

Precalculus · Exponential and Logarithmic Functions and Models · Worksheet Answer Key

Common Core F-LE.A.4, F-BF.B.5, F-IF.C.8b · mathovore.fr

This precalculus exponential log worksheet answer key solves all 24 problems of the exponential and logarithmic functions and models worksheet step by step.

Full worked solutions for every problem on the exponential and logarithmic functions worksheet. Final answers are in bold. Compare your steps, not only the result, and note where a solution had to be rejected because it made a logarithm argument negative.

Precalculus Common Core: F-LE.A.4 · F-BF.B.5 · F-IF.C.8b

Precalculus Exponential Log Worksheet Answer Key: problems 1-5

Problem 1: Exponential to logarithmic form

- a. $\log_2 128 = 7$
- b. $\log 0.001 = -3$
- c. $\log_9 3 = \frac{1}{2}$
- d. $\ln 1 = 0$

Problem 2: Logarithmic to exponential form

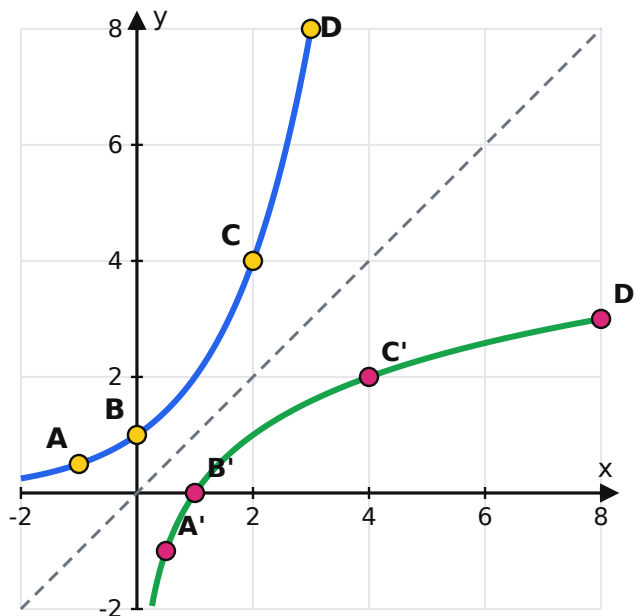
- a. $5^3 = 125$
- b. $4^{-3} = \frac{1}{64}$
- c. $36^{1/2} = 6$
- d. $e^4 = 20$

Problem 3: Evaluate logarithms

- a. $3^4 = 81$, so **4**
- b. $2^{-5} = \frac{1}{32}$, so **-5**
- c. $25^{1/2} = 5$, so $\frac{1}{2}$
- d. $\left(\frac{1}{4}\right)^{-2} = 16$, so **-2**
- e. $\sqrt{e} = e^{1/2}$, so $\frac{1}{2}$

Problem 4: Points on an inverse graph

- a. Swap the coordinates: $A'\left(\frac{1}{2}, -1\right)$, $B'(1, 0)$, $C'(4, 2)$, $D'(8, 3)$
- b. $f^{-1}(x) = \log_2 x$
- c. The domain of the inverse is the range of f , so $x > 0$



Reflecting the points across $y = x$ gives the graph of the inverse.

Problem 5: Finding inverse functions

- $y = 4^x$ becomes $x = 4^y$, so $f^{-1}(x) = \log_4 x$
- $x = 2 \cdot 3^y$ gives $3^y = \frac{x}{2}$, so $f^{-1}(x) = \log_3 \frac{x}{2}$
- $x = \ln(y+3)$ gives $y+3 = e^x$, so $f^{-1}(x) = e^x - 3$
- $x-1 = \log_2 y$ gives $y = 2^{x-1}$, so $f^{-1}(x) = 2^{x-1}$

