

Precalculus · Inverse Trigonometric Functions and Trigonometric Equations · Worksheet Answer Key

Common Core F-TF.B.6, F-TF.B.7 · mathovore.fr

This precalculus inverse trig worksheet answer key solves all 24 problems of the inverse trigonometric functions and trigonometric equations worksheet step by step.

Each solution names the restricted range, the reference angle, and the quadrant reasoning. If your answer differs, check whether it lies in the correct interval and whether your calculator was in the right mode.

Precalculus Common Core: F-TF.B.6 · F-TF.B.7

Precalculus Inverse Trig Worksheet Answer Key: problems 1-5

Problem 1: Evaluate arcsin

The answer must lie in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

- a. $\frac{\pi}{6}$
- b. $-\frac{\pi}{6}$
- c. $\frac{\pi}{3}$
- d. $-\frac{\pi}{2}$

Problem 2: Evaluate arccos

The answer must lie in $[0, \pi]$.

- a. $\frac{\pi}{4}$
- b. $\frac{2\pi}{3}$ (cosine is negative in Quadrant II)
- c. $\frac{\pi}{2}$
- d. π

Problem 3: Evaluate arctan

The answer must lie in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

- a. $\frac{\pi}{4}$
- b. $-\frac{\pi}{4}$
- c. $\frac{\pi}{3}$

d. $-\frac{\pi}{6}$

Problem 4: Graph of arctan

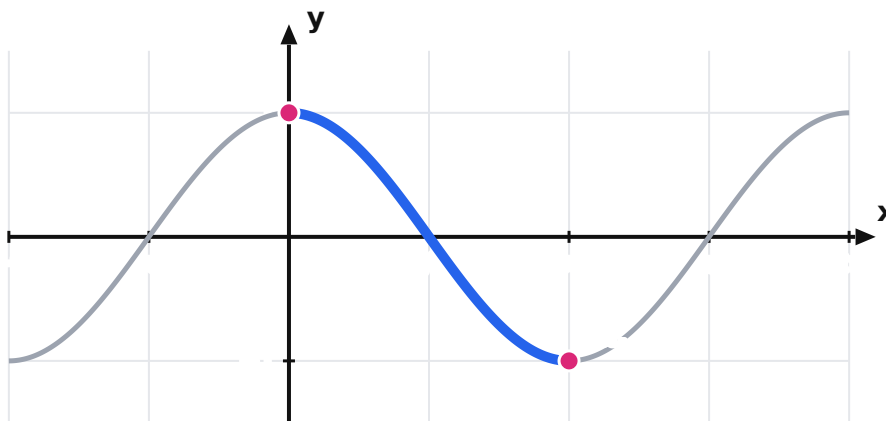
Every real number can be an input, and the curve stays between the dashed lines at $y = \pm \frac{\pi}{2}$.

Domain: all real numbers. **Range:** $(-\frac{\pi}{2}, \frac{\pi}{2})$. **Asymptotes:** $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$. $\tan^{-1}(0) = 0$.

Problem 5: Restricted cosine

The thick piece is $[0, \pi]$.

- a. $\frac{\pi}{3}$ (already in $[0, \pi]$)
- b. $\cos(-\frac{\pi}{3}) = \frac{1}{2}$, and $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$
- c. $\cos\frac{4\pi}{3} = -\frac{1}{2}$, and $\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$



Endpoints of the thick piece.

Precalculus Inverse Trig Worksheet Answer Key: problems 6-10

Problem 6: Compositions that cancel

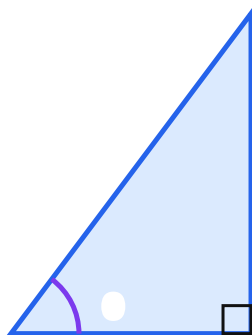
- a. **0.3**
- b. **-0.8**
- c. **25** (every real number is allowed for $\tan^{-1}$)
- d. **undefined**, because 1.2 is not in $[-1, 1]$

Problem 7: Arcsine of a sine

- a. $\frac{\pi}{4}$ (already in the range)
- b. $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$, so $\frac{\pi}{4}$
- c. $\sin\frac{7\pi}{6} = -\frac{1}{2}$, so $-\frac{\pi}{6}$
- d. $\sin(-\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}$, so $-\frac{\pi}{3}$

Problem 8: Triangle compositions

- a. The missing leg is $\sqrt{5^2 - 3^2} = 4$, so $\sin \theta = \frac{4}{5}$
- b. With opposite 5 and hypotenuse 13 the adjacent side is 12, so $\frac{12}{13}$
- c. Adjacent 4, hypotenuse 5, opposite 3: $\frac{3}{4}$
- d. The angle is in Quadrant IV where cosine is positive: opposite -3 , adjacent 4, so $-\frac{3}{4}$



Completed triangle for part (a).

