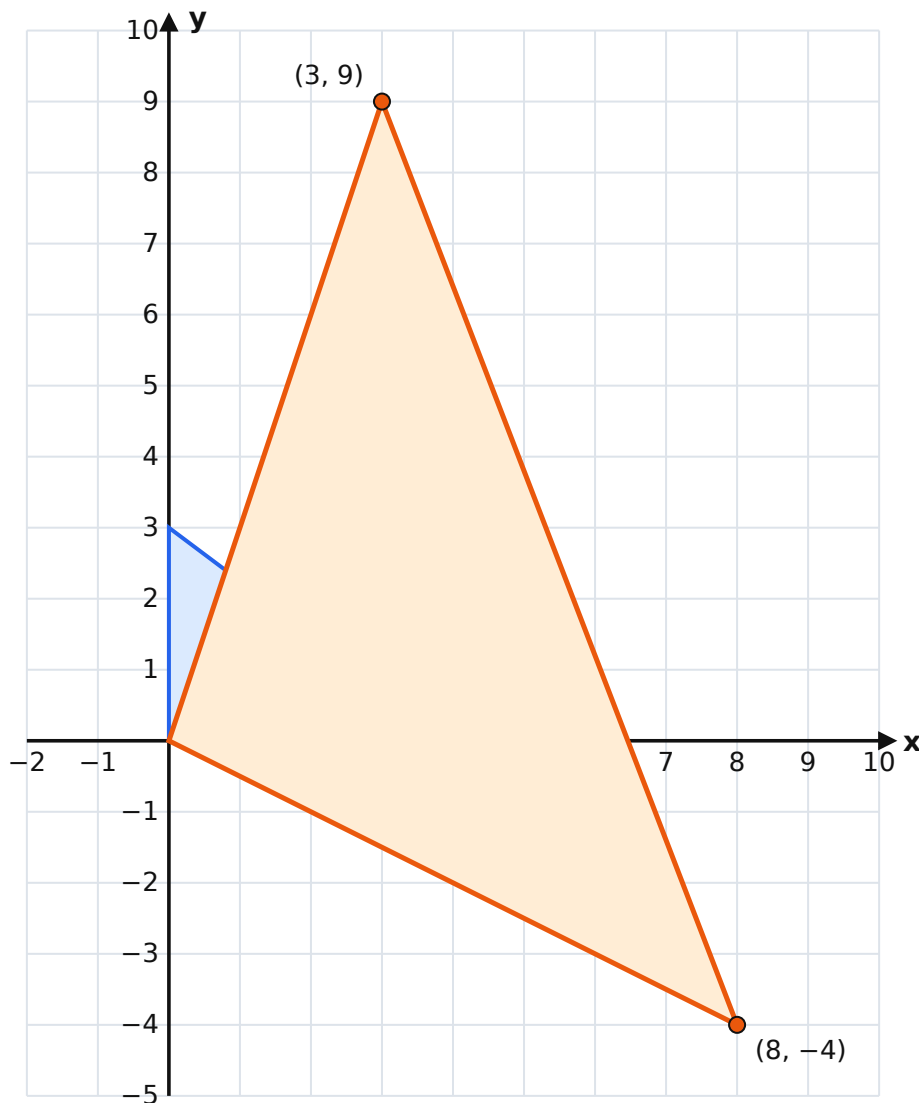
Problem 22: Scaling a game sprite

The matrix multiplies x by 3 and y by 2: the corner $(5, 4)$ goes to $(15, 8)$. New size: 15 by 8, area 120.

Old area 20; $\det = 6 - 0 = 6$; $6 \times 20 = 120$. **New size 15 by 8, area 120**

Problem 23: Mapping a triangular lot

Images: $(4, 0) \rightarrow (8, -4)$; $(0, 3) \rightarrow (0+3, 0+9) = (3, 9)$; $(0, 0) \rightarrow (0, 0)$.



Blue: original, area 6. Orange: image.

Way 1: $\det = 6+1=7$, original area $\frac{1}{2}(4)(3)=6$, image area $7 \times 6 = 42$.

Way 2 (shoelace): $\frac{1}{2}|8 \cdot 9 - (-4) \cdot 3| = \frac{1}{2}(84) = 42$.

Vertices $(0, 0)$, $(8, -4)$, $(3, 9)$; **area** 42

Problem 24: Undo a transformation

$\det H = 2 - 1 = 1$, so H^{-1} has rows 1, -1 and -1, 2. Then

$P = H^{-1}(11, 8) = (11 - 8, -11 + 16) = (3, 5)$.

Check: $H(3, 5) = (6+5, 3+5) = (11, 8)$. Since $|\det H| = 1$, areas keep their size. $P = (3, 5)$

How to use this precalculus matrix systems worksheet answer key

Read this precalculus matrix systems worksheet answer key after trying each problem yourself, and redo any problem you missed.

