

Precalculus · Laws of Sines and Cosines and Triangle Area · Lesson

Common Core G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · mathovore.fr

This precalculus law sines cosines lesson explains laws of sines and cosines and triangle area in one printable page, with definitions, a method and a worked example.

The area of a triangle is half the product of two sides and the sine of the angle between them. The Law of Sines, $a/\sin A = b/\sin B = c/\sin C$, and the Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, let you solve any triangle, and Heron's formula finds the area from three sides.

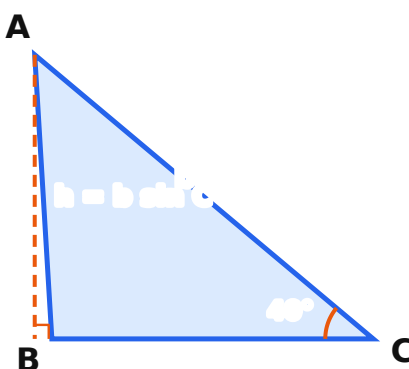
Precalculus Common Core: G-SRT.D.9 · G-SRT.D.10 · G-SRT.D.11

Precalculus Law Sines Cosines Lesson: key facts

- Standards G-SRT.D.9, G-SRT.D.10 and G-SRT.D.11: triangle area with sine, the Law of Sines and the Law of Cosines in applied problems.
- Area: $\frac{1}{2}ab\sin C$; Heron: $\sqrt{s(s-a)(s-b)(s-c)}$ with $s = \frac{a+b+c}{2}$.
- Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$; use it for AAS, ASA and SSA.
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$; use it for SAS and SSS.
- SSA can give 0, 1 or 2 triangles, depending on how a compares with $b\sin A$ and b .

How do you find a triangle's area with sine?

Drop an altitude from one vertex. In the figure the height is $h = b\sin C$, so the area $\frac{1}{2}ab\sin C$ needs only two sides and the angle between them. For $a=8$, $b=11$ and $C=40^\circ$, the area is $\frac{1}{2}(8)(11)\sin 40^\circ \approx 28.28$ square units.



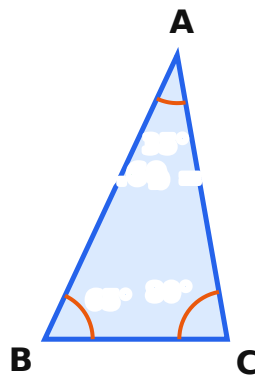
What is the Law of Sines?

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use it when you know two angles and a side (AAS or ASA), or two sides and an angle opposite one of them (SSA). If $A = 35^\circ$, $B = 65^\circ$ and $a = 12$, then $C = 80^\circ$ and $b = \frac{12 \sin 65^\circ}{\sin 35^\circ} \approx 18.96$.

Always pair each side with the angle opposite it. Find the third angle first with $A + B + C = 180^\circ$, because it gives you a complete ratio to work with.



AAS: both remaining sides follow from the ratio.

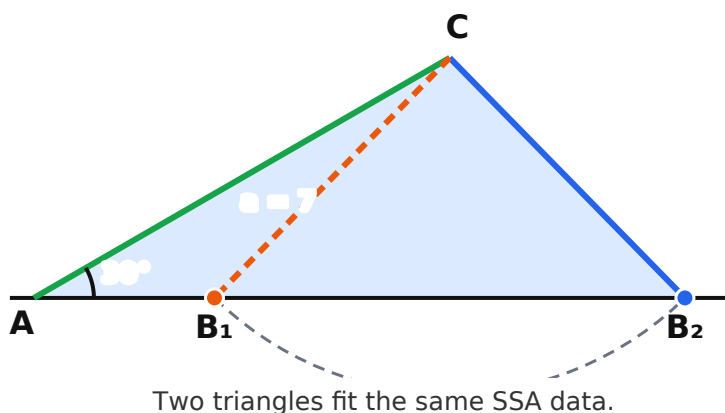
What is the ambiguous case?

