

Precalculus · Matrices and Matrix Operations · Lesson

Common Core N-VM.C.6, N-VM.C.7, N-VM.C.8, N-VM.C.9, N-VM.C.10 · mathovore.fr

This precalculus matrices lesson explains matrices and matrix operations in one printable page, with definitions, a method and a worked example.

A matrix is a rectangular array of numbers in rows and columns that organizes data. Matrices of equal dimensions add entry by entry, any matrix can be scaled, and AB multiplies rows by columns. A 2×2 matrix has an inverse exactly when its determinant $ad-bc$ is not zero.

Precalculus Common Core: N-VM.C.6 · N-VM.C.7 · N-VM.C.8 · N-VM.C.9 · N-VM.C.10

Precalculus Matrices Lesson: key facts

- An $m \times n$ matrix has m rows and n columns; entry a_{ij} sits in row i , column j .
- Add or subtract matrices only when dimensions match; scalar multiplication scales every entry.
- AB exists when columns of A equal rows of B , and in general $AB \neq BA$.
- $A+O=A$ and $AI=IA=A$ for the zero matrix O and identity matrix I .
- For a 2×2 matrix, $\det A = ad-bc$, and A^{-1} exists exactly when $\det A \neq 0$.

What is a matrix and how does it organize data?

A **matrix** is a rectangular array of numbers arranged in rows and columns. A matrix with m rows and n columns has dimensions $m \times n$, and the number in row i and column j is its entry a_{ij} . Matrices store tables of data compactly. Below, a school store records pencils, erasers and notebooks sold on Monday and Tuesday, so row 2, column 3 says that 9 notebooks sold on Tuesday.

		Pencils	Erasers	Notebooks
S =	Mon	12	8	5
	Tue	15	6	9

2 × 3 matrix

Daily sales of three school store items. The highlighted entry is $a_{23} = 9$.

How do you add and scale matrices?

Matrices with the *same dimensions* add or subtract entry by entry. A **scalar multiple** kA multiplies every entry of A by the number k . For the matrix A below, $3A$ has entries 6, -3 , 0 and 15.

$$\begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & 6 \end{bmatrix}$$

$A \qquad B \qquad A + B$

Add matching entries: $2 + 3 = 5$. Both matrices must be 2×2 .

Zero and identity matrices

The **zero matrix** O has every entry 0, so $A + O = A$. The **identity matrix** I has 1s on the main diagonal and 0s elsewhere, so $AI = IA = A$.

How do you multiply matrices?

Row-by-column rule

The product AB exists only when the number of columns of A equals the number of rows of B . An $m \times n$ matrix times an $n \times p$ matrix is an $m \times p$ matrix. Entry c_{ij} comes from row i of A and column j of B : multiply matching entries, then add.

$$\begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ -10 & 5 \end{bmatrix}$$

$A \qquad B \qquad AB$

Row 1 of A times column 2 of B : $2(4) + (-1)(1) = 7$.

Matrix multiplication is **not commutative**: AB and BA usually differ, and one may exist when the other does not. It is distributive, so $A(B+C) = AB+AC$.

$$AB = \begin{bmatrix} 8 & 7 \\ -10 & 5 \end{bmatrix} \quad BA = \begin{bmatrix} 6 & 17 \\ -4 & 7 \end{bmatrix} \neq$$

The same two matrices multiplied in both orders give different products.

How do you find the determinant and inverse of a 2×2 matrix?

For a matrix with top row a, b and bottom row c, d , the **determinant** is $\det A = ad - bc$. The inverse A^{-1} satisfies $AA^{-1} = A^{-1}A = I$. It exists exactly when $\det A \neq 0$.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The inverse formula: swap a and d , negate b and c , divide by the determinant.

1. Compute $ad - bc$. If it is 0, there is no inverse.
2. Swap a and d , and change the signs of b and c .
3. Multiply every entry by $\frac{1}{\det A}$.
4. Check that $AA^{-1} = I$.

Example: for D with top row 4, 3 and bottom row 2, 2, $\det D = 4 \cdot 2 - 3 \cdot 2 = 2$, so the inverse exists.

$$D = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} D^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}$$

For D , the determinant is $4(2) - 3(2) = 2$, so each entry of the swapped matrix is halved.

$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\underset{D}{\quad} \quad \quad \underset{D^{-1}}{\quad} \quad \quad \underset{I}{\quad}$

Check: D times its inverse gives the identity matrix.

Matrices and Matrix Operations: frequently asked questions

What is a matrix used for?

A matrix organizes data, such as sales by day and item, so it can be added, scaled or combined with another table. Multiplying a sales matrix by a price matrix, for example, gives total revenue for each day in one step.

When can you multiply two matrices?

You can multiply AB only when the number of columns in A equals the number of rows in B . A 2×3 matrix times a 3×4 matrix is a 2×4 matrix, but a 2×3 times a 2×3 is not defined.

Why is matrix multiplication not commutative?

Row-by-column products depend on which matrix supplies the rows. Swapping the order changes which entries pair up, so AB and BA often differ. Counterexamples with two 2×2 matrices show this.

How do you know if a 2×2 matrix has an inverse?

Compute the determinant $ad - bc$. If it is not zero, the inverse exists and equals $\frac{1}{ad - bc}$ times the matrix with a and d swapped and b, c negated. If it is zero, there is no inverse.

What do the zero and identity matrices do?

The zero matrix O acts like 0: $A + O = A$ and $AO = O$. The identity matrix I acts like 1: $AI = IA = A$. Both must have dimensions that match A .

How to use this precalculus matrices lesson

Use this precalculus matrices lesson before the worksheet, then check what you learned with the quiz.

