

Precalculus · Solving Systems with Matrices · Worksheet

Common Core A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This precalculus matrix systems worksheet gives 24 problems on solving systems with matrices, ordered from easy to hard, with a separate answer key.

These 24 problems build the matrix toolkit step by step: writing a system as $AX = B$, computing determinants and inverses, solving systems with $X = A^{-1}B$, and using matrices as transformations of the plane (A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12).

The last six are word problems, including a three-item bake sale that needs technology. Every problem has a worked solution in the answer key.

Precalculus Common Core: A-REI.C.8 · A-REI.C.9 · N-VM.C.11 · N-VM.C.12

Precalculus Matrix Systems Worksheet: problems 1-5

Problem 1: Write a system as $AX = B$

Write the system $3x + y = 7$, $2x - y = 3$ as a matrix equation $AX = B$. Name the matrices A , X and B .

Problem 2: Read a matrix equation

Matrix A has rows 4, -1 and 2, 5. Write the equations of $AX = B$ when $B = (9, -1)$, then verify that $(2, -1)$ is a solution.

Problem 3: Determinants of 2 by 2 matrices

Find the determinant of each matrix. Each is written as its rows.

- a. rows 3, 5 and 2, 4
- b. rows -2, 6 and 1, -3
- c. rows 4, -1 and 3, 2
- d. rows $\frac{1}{2}$, 3 and 2, 8

Problem 4: Does the inverse exist?

Decide whether each matrix has an inverse. Explain using the determinant.

- a. rows 6, 9 and 2, 3
- b. rows 5, 2 and 3, 1
- c. rows -4 , 8 and 1, -2
- d. rows 7, 1 and 0, 3

Problem 5: Inverse with determinant 1

Find the inverse of the matrix with rows 3, 5 and 1, 2.

Precalculus Matrix Systems Worksheet: problems 6-10

Problem 6: Inverses and the identity

Find the inverse of each matrix (rows listed) and show that $AA^{-1} = I$ for part (a).

- a. rows 4, 3 and 2, 2
- b. rows 5, 2 and -3 , 4
- c. rows 1, 2 and 3, 5

Problem 7: Are these inverses?

Matrix A has rows 2, 3 and 1, 2. Matrix C has rows 2, -3 and -1 , 2. Multiply AC and CA to decide whether $C = A^{-1}$.

Problem 8: Solve with a given inverse

The inverse of the matrix with rows 2, 5 and 1, 3 has rows 3, -5 and -1 , 2. Use it to solve $2x+5y=7$, $x+3y=4$.

Problem 9: Solve a system with the inverse

Solve $3x+4y=10$, $2x+3y=7$ by writing $AX=B$ and using A^{-1} .

Problem 10: Determinant 11

Solve $5x-2y=4$, $3x+y=9$ with the inverse matrix.

Precalculus Matrix Systems Worksheet: problems 11-15

Problem 11: A fractional solution

Solve $4x+y=3$, $6x+2y=5$ with an inverse matrix.

Problem 12: When the determinant is zero

For each system, find $\det A$, then decide whether the system has one solution, none or infinitely many.