With SSA the given side may swing to two positions. Let $h = b \sin A$ for acute angle A .

Condition	Triangles
$a < h$	0
$a = h$	1 (right triangle)
$h < a < b$	2
$a \geq b$	1

For $A = 30^\circ$, $a = 7$ and $b = 10$, $h = 5$ and $5 < 7 < 10$, so $B \approx 45.6^\circ$ or $B \approx 134.4^\circ$.

When the second angle is $180^\circ - B$, check that it fits with A inside a total of 180° . Always test for a second solution before you stop.

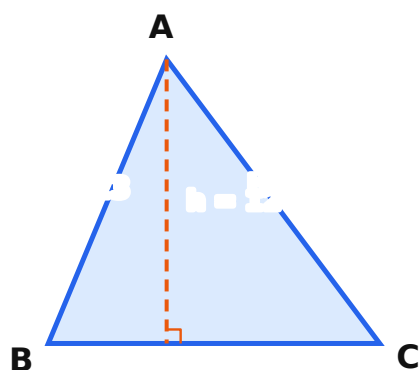


What are the Law of Cosines and Heron's formula?

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Use it for SAS or SSS. With $a=9$, $b=14$ and $C=38^\circ$, $c \approx 8.86$. When all three sides are known, **Heron's formula** gives the area: with $s = \frac{a+b+c}{2}$, $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$. For sides 13, 14 and 15, $s=21$ and the area is $\sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84$.

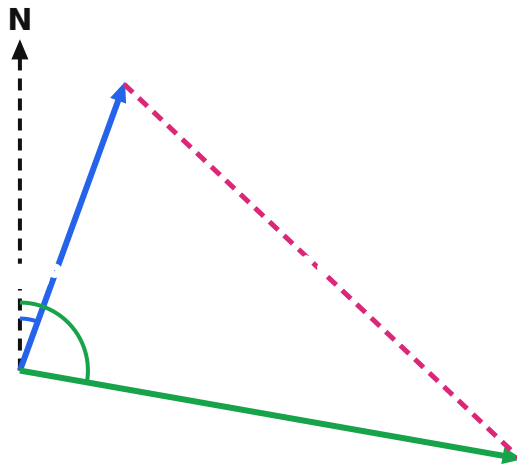


Heron's formula gives 84, matching half of 14 times 12.

How do bearings work in navigation?

A **bearing** is a clockwise angle from north, written with three digits. Two ships leave a port, one 30 miles on bearing 020° and one 50 miles on bearing 100° . The angle between the paths is $100^\circ - 20^\circ = 80^\circ$, so $d^2 = 30^2 + 50^2 - 2(30)(50)\cos 80^\circ$ and $d \approx 53.66$ miles.

Draw a north line at each starting point and mark every bearing clockwise from it before applying a law.



Bearings turn clockwise from north.

Laws of Sines and Cosines and Triangle Area: frequently asked questions

How do I decide between the Law of Sines and the Law of Cosines?

Use the Law of Cosines when you know two sides and the included angle (SAS) or all three sides (SSS). Use the Law of Sines when you know a side and its opposite angle plus one more angle or side.

Why does the SSA case sometimes give two triangles?

The side opposite the given angle can swing to two positions on the base line when it is longer than the height but shorter than the adjacent side. Sine gives both an acute angle and its supplement as solutions.

How can I tell whether the second angle from the Law of Sines is valid?

Take the supplement $180^\circ - B$ and check that it plus the given angle is less than 180° . If it is, a second triangle exists.

What is s in Heron's formula?

The letter s is the semiperimeter, half the perimeter: $s = \frac{a+b+c}{2}$. For sides 7, 8 and 9, $s = 12$.

How do bearings differ from ordinary angles?

A bearing is measured clockwise from north with three digits, such as 035° , while ordinary angles are measured counterclockwise from the positive x-axis. Subtract two bearings to find

the angle between two paths.

How to use this precalculus law sines cosines lesson

Use this precalculus law sines cosines lesson before the worksheet, then check what you learned with the quiz.

