

Precalculus · Exponential and Logarithmic Functions and Models · Test Answer Key

Common Core F-LE.A.4, F-BF.B.5, F-IF.C.8b · mathovore.fr

This precalculus exponential log test answer key gives the full solution and the scoring guide for each of the 9 test questions on exponential and logarithmic functions and models.

Scoring guide and worked solutions for the exponential and logarithmic functions test. Partial credit is listed in each scoring note; award half points for correct setup with an arithmetic slip. The total is 20 points.

Precalculus Common Core: F-LE.A.4 · F-BF.B.5 · F-IF.C.8b

Precalculus Exponential Log Test Answer Key: questions 1-5

Question 1: Logarithm basics (2 points)

a. $\log_7 343 = 3$

b. $\left(\frac{1}{3}\right)^y = 9$ gives $3^{-y} = 3^2$, so $y = -2$

Scoring: 1 point for each correct part.

Question 2: Laws of logarithms (2 points)

a. $3\ln x + \frac{1}{2}\ln y - 2\ln z$

b. $\log a^2 + \log b^3 - \log c = \log \frac{a^2 b^3}{c}$

Scoring: 1 point for each part: the power law, the quotient law and the product law all applied correctly.

Question 3: Inverse function (2 points)

a. Solve $x = 3\ln(y-1) + 2$: $\frac{x-2}{3} = \ln(y-1)$, so $f^{-1}(x) = e^{(x-2)/3} + 1$

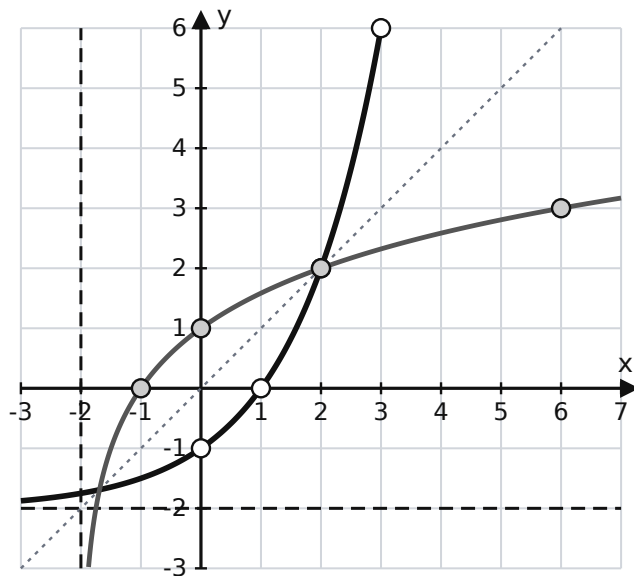
b. The range of f is all real numbers, so the domain of f^{-1} is **all real numbers**.

Scoring: 1 point for the inverse (0.5 for isolating the logarithm, 0.5 for exponentiating), 1 point for the domain.

Question 4: Graph and inverse (2 points)

a. The asymptote $y = -2$ and the point $(1,0)$ fit $f(x) = 2^x - 2$: $2^1 - 2 = 0$, $2^3 - 2 = 6$. Answer:
 $f(x) = 2^x - 2$

b. $x = 2^y - 2$ gives $2^y = x + 2$, so $f^{-1}(x) = \log_2(x + 2)$ with asymptote $x = -2$



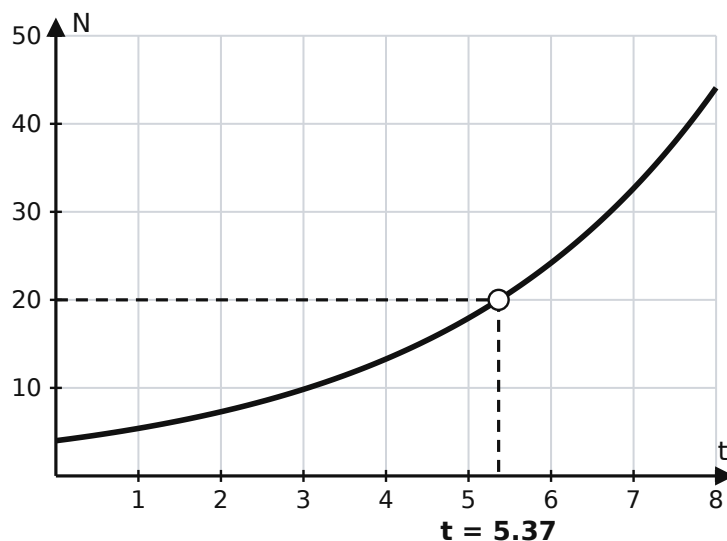
The inverse (gray curve) has its asymptote at $x = -2$.