Precalculus Exponential Log Worksheet Answer Key: problems 6-10

Problem 6: Verify inverses by composition

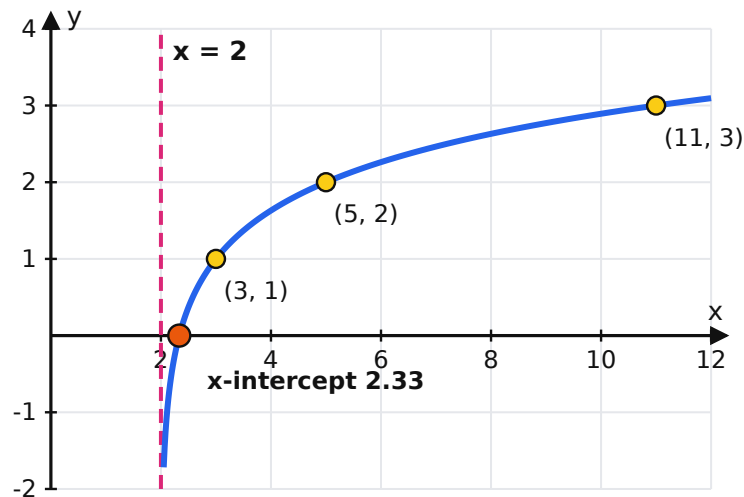
$$f(g(x)) = e^{2 \cdot \frac{1}{2} \ln x} = e^{\ln x} = x \text{ for } x > 0.$$

$$g(f(x)) = \frac{1}{2} \ln e^{2x} = \frac{1}{2} (2x) = x \text{ for all } x.$$

Both compositions return x , so **the functions are inverses**.

Problem 7: Features of a log graph

- The asymptote is $x = 2$.
- We need $x-2 > 0$, so the domain is $x > 2$ and the range is **all real numbers**.
- $\log_3(x-2) = -1$ gives $x-2 = 3^{-1} = \frac{1}{3}$, so $x = \frac{7}{3}$
- $g(29) = \log_3 27 + 1 = 3 + 1 = 4$



The x-intercept $(7/3, 0)$ is about 2.33.

Problem 8: Expand logarithms

- $3\log_b x + 2\log_b y$
- $\ln 5 + \ln x - 4\ln y$
- $\frac{1}{2}(\log_2 8 + \log_2 x - \log_2 z) = \frac{3}{2} + \frac{1}{2}\log_2 x - \frac{1}{2}\log_2 z$

Problem 9: Condense logarithms

- $\log 3^2 + \log 5 = \log 45$
- $\ln \frac{12}{4} + \ln x^2 = \ln(3x^2)$
- $\log_2 \sqrt{x} + \log_2 y^2 - \log_2 z^3 = \log_2 \frac{\sqrt{x} y^2}{z^3}$

Problem 10: Using given logarithm values

- $\log(2 \cdot 3) = 0.3010 + 0.4771 = \mathbf{0.7781}$
- $\log(2 \cdot 3^2) = 0.3010 + 2(0.4771) = \mathbf{1.2552}$
- $\log_2 \frac{3}{2} = 0.4771 - 0.3010 = \mathbf{0.1761}$
- $\log 2^3 = 3(0.3010) = \mathbf{0.9030}$
- $\log 2 - 2\log 3 = 0.3010 - 0.9542 = \mathbf{-0.6532}$

Precalculus Exponential Log Worksheet Answer Key: problems 11-15

Problem 11: Change of base

- $\frac{\ln 40}{\ln 5} \approx \frac{3.6889}{1.6094} = \mathbf{2.292}$
- $\frac{\ln 3}{\ln 7} \approx \frac{1.0986}{1.9459} = \mathbf{0.565}$
- $\frac{\ln 100}{\ln 2} \approx \frac{4.6052}{0.6931} = \mathbf{6.644}$

Problem 12: Same-base exponential equations

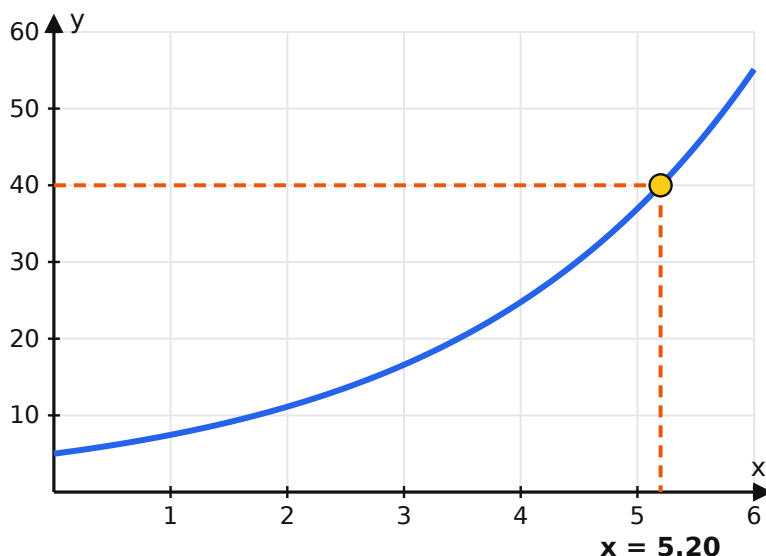
- a. $2^{2x+2} = 2^5$, so $2x+2=5$ and $x = \frac{3}{2}$
- b. $3^{2x} = 3^{3x-6}$, so $2x=3x-6$ and $x=6$
- c. $2^{-x} = 2^4$, so $x = -4$
- d. $5^{2x-2} = 5^{3x}$, so $2x-2=3x$ and $x = -2$

