

Precalculus · Functions: Domain, Range, Composition and Inverses · Lesson

Common Core F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d · mathovore.fr

This precalculus functions review lesson explains functions: domain, range, composition and inverses in one printable page, with definitions, a method and a worked example.

A function's domain is the set of allowed inputs, a composition $f(g(x))$ feeds the output of g into f , and an inverse function undoes f by swapping inputs and outputs. Only one-to-one functions have inverses, so a function such as x squared needs a restricted domain like x at least 0 first.

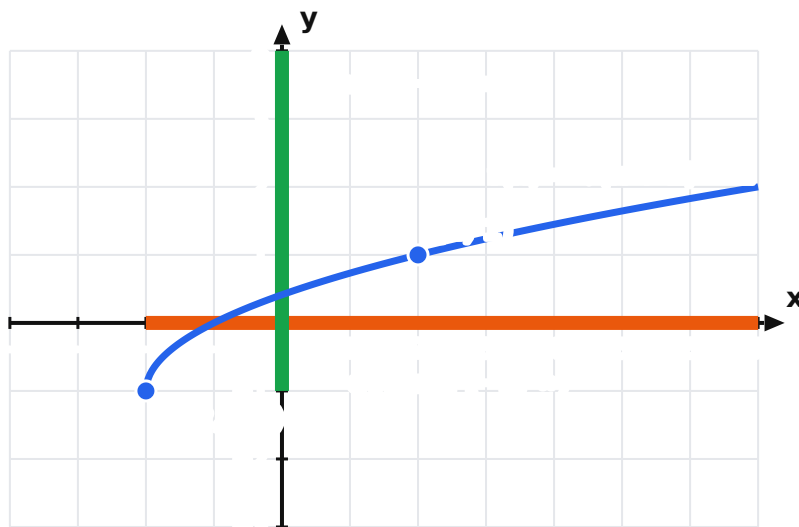
Precalculus Common Core: F-IF.A.1 · F-BF.A.1c · F-BF.B.4b · F-BF.B.4c · F-BF.B.4d

Precalculus Functions Review Lesson: key facts

- Standards F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c and F-BF.B.4d: notation, composition, inverses and restricted domains.
- Domain rules: no zero denominators, no negative numbers under even roots, and only positive inputs for $\ln$.
- In $f(g(x))$, the input must be in the domain of g and $g(x)$ must be in the domain of f .
- $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$; the domain of f is the range of f^{-1} and the reverse.
- A function has an inverse function only if it passes the horizontal line test; restrict the domain when it fails.

What are function notation, domain and range?

The symbol $f(x)$ names the output of function f for the input x . The **domain** is every input that gives a real output, and the **range** is every output the function produces. Two rules shrink a domain: never divide by zero, and never take an even root (or a logarithm) of a quantity that is not allowed.



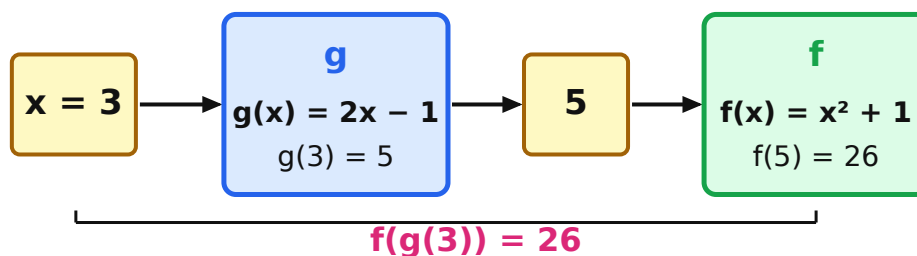
Domain (orange) is the set of inputs that work; range (green) is the set of outputs produced.

Interval notation

A square bracket includes an endpoint and a parenthesis excludes it. $[3, \infty)$ means $x \geq 3$; $(-\infty, 2)$ means $x < 2$. The symbol $\cup$ joins separate pieces, so $x \neq 4$ becomes $(-\infty, 4) \cup (4, \infty)$.

How do you compose functions?

The composition $f(g(x))$ (also written $f \circ g$) feeds the output of g into f . Its domain contains each x that is allowed in g and whose value $g(x)$ is allowed in f .



In $f(g(x))$ the output of g becomes the input of f .

Order matters

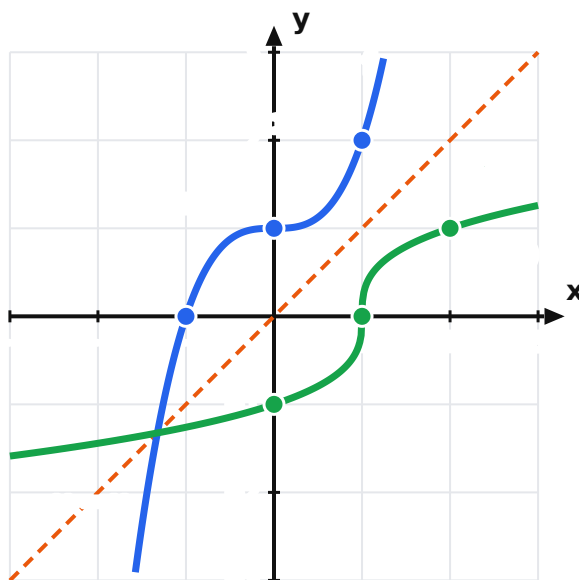
With $f(x) = x^2 + 1$ and $g(x) = 2x - 1$, we get $f(g(x)) = (2x - 1)^2 + 1 = 4x^2 - 4x + 2$ but $g(f(x)) = 2x^2 + 1$. To decompose a function, find an inner piece and an outer piece, for example $h(x) = \sqrt{x^2 + 9}$ is $f(x) = \sqrt{x}$ after $g(x) = x^2 + 9$.