Scoring: 1 point for the equation of f (0.5 if the shift is right but the base is wrong); 1 point for the inverse (0.5) and its asymptote (0.5).

Question 5: Exponential equations (2 points)

a. $e^{0.3t} = 5$, so $t = \frac{\ln 5}{0.3} \approx \mathbf{5.36 \text{ hours}}$

b. $(x+1)\ln 2 = x\ln 3$, so $x(\ln 3 - \ln 2) = \ln 2$ and $x = \frac{\ln 2}{\ln 3 - \ln 2} \approx \mathbf{1.710}$



The culture reaches 20 thousand cells at $t = 5.37$ hours.

Scoring: 1 point per part: 0.5 for the correct setup with logarithms, 0.5 for the correct rounded value.

Precalculus Exponential Log Test Answer Key: questions 6-9

Question 6: Logarithmic equation (2 points)

$$\log_3((x+2)(x-4))=3, \text{ so } (x+2)(x-4)=27.$$

$$x^2-2x-35=0 \text{ gives } (x-7)(x+5)=0.$$

For $x=-5$, the argument $x-4=-9$ is negative, so it is rejected. Check $x=7$:

$$\log_3 9 + \log_3 3 = 2 + 1 = 3. \text{ The solution is } x=7.$$

Scoring: 1 point for combining into a single equation and solving the quadratic; 1 point for rejecting $x = -5$ with a correct reason and stating $x = 7$.

Question 7: Continuous growth (2 points)

a. $P(10)=12000e^{0.24} \approx 12000(1.27125) \approx \mathbf{15,255 \text{ people}}$

b. $e^{0.024t}=\frac{5}{3}$, so $t=\frac{\ln(5/3)}{0.024} \approx \mathbf{21.28 \text{ years}}$

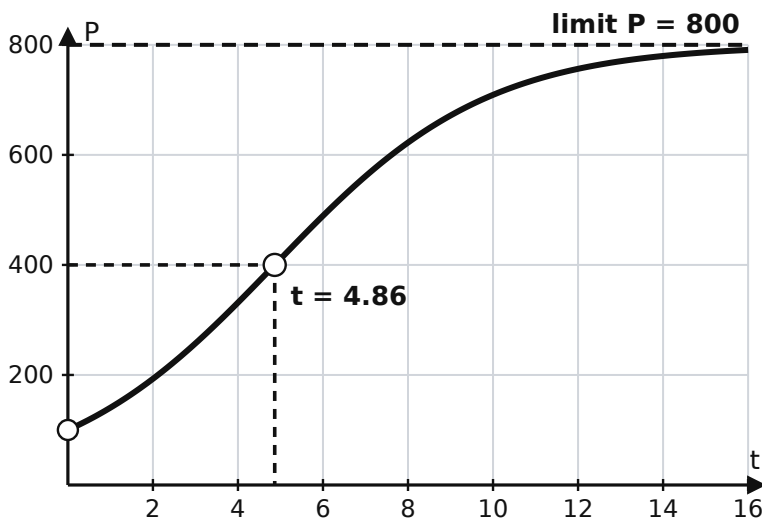
Scoring: 1 point per part: 0.5 for the correct substitution or setup, 0.5 for the correct value.

Question 8: Logistic model (3 points)

a. $P(0)=\frac{800}{1+7}=100$ **fish**

b. As t grows, $e^{-0.4t} \rightarrow 0$, so $P \rightarrow 800$: **800 fish**

c. $1+7e^{-0.4t}=2$, so $e^{-0.4t}=\frac{1}{7}$ and $t=\frac{\ln 7}{0.4} \approx \mathbf{4.86 \text{ years}}$



The population reaches 400 fish at $t = 4.86$ years.

Scoring: 1 point per part. In part (c), award 0.5 for setting the denominator equal to 2 and 0.5 for the final value.

Question 9: Logarithmic scales (3 points)

- a. $10^{6.8-5.6} = 10^{1.2} \approx \mathbf{15.8 \text{ times}}$
- b. $\text{pH} = -\log(2.5 \times 10^{-4}) \approx \mathbf{3.60}$
- c. $10^{(95-80)/10} = 10^{1.5} \approx \mathbf{31.6 \text{ times}}$

Scoring: 1 point per part: 0.5 for the correct power of 10 or logarithm, 0.5 for the correct rounded value.

Scoring summary

Total
20 points: add the points of every question of this precalculus exponential log test answer key.

How to use this precalculus exponential log test answer key

Use this precalculus exponential log test answer key to score the test fairly and to see which skills need more practice.