- a. $x+2y=3$, $2x+4y=6$
- b. $x+2y=3$, $2x+4y=7$

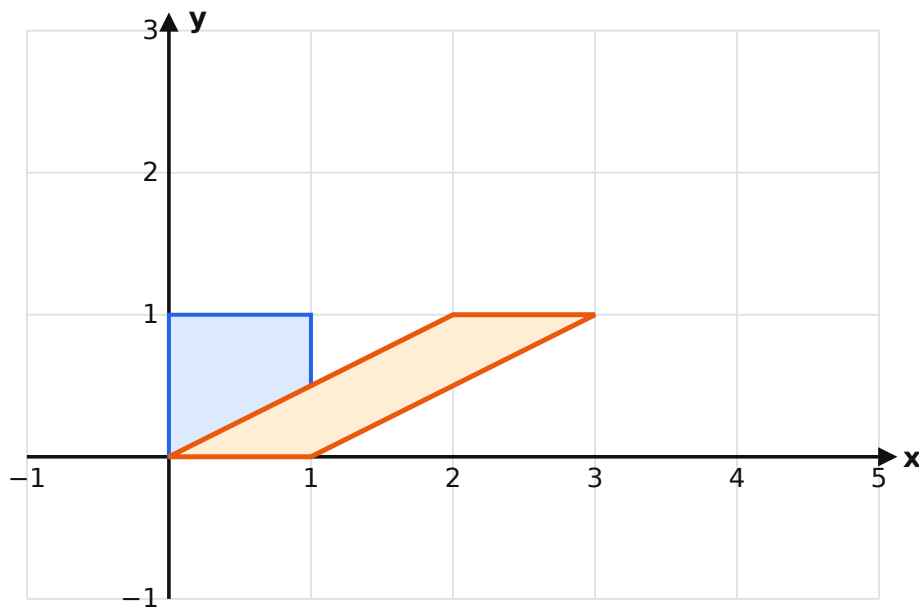
Problem 13: Apply a matrix to a point

Matrix P has rows 2, 0 and 0, 3. Matrix R has rows 0, -1 and 1, 0. Find

- a. P times $(4, -1)$
- b. R times $(2, 5)$
- c. R times $(-3, 1)$

Problem 14: Shear the unit square

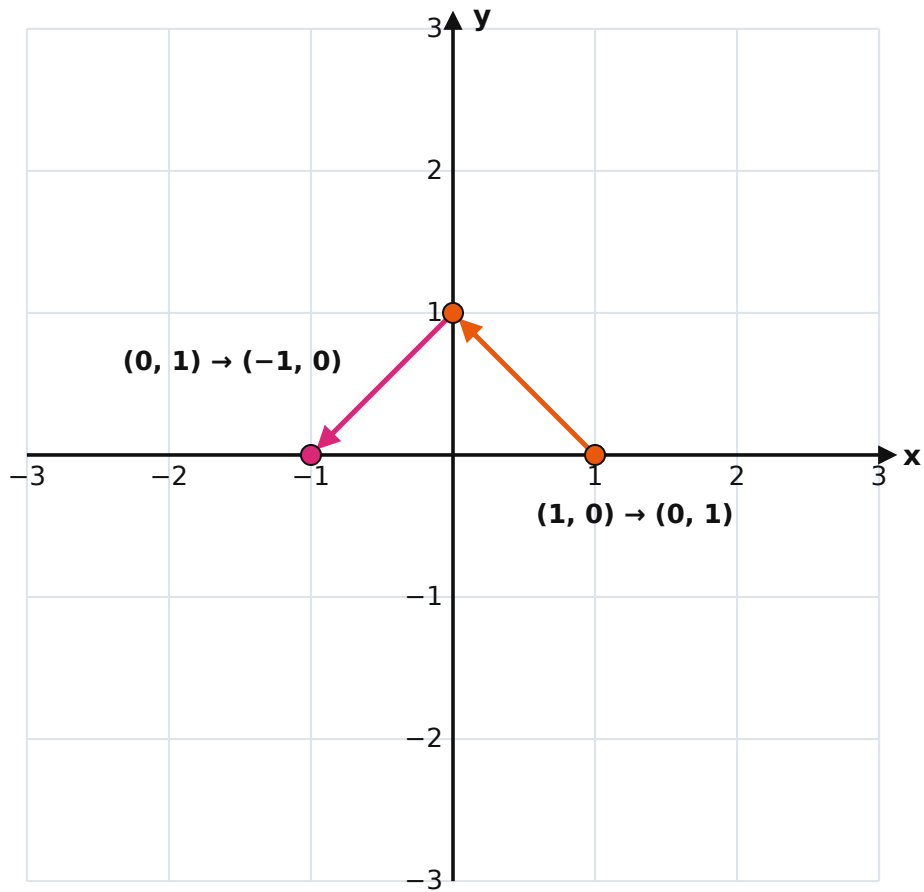
A shear matrix S has rows 1, 2 and 0, 1. Find the images of the vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$ of the unit square shown, and sketch the image.