Problem 9: Calculator practice

- a. **0.36**
- b. **1.37**
- c. **1.33**
- d. **2.28**

Problem 10: Solve a sine equation

The reference angle is $\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$. Sine is positive in Quadrants I and II, so the second solution is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

$$x = \frac{\pi}{3} \text{ or } x = \frac{2\pi}{3}$$

Precalculus Inverse Trig Worksheet Answer Key: problems 11-15

Problem 11: Solve a cosine equation

The reference angle is $\frac{\pi}{4}$. Cosine is negative in Quadrants II and III: $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$ and $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$.

$$x = \frac{3\pi}{4} \text{ or } x = \frac{5\pi}{4}$$

Problem 12: Solve a tangent equation

$\tan^{-1}(-1) = -\frac{\pi}{4}$. Tangent repeats every π , so add π and 2π to reach the interval: $-\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

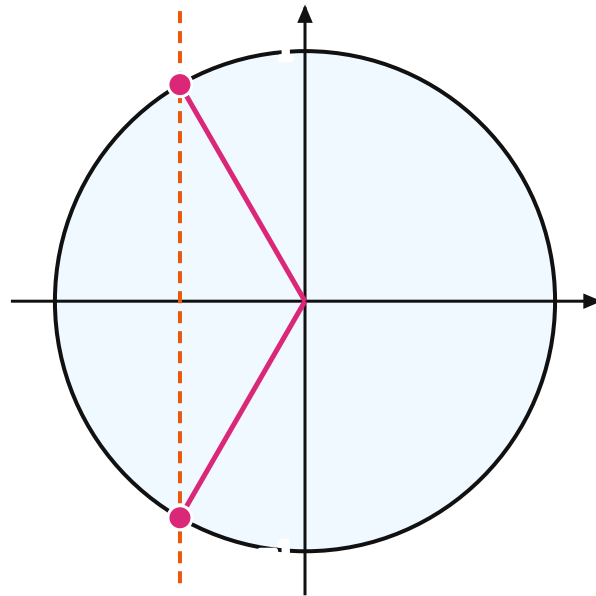
and $-\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$.

$$x = \frac{3\pi}{4} \text{ or } x = \frac{7\pi}{4}$$

Problem 13: Equation with a unit circle

Subtract and divide: $\cos x = -\frac{1}{2}$. The line $x = -\frac{1}{2}$ meets the circle in Quadrants II and III.
 $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ and the other angle is $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$.

$$x = \frac{2\pi}{3} \text{ or } x = \frac{4\pi}{3}$$



The two solution angles.

Problem 14: Sine equation by calculator

$\sin x = 0.4$. $x = \sin^{-1}(0.4) \approx 0.41$. The second solution is $\pi - 0.4115 \approx 2.73$.

$$x \approx 0.41 \text{ or } x \approx 2.73$$

Problem 15: Cosine equation by calculator

$x = \cos^{-1}(0.3) \approx 1.27$. Cosine is also positive in Quadrant IV: $2\pi - 1.2661 \approx 5.02$.

$$x \approx 1.27 \text{ or } x \approx 5.02$$

Precalculus Inverse Trig Worksheet Answer Key: problems 16-20

Problem 16: General solutions

$$\text{a. } x = \frac{\pi}{6} + 2\pi k \text{ or } x = \frac{5\pi}{6} + 2\pi k$$

b. $x = \frac{\pi}{2} + \pi k$ (both $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are included)

c. $x = \frac{\pi}{3} + \pi k$

Problem 17: Quadratic in sine

Factor: $(2\sin x + 1)(\sin x - 1) = 0$. If $\sin x = 1$ then $x = \frac{\pi}{2}$. If $\sin x = -\frac{1}{2}$ then $x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$.

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Problem 18: Multiple-angle equation

Let $u = 2x$, so $0 \leq u < 4\pi$. $\sin u = \frac{\sqrt{2}}{2}$ gives $u = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$. Divide by 2.

