

Precalculus · Exponential and Logarithmic Functions and Models · Lesson

Common Core F-LE.A.4, F-BF.B.5, F-IF.C.8b · mathovore.fr

This precalculus exponential log lesson explains exponential and logarithmic functions and models in one printable page, with definitions, a method and a worked example.

A logarithm is an exponent: log base b of x equals y exactly when b to the power y equals x . Exponential and logarithmic functions are inverses, so their graphs mirror each other in the line $y = x$, and the laws of logarithms turn products into sums and powers into multipliers, which lets you solve for exponents.

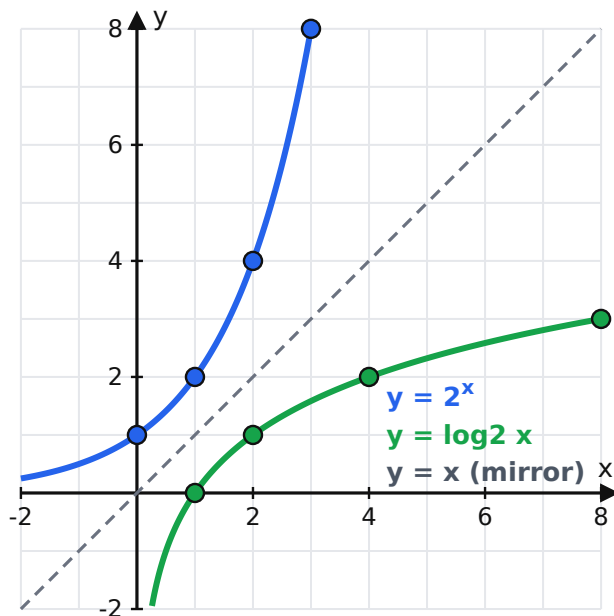
Precalculus Common Core: F-LE.A.4 · F-BF.B.5 · F-IF.C.8b

Precalculus Exponential Log Lesson: key facts

- $\log_b x = y$ means $b^y = x$, with $b > 0$, b not equal to 1 and $x > 0$.
- Product, quotient and power laws: $\log(MN) = \log M + \log N$, $\log(M/N) = \log M - \log N$, $\log(M^k) = k \log M$.
- Solve $b^t = c$ with $t = \ln c / \ln b$, and always check log equations for extraneous answers.
- Continuous growth is $A = Pe^{(rt)}$; a logistic model $P = K/(1 + Ae^{(-rt)})$ levels off at K .
- Log scales (pH, decibels, magnitude): a difference of d units means a ratio of 10^d .

How are exponents and logarithms inverses?

A **logarithm** is an exponent. The statement $\log_b x = y$ means $b^y = x$, where $b > 0$, $b \neq 1$ and $x > 0$. Because the two functions undo each other, $\log_b b^x = x$ and $b^{\log_b x} = x$. Their graphs are mirror images in the line $y = x$: every point (a, b) on one graph matches (b, a) on the other, so domain and range swap and the horizontal asymptote $y = 0$ becomes the vertical asymptote $x = 0$.



The graphs of $y = 2^x$ and $y = \log_2 x$ reflect across the line $y = x$.

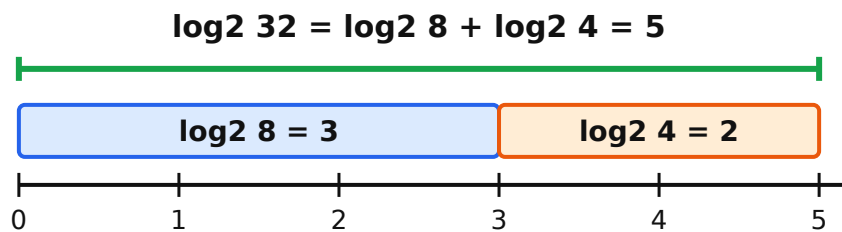
Vocabulary

Common logarithm: base 10, written $\log x$. **Natural logarithm:** base $e \approx 2.718$, written $\ln x$.

Carrying capacity: the limiting value K that a logistic model approaches.

What are the laws of logarithms?

Law	Rule
Product	$\log_b(MN) = \log_b M + \log_b N$
Quotient	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power	$\log_b M^k = k \log_b M$
Change of base	$\log_b M = \frac{\ln M}{\ln b}$



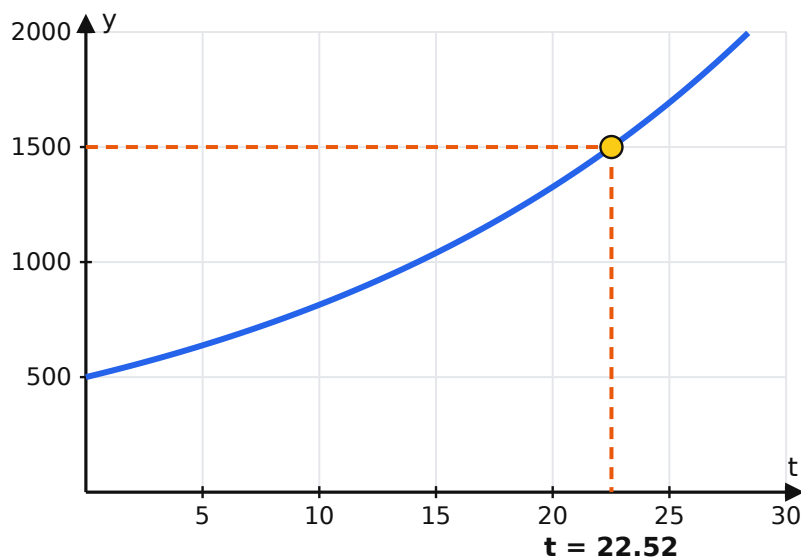
Multiplying 8 by 4 adds the exponents: 2^3 times $2^2 = 2^5$.

