

Precalculus · Laws of Sines and Cosines and Triangle Area · Worksheet Answer Key

Common Core G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · mathovore.fr

This precalculus law sines worksheet answer key solves all 25 problems of the laws of sines and cosines and triangle area worksheet step by step.

Full worked solutions for every problem. Rounded values come from unrounded intermediate results, so a last digit can differ slightly if you round earlier. Check the quadrant or case before accepting an angle.

Precalculus Common Core: G-SRT.D.9 · G-SRT.D.10 · G-SRT.D.11

Precalculus Law Sines Worksheet Answer Key: problems 1-5

Problem 1: Area from two sides and an angle

- a. $\frac{1}{2}(10)(7)\sin 30^\circ = 17.5$
- b. $\frac{1}{2}(12)(9)(1) = 54$
- c. $\frac{1}{2}(8)(5)\sin 150^\circ = 20 \cdot \frac{1}{2} = 10$
- d. $\frac{1}{2}(6)(6)\sin 60^\circ = 18 \cdot \frac{\sqrt{3}}{2} = 9\sqrt{3} \approx 15.59$

Problem 2: Area of a triangular sign

$$\text{Area} = \frac{1}{2}(9)(14)\sin 38^\circ = 63\sin 38^\circ \approx 38.79 \text{ in}^2.$$

Problem 3: Find the angle from the area

$\frac{1}{2}(10)(12)\sin C = 30$ gives $60\sin C = 30$, so $\sin C = \frac{1}{2}$. Therefore $C = 30^\circ$ **or** $C = 150^\circ$, and both angles give a valid triangle.

Problem 4: Law of Sines with two angles

$$C = 180^\circ - 40^\circ - 60^\circ = 80^\circ. \quad b = \frac{10\sin 60^\circ}{\sin 40^\circ} \approx 13.47 \text{ and } c = \frac{10\sin 80^\circ}{\sin 40^\circ} \approx 15.32.$$

Problem 5: Find the third angle and both sides

$$B = 180^\circ - 48^\circ - 71^\circ = 61^\circ. \quad a = \frac{20\sin 48^\circ}{\sin 71^\circ} \approx 15.72 \text{ and } b = \frac{20\sin 61^\circ}{\sin 71^\circ} \approx 18.50.$$

Precalculus Law Sines Worksheet Answer Key: problems 6-10

Problem 6: Law of Sines with the side between two angles

$$A = 180^\circ - 52^\circ - 67^\circ = 61^\circ. \quad b = \frac{25 \sin 52^\circ}{\sin 61^\circ} \approx 22.52 \text{ ft and } c = \frac{25 \sin 67^\circ}{\sin 61^\circ} \approx 26.31 \text{ ft.}$$

Problem 7: SSA with one solution

Since $a \geq b$, only one triangle exists. $\sin B = \frac{10 \sin 70^\circ}{15} \approx 0.6265$, so $B \approx 38.8^\circ$. Then $C \approx 71.2^\circ$ and $c = \frac{15 \sin 71.2^\circ}{\sin 70^\circ} \approx 15.11$.

Problem 8: How many triangles?

- a. $h = 10 \sin 40^\circ \approx 6.43$ and $5 < h$: **no triangle**
- b. $h = 10 \sin 30^\circ = 5 = a$: **one right triangle**
- c. $h = 5 < 7 < 10$: **two triangles**
- d. $a = 12 \geq b = 10$: **one triangle**

Problem 9: Solve both triangles

$\sin B = \frac{10 \sin 30^\circ}{7} = \frac{5}{7} \approx 0.7143$. Triangle 1: $B_1 \approx 45.6^\circ$, $C_1 \approx 104.4^\circ$, $c_1 = \frac{7 \sin 104.4^\circ}{\sin 30^\circ} \approx 13.56$. Triangle 2: $B_2 = 180^\circ - 45.6^\circ \approx 134.4^\circ$, $C_2 \approx 15.6^\circ$, $c_2 = \frac{7 \sin 15.6^\circ}{\sin 30^\circ} \approx 3.76$.