How do you find an inverse function?

An inverse function f^{-1} undoes f : if $f(a)=b$ then $f^{-1}(b)=a$. Its graph is the mirror image of the graph of f across $y=x$.

1. Write $y = f(x)$.
2. Swap x and y .
3. Solve the new equation for y .
4. Name the result $f^{-1}(x)$ and verify that $f^{-1}(f(x)) = x$.

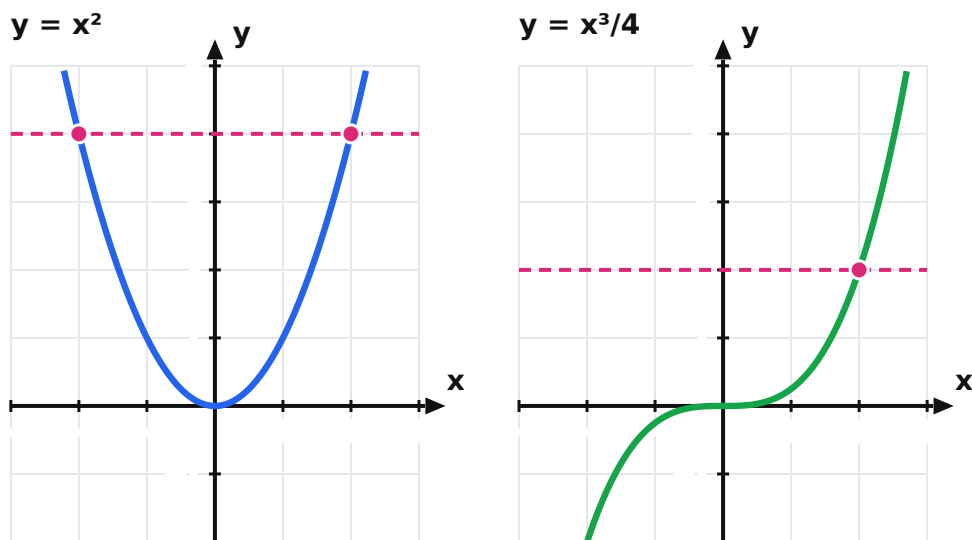
Example: for $f(x) = \frac{x}{x+2}$ swap to get $x = \frac{y}{y+2}$, so $xy+2x=y$ and $y = \frac{2x}{1-x}$. Check: $f(1) = \frac{1}{3}$ and $f^{-1}(\frac{1}{3}) = 1$.



The graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

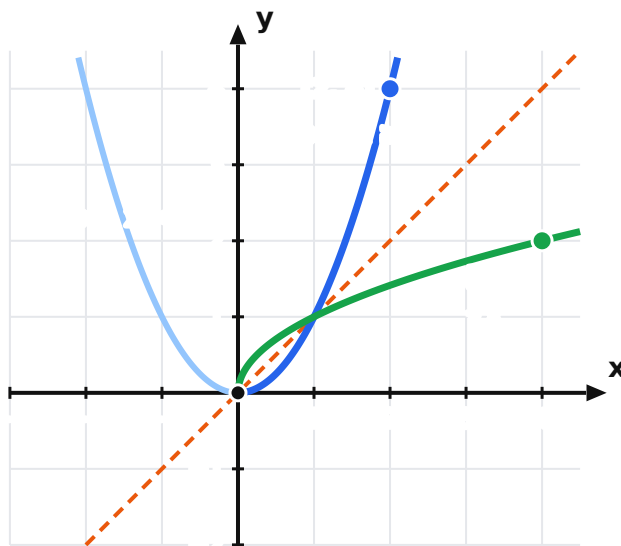
Why must a domain sometimes be restricted?

Only **one-to-one** functions have inverse functions. The horizontal line test checks this: a horizontal line must never meet the graph twice.



A horizontal line that meets a graph twice shows that the function is not one-to-one.

The parabola $y = x^2$ fails the test, but keeping only $x \geq 0$ passes it and gives the inverse $y = \sqrt{x}$.



Keeping only $x \geq 0$ makes $y = x^2$ one-to-one, so its inverse is $y = \sqrt{x}$.

Functions: Domain, Range, Composition and Inverses: frequently asked questions

How do you write a domain in interval notation?

List the allowed x-values from left to right, using brackets for included endpoints and parentheses for excluded ones or for infinity. For $f(x) = \sqrt{x-1}$ the domain is $[1, \infty)$.

Is $f(g(x))$ the same as $g(f(x))$?

Usually not. With $f(x) = x+3$ and $g(x) = 2x$, $f(g(x)) = 2x+3$ but $g(f(x)) = 2x+6$.

How do you check that two functions are inverses?

Show that $f(g(x)) = x$ and $g(f(x)) = x$ for every x in the proper domains. One composition alone is not enough to be sure.

Why does $y = x$ squared have no inverse function?

Two different inputs such as -2 and 2 give the same output 4 , so reversing the rule would give two answers. Restricting to $x \geq 0$ fixes that.

How do you read an inverse value from a graph or table?

To find $f^{-1}(b)$, locate the output b and read the matching input. If the table shows $f(4) = 9$, then $f^{-1}(9) = 4$.

How to use this precalculus functions review lesson

Use this precalculus functions review lesson before the worksheet, then check what you learned with the quiz.