Problem 13: Solve using logarithms

- a. $2^x = 20$, so $x = \frac{\ln 20}{\ln 2} \approx 4.322$
- b. $x-1 = \frac{\ln 50}{\ln 7}$, so $x = 1 + \frac{\ln 50}{\ln 7} \approx 3.010$
- c. $e^{2x} = 9$, so $x = \frac{\ln 9}{2} = \ln 3 \approx 1.099$
- d. $0.9^x = 0.25$, so $x = \frac{\ln 0.25}{\ln 0.9} \approx 13.158$

Problem 14: Estimate, then solve

- a. Draw the line $y=40$: it meets the curve near $x \approx 5.2$.
- b. $e^{0.4x} = 8$, so $0.4x = \ln 8$ and $x = \frac{\ln 8}{0.4} \approx 5.199$.



The curve reaches 40 at $x = 5.20$.

Problem 15: Logarithmic equations

- a. $\log_2(x^2-9) = 4$, so $x^2-9=16$ and $x = \pm 5$. The value $x = -5$ makes $x+3$ negative, so the solution is $x=5$
- b. $x(x-2)=8$ gives $x^2-2x-8=0$, so $(x-4)(x+2)=0$. Reject $x = -2$: $x=4$
- c. $2x-1=3^2=9$, so $x=5$
- d. $x+6=x^2$ gives $(x-3)(x+2)=0$. Reject $x = -2$: $x=3$

Precalculus Exponential Log Worksheet Answer Key: problems 16-20

Problem 16: Quadratic form

- a. Let $u = e^x$: $u^2 - 5u + 6 = 0$, so $u = 2$ or $u = 3$. Then $x = \ln 2 \approx 0.693$ or $x = \ln 3 \approx 1.099$
- b. Let $u = 2^x$: $u^2 - 6u + 8 = 0$, so $u = 2$ or $u = 4$. Then $x = 1$ or $x = 2$

Problem 17: Transformations of an exponential

- a. Shift right 1 unit, reflect over the x-axis, then shift up 3 units.
- b. The asymptote moves up 3: $y = 3$
- c. $h(0) = -2^{-1} + 3 = \frac{5}{2}$
- d. $2^{x-1} = 3$, so $x = 1 + \log_2 3 \approx 2.585$: $x = 1 + \log_2 3$
- e. The graph lies below its asymptote: $y < 3$

Problem 18: Rewrite growth and decay models

- a. $0.8^t = e^{t \ln 0.8}$, so $A = 200e^{kt}$ with $k = \ln 0.8 \approx -0.2231$. Negative k means decay.
- b. $e^{0.07} \approx 1.0725$, so the growth is **about 7.25%** per unit, a bit more than 7%.
- c. $2^{t/3} = e^{(t/3) \ln 2}$, so $k = \frac{\ln 2}{3} \approx 0.2310$

Problem 19: Compound interest time

$$3000 = 2000(1.01)^{4t}, \text{ so } 1.01^{4t} = 1.5.$$

$$4t = \frac{\ln 1.5}{\ln 1.01} \approx \frac{0.40547}{0.00995} = 40.75$$

$$t \approx \mathbf{10.19 \text{ years.}}$$

Problem 20: Continuous compounding

$$10000 = 5000e^{0.035t} \text{ gives } e^{0.035t} = 2, \text{ so } t = \frac{\ln 2}{0.035} \approx \mathbf{19.80 \text{ years.}}$$

$$e^{0.035t} = 2.4, \text{ so } t = \frac{\ln 2.4}{0.035} \approx \mathbf{25.01 \text{ years.}}$$

Precalculus Exponential Log Worksheet Answer Key: problems 21-24

Problem 21: Radioactive decay dating

$$0.3 = \left(\frac{1}{2}\right)^{t/5730}.$$

$$\text{Take natural logs: } \frac{t}{5730} \ln 0.5 = \ln 0.3, \text{ so } t = 5730 \cdot \frac{\ln 0.3}{\ln 0.5} \approx 5730(1.7370)$$