Blue: the unit square. Only the blue square belongs to the problem.

Problem 15: Build the matrix from two images

A matrix sends $(1, 0)$ to $(0, 1)$ and $(0, 1)$ to $(-1, 0)$, as shown. Write the matrix, then find the image of $(3, 2)$.

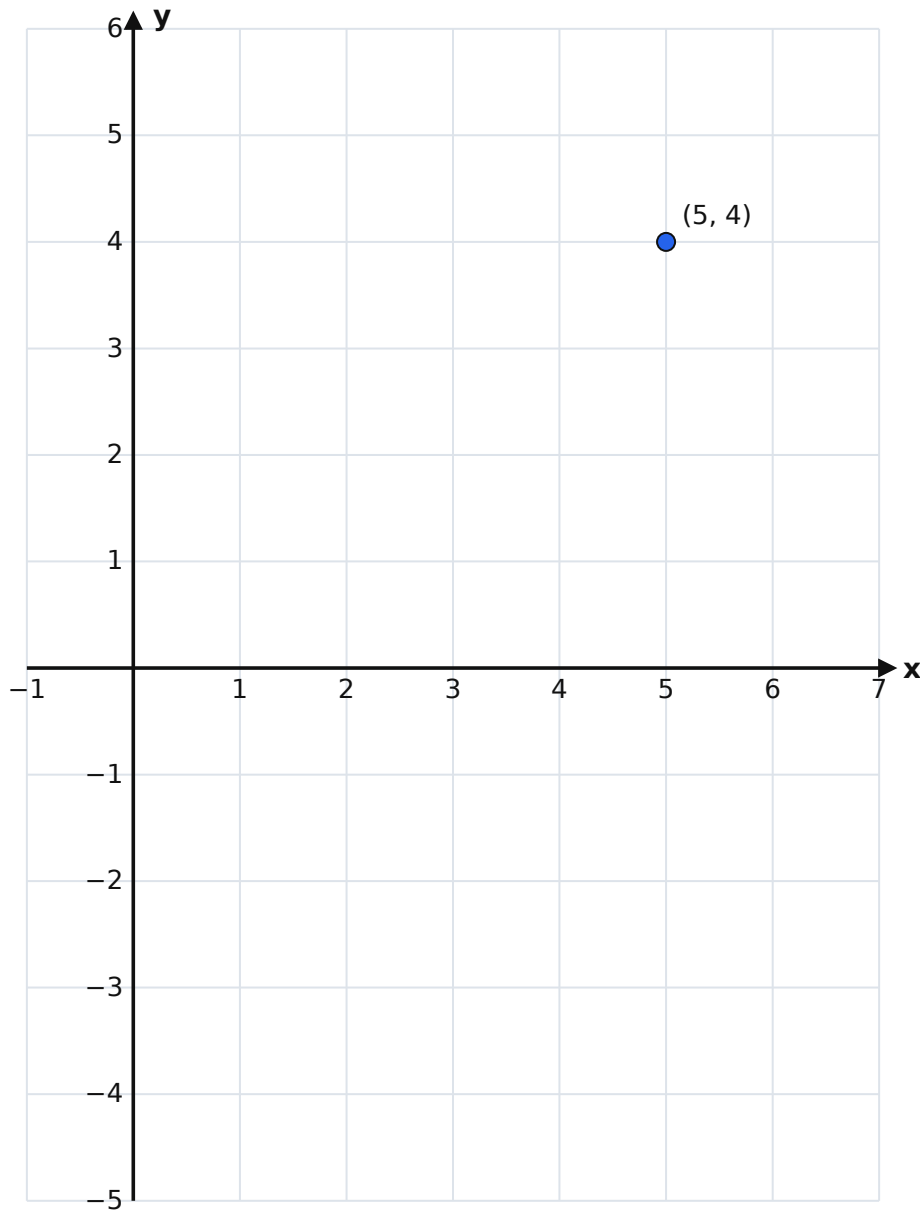


The arrows show where the two basis points land.

Precalculus Matrix Systems Worksheet: problems 16-20

Problem 16: Reflections

Matrix T has rows $1, 0$ and $0, -1$. The point $(5, 4)$ is plotted.