Problem 10: Another ambiguous case

$h = 24 \sin 42^\circ \approx 16.06$, and $16.06 < 18 < 24$, so two triangles exist. $\sin B = \frac{24 \sin 42^\circ}{18} \approx 0.8922$: $B_1 \approx 63.1^\circ$ gives $C_1 \approx 74.9^\circ$ and $c_1 \approx 25.97$; $B_2 \approx 116.9^\circ$ gives $C_2 \approx 21.1^\circ$ and $c_2 \approx 9.71$.

Precalculus Law Sines Worksheet Answer Key: problems 11-15

Problem 11: Law of Cosines: SAS

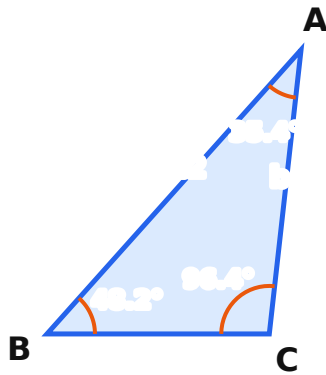
$$c^2 = 12^2 + 7^2 - 2(12)(7) \cos 55^\circ = 193 - 168 \cos 55^\circ \approx 96.64, \text{ so } c \approx 9.83.$$

Problem 12: Law of Cosines with an obtuse angle

$$c^2 = 36 + 121 - 2(6)(11) \cos 120^\circ = 157 - 132 \left(-\frac{1}{2}\right) = 223, \text{ so } c = \sqrt{223} \approx 14.93.$$

Problem 13: Law of Cosines: SSS

The largest angle is opposite 12: $\cos C = \frac{7^2 + 9^2 - 12^2}{2(7)(9)} = \frac{-14}{126} \approx -0.1111$, so $C \approx 96.4^\circ$. Then $\sin A = \frac{7 \sin 96.4^\circ}{12}$ gives $A \approx 35.4^\circ$ and $B \approx 48.2^\circ$.



Angles add to 180 degrees.

Problem 14: Law of Cosines and the Pythagorean theorem

- $\cos C = \frac{25+144-169}{2(5)(12)} = 0$, so $C = 90^\circ$
- $\cos C = \frac{64+225-289}{2(8)(15)} = 0$, so $C = 90^\circ$
- When $C = 90^\circ$, $\cos C = 0$ and the Law of Cosines becomes $c^2 = a^2 + b^2$. **The Pythagorean theorem is a special case.**

Problem 15: Choose the law

- Law of Sines:** two angles and a side (AAS).
- Law of Cosines:** two sides and the included angle (SAS).
- Law of Cosines:** three sides (SSS).
- Law of Sines:** a side, its opposite angle and another side (SSA); check for two solutions.

Precalculus Law Sines Worksheet Answer Key: problems 16-20

Problem 16: Heron's formula

$$s = \frac{7+8+9}{2} = 12. \text{ Area} = \sqrt{12(5)(4)(3)} = \sqrt{720} = 12\sqrt{5} \approx 26.83.$$

Problem 17: Heron's formula on a flat triangle

$$s = \frac{9+10+17}{2} = 18. \text{ Area} = \sqrt{18(9)(8)(1)} = \sqrt{1296} = \mathbf{36} \text{ square units.}$$

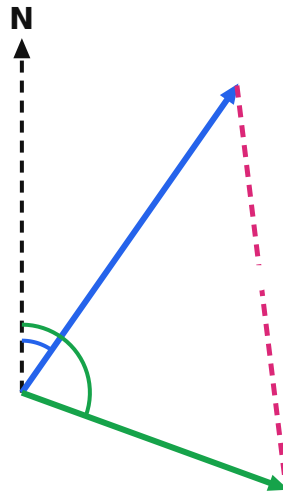
Problem 18: Two ways to find an area

Heron: $s = 12$ and $\sqrt{12(6)(4)(2)} = \sqrt{576} = \mathbf{24}$. The angle between 6 and 8 satisfies $\cos C = \frac{36+64-100}{96} = 0$, so $C = 90^\circ$ and $\frac{1}{2}(6)(8)\sin 90^\circ = \mathbf{24}$. The two methods agree.