Multiplying adds exponents, so the log of a product is a sum of logs.

How do you solve exponential and logarithmic equations?

1. Isolate the exponential (or the logarithm) on one side.
2. Take a logarithm of both sides, or rewrite the log equation in exponential form.
3. Solve the resulting equation, then check every answer in the original equation.

Example. A \$500 deposit earns 5% interest per year, so $500(1.05)^t = 1500$ tells when it reaches \$1,500. Divide by 500: $1.05^t = 3$. Then $t = \frac{\ln 3}{\ln 1.05} \approx 22.52$ years.



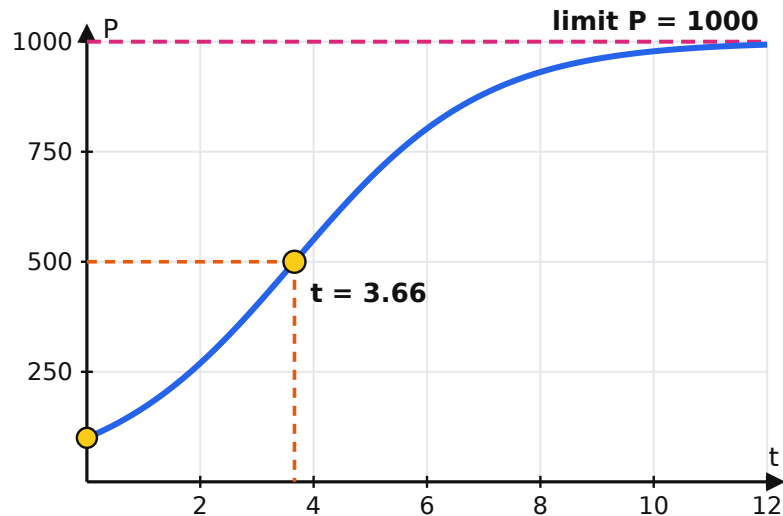
The curve $y = 500(1.05)^t$ crosses $y = 1500$ at $t = 22.52$ years.

A logarithm accepts only positive inputs, so an answer that makes any log argument zero or negative is *extraneous* and must be rejected.

How do continuous growth and logistic models work?

Any growth factor b can be written with base e : $b^t = e^{t \ln b}$. **Continuous growth** uses $A = Pe^{rt}$, and the doubling time is $\frac{\ln 2}{r}$. Real populations cannot grow forever, so a **logistic model**

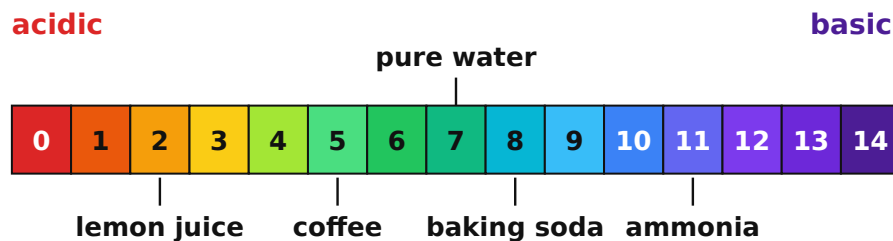
$P(t) = \frac{K}{1 + Ae^{-rt}}$ levels off at the carrying capacity K . It grows fastest when $P = \frac{K}{2}$, at $t = \frac{\ln A}{r}$.



A logistic curve rises fastest at half the limit, then levels off at $P = 1000$.

How do logarithmic scales work?

On a logarithmic scale each step multiplies the quantity by 10. For acidity, $\text{pH} = -\log[H^+]$; for sound, $L = 10 \log \frac{I}{I_0}$ decibels; for earthquakes, one magnitude unit means 10 times the wave amplitude. A difference of d units therefore means a ratio of 10^d .



One step down the scale means 10 times more hydrogen ions.

The pH scale is logarithmic: each whole step is a factor of 10.

Exponential and Logarithmic Functions and Models: frequently asked questions

Why are exponential and logarithmic functions inverses?

Exponentiating undoes taking a logarithm and the other way around, since $\log_b(b^x) = x$ and $b^{(\log_b x)} = x$. Graphically, swapping x and y reflects one graph across the line $y = x$.

Why can you not take the log of a negative number?

Every output of b^y is positive, so no exponent produces a negative or zero number. The domain of $\log_b x$ is $x > 0$, which is why some solutions of log equations must be rejected.

How do you solve an equation with the unknown in an exponent?

Isolate the exponential part, take $\ln$ of both sides and use the power law to bring the exponent down. For $5^x = 40$, $x = \ln 40 / \ln 5$, about 2.29.

What is the difference between $\ln$ and $\log$?

The symbol $\log$ means base 10 and $\ln$ means base e , about 2.718. Both obey the same laws, and the change-of-base formula converts between them.

What does the carrying capacity K in a logistic model mean?

K is the largest value the quantity can approach, such as the number of fish a pond can support. The graph rises toward the horizontal line $y = K$ but never crosses it.

How to use this precalculus exponential log lesson

Use this precalculus exponential log lesson before the worksheet, then check what you learned with the quiz.