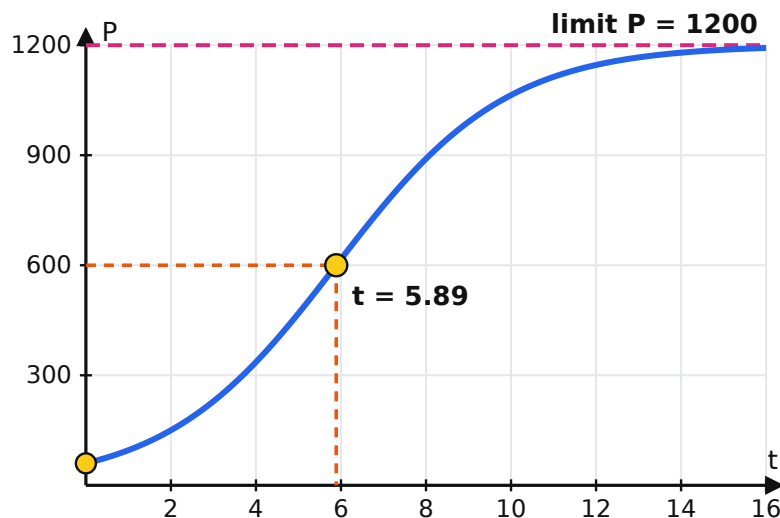
The bone is about **9953 years** old.

Problem 22: Logistic population model

- a. $P(0) = \frac{1200}{1+19} = \mathbf{60 \text{ students}}$
- b. **1,200 students**, the numerator K

c. $1+19e^{-0.5t}=2$, so $e^{-0.5t}=\frac{1}{19}$ and $t=2\ln 19 \approx \mathbf{5.89 \text{ hours}}$

d. $P(8)=\frac{1200}{1+19e^{-4}} \approx \frac{1200}{1.348} \approx \mathbf{890 \text{ students}}$



$P = 600$ at $t = 5.89$ hours, the steepest point of the curve.

Problem 23: pH calculations

a. $-\log(3.2 \times 10^{-5}) = -(0.5051 - 5) \approx \mathbf{4.49}$

b. $[H^+] = 10^{-8.2} \approx 6.31 \times 10^{-9} \text{ mol/L}$

c. 10^{-3} versus 10^{-6} : the ratio is $10^3 = \mathbf{1,000 \text{ times}}$

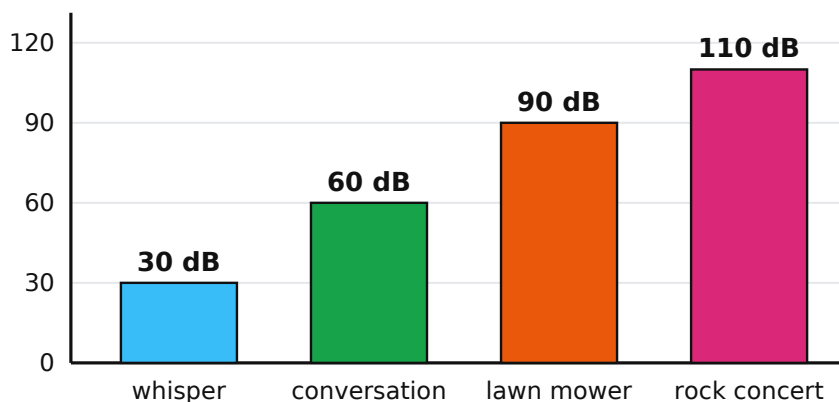
Problem 24: Decibels and earthquakes

a. $90 - 60 = 30 \text{ dB}$ is 3 steps of 10 dB, so the intensity ratio is $10^3 = \mathbf{1,000}$

b. $110 = 10 \log \frac{I}{I_0}$ gives $\log \frac{I}{I_0} = 11$, so $\frac{I}{I_0} = 1 \times 10^{11}$

c. The difference is $7.1 - 5.4 = 1.7$, so the factor is $10^{1.7} \approx 50.12$, about **50**.

decibels (dB)



The 30 dB gap between 60 dB and 90 dB means a factor of 1,000.

How to use this precalculus exponential log worksheet answer key

Read this precalculus exponential log worksheet answer key after trying each problem yourself, and redo any problem you missed.