Problem 19: Two boats leave a dock

After 2 hours the boats are 48 mi and 36 mi from the dock. The angle between the paths is

$$110^\circ - 35^\circ = 75^\circ. d^2 = 48^2 + 36^2 - 2(48)(36)\cos 75^\circ \approx 2705.52, \text{ so } d \approx 52.01 \text{ mi.}$$



The distance between the boats.

Problem 20: Width of a lake

$$AB^2 = 410^2 + 560^2 - 2(410)(560)\cos 72^\circ \approx 339,799, \text{ so } AB \approx 583 \text{ ft.}$$

Precalculus Law Sines Worksheet Answer Key: problems 21-25

Problem 21: A lot with two possible shapes

$$h = 90\sin 28^\circ \approx 42.25 < 50 < 90, \text{ so two lots fit. } \sin B = \frac{90\sin 28^\circ}{50} \approx 0.8450: B_1 \approx 57.7^\circ, C_1 \approx 94.3^\circ, AB \approx 106.2 \text{ ft}; B_2 \approx 122.3^\circ, C_2 \approx 29.7^\circ, AB \approx 52.7 \text{ ft.}$$

Problem 22: Cost of sod

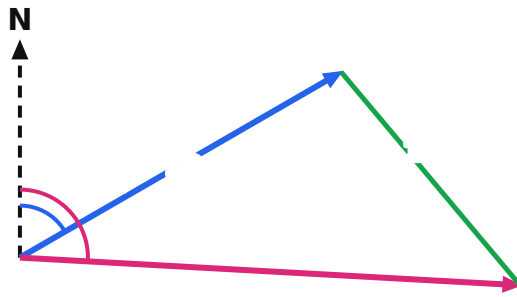
$$\text{Area} = \frac{1}{2}(120)(150)\sin 65^\circ = 9000\sin 65^\circ \approx 8,157 \text{ ft}^2. \text{ Cost: } 8,157 \times \$0.65 \approx 5,302, \text{ so about } \mathbf{\$5,302}.$$

Problem 23: Fertilizing a field

$$s = \frac{300+400+500}{2} = 600, \text{ Area} = \sqrt{600(300)(200)(100)} = \sqrt{3,600,000,000} = 60,000 \text{ ft}^2. \text{ Bags: } 60,000 \div 5,000 = \mathbf{12}. \text{ Cost: } 12 \times \$18 = \$216.$$

Problem 24: Plane flight on two bearings

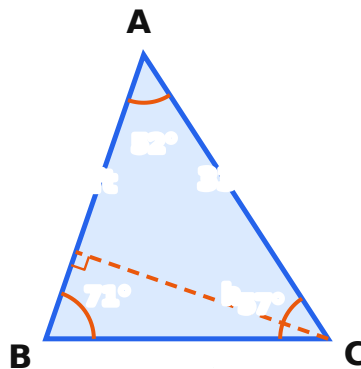
The turn changes the heading by $140^\circ - 60^\circ = 80^\circ$, so the interior angle at the turn is $180^\circ - 80^\circ = 100^\circ$. $d^2 = 200^2 + 150^2 - 2(200)(150)\cos 100^\circ \approx 72,919$, so $d \approx 270.0$ mi. The angle at the airport satisfies $\sin A = \frac{150\sin 100^\circ}{270.0}$, so $A \approx 33.2^\circ$. The plane lies on bearing $60^\circ + 33.2^\circ$, about 93.2° (the bearing back is 273.2°).



The two legs and the direct path.

Problem 25: Distance to a sailboat

The angle at the boat is $180^\circ - 52^\circ - 71^\circ = 57^\circ$. Distance from Mei: $\frac{300 \sin 71^\circ}{\sin 57^\circ} \approx 338.2$ ft. Shortest distance to the shore: $h = 338.2 \sin 52^\circ \approx 266.5$ ft.



The altitude from the boat is the shortest distance to shore.

How to use this precalculus law sines worksheet answer key

Read this precalculus law sines worksheet answer key after trying each problem yourself, and redo any problem you missed.