The point before the transformation.

- Find the image of $(5, 4)$.
- Describe the transformation.
- Find $\det T$ and explain what the sign tells you.

Problem 17: Area scale factor

Let A have rows $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

- Find the area scale factor.
- A triangle has area 6. What is the area of its image?
- A region maps to an image of area 35. What was its area?

Problem 18: Negative determinant

Matrix B has rows 1, 2 and 3, 1. A triangle of area 4 is transformed by B . Find the area of the image and state what the sign of $\det B$ means.

Problem 19: School store prices

At the school store, Ethan buys 3 pens and 2 notebooks for \$7.25. Maya buys 2 pens and 5 notebooks for \$11.25. Write a matrix equation and use the inverse to find the price of one pen and one notebook.

Problem 20: Farmers market

At a farmers market, 4 pounds of apples and 3 pounds of pears cost \$19.00. Also, 3 pounds of apples and 2 pounds of pears cost \$13.50. Use $X = A^{-1}B$ to find each price per pound.

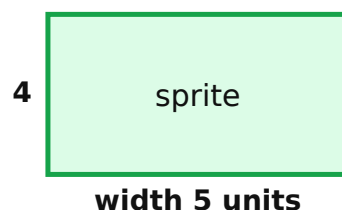
Precalculus Matrix Systems Worksheet: problems 21-24

Problem 21: Bake sale with three items

At a bake sale, Priya sells cookies, brownies and muffins at three prices. Day 1: 4 cookies, 3 brownies and 2 muffins earn \$17.00. Day 2: 2 cookies, 5 brownies and 1 muffin earn \$15.50. Day 3: 1 cookie, 2 brownies and 6 muffins earn \$20.50. Write $AX = B$ and use technology to find A^{-1} and each price.

Problem 22: Scaling a game sprite

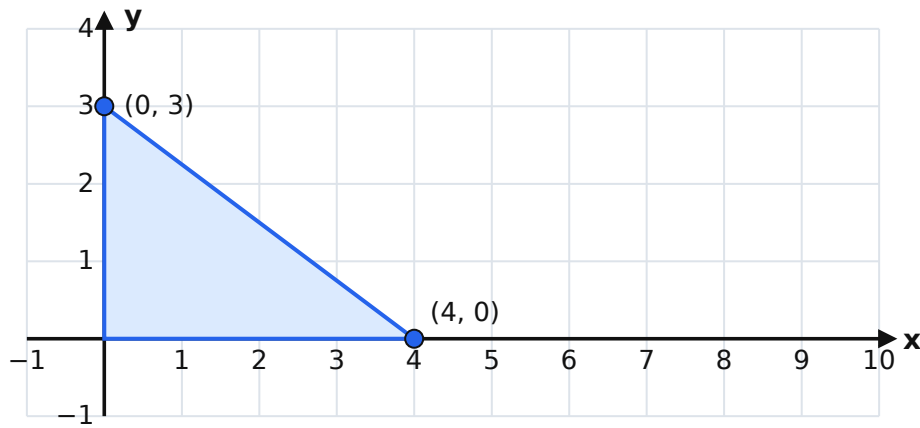
A game designer enlarges a rectangular sprite measuring 5 units by 4 units with the matrix with rows 3, 0 and 0, 2. Find the new dimensions and show that the area changes by $|\det|$.



The original sprite.

Problem 23: Mapping a triangular lot

A surveyor transforms a triangular lot with vertices $(0, 0)$, $(4, 0)$ and $(0, 3)$ using the matrix with rows 2, 1 and -1 , 3. Find the image vertices. Then find the area of the image two ways.



The original lot (units are hundreds of feet).

Problem 24: Undo a transformation

Matrix H has rows 2, 1 and 1, 1. A point P is sent to $(11, 8)$ by H . Find P , and explain why the area of the figure is unchanged.

How to use this precalculus matrix systems worksheet

Print this precalculus matrix systems worksheet as a PDF, solve every problem on paper, then compare with the answer key.