$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$$

Problem 19: Wheelchair ramp

$$\sin \theta = \frac{1}{12}, \text{ so } \theta = \sin^{-1}\left(\frac{1}{12}\right) \approx 0.0834 \text{ rad.}$$

$$\theta \approx 4.78^\circ$$

Problem 20: Ladder angle

a. $\cos \theta = \frac{4}{16} = 0.25$, so $\theta = \cos^{-1}(0.25) \approx 75.5^\circ$

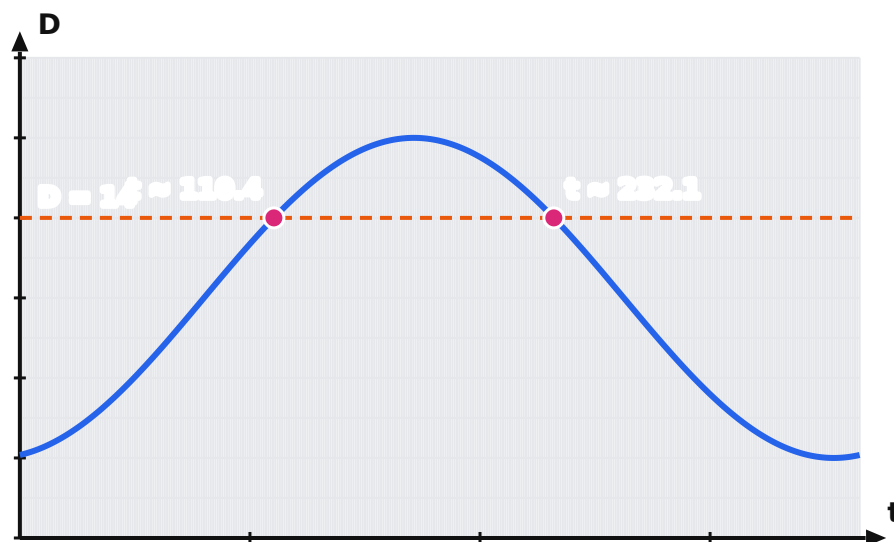
b. $16 \cos 70^\circ \approx 5.47$, so about **5.5 ft**

Precalculus Inverse Trig Worksheet Answer Key: problems 21-24

Problem 21: Daylight hours

$$4 \sin\left(\frac{2\pi}{365}(t-80)\right) = 2, \text{ so } \sin u = \frac{1}{2} \text{ with } u = \frac{2\pi}{365}(t-80). \text{ Then } u = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}. t-80 = \frac{365}{12} \approx 30.4 \text{ gives } t \approx 110.4; t-80 = \frac{5 \cdot 365}{12} \approx 152.1 \text{ gives } t \approx 232.1.$$

About day 110 (April 20) to day 232 (August 20) has more than 14 hours of daylight.



Daylight curve with the line $D = 14$.

Problem 22: Spring height

Height 23: $\cos \frac{2\pi t}{3} = \frac{1}{2}$, so $\frac{2\pi t}{3} = \frac{\pi}{3}$ or $\frac{5\pi}{3}$, giving $t = 0.5$ and $t = 2.5$.

Height 16: $\cos \frac{2\pi t}{3} = -\frac{2}{3}$. $\cos^{-1}\left(-\frac{2}{3}\right) \approx 2.3005$, so $t = \frac{3(2.3005)}{2\pi} \approx 1.10$; the other angle is $2\pi - 2.3005$, giving $t \approx 1.90$.

23 in at $t = 0.5$ and $t = 2.5$ s; 16 in at $t \approx 1.10$ and $t \approx 1.90$ s

Problem 23: Household voltage

$\sin(120\pi t) = \frac{100}{170} \approx 0.5882$. $\sin^{-1}(0.5882) \approx 0.6288$. So $t = \frac{0.6288}{120\pi} \approx 0.001668$ s and $t = \frac{\pi - 0.6288}{120\pi} \approx 0.006665$ s.

$t \approx 1.67$ ms and $t \approx 6.67$ ms; one cycle is $\frac{2\pi}{120\pi} = \frac{1}{60}$ s, about 16.67 ms

Problem 24: Sun angle

a. $\tan \theta = \frac{40}{55}$, so $\theta = \tan^{-1}\left(\frac{40}{55}\right) \approx 36.0^\circ$

b. $\text{shadow} = \frac{40}{\tan 60^\circ} = \frac{40}{\sqrt{3}} \approx 23.1$ ft

How to use this precalculus inverse trig worksheet answer key

Read this precalculus inverse trig worksheet answer key after trying each problem yourself, and redo any problem you missed.

