

Precalculus · Expected Value and Decisions · Lesson

Common Core S-MD.A.3, S-MD.A.4, S-MD.B.5 · mathovore.fr

A probability distribution lists every value a random variable can take together with its probability, and the probabilities add to 1. The expected value is the long-run average, found by multiplying each value by its probability and adding the products. Comparing expected values helps you judge games, insurance, and warranties.

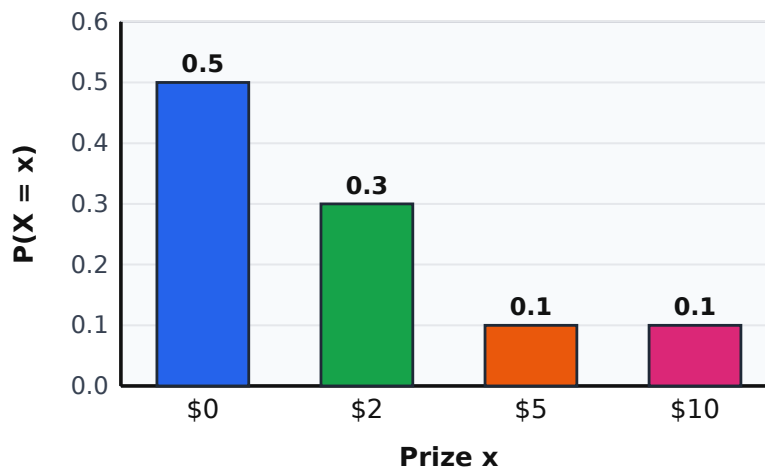
PrecalculusCommon Core: S-MD.A.3 · S-MD.A.4 · S-MD.B.5

Key facts:

- In a probability distribution, every probability is between 0 and 1 and the total is exactly 1.
- Theoretical probabilities come from equally likely outcomes; empirical probabilities are relative frequencies from data.
- Expected value = sum of (value times probability); it need not be a possible outcome.
- A game is fair when its expected net result is 0; negative means a long-run loss for the player.
- To choose between options, compare expected values and also consider the risk of a large loss.

What is a probability distribution?

A **random variable** X turns each outcome of a chance process into a number. A **probability distribution** lists every value of X with its probability. At the school carnival, Maya spins a wheel with 10 equal sections: five pay \$0, three pay \$2, one pays \$5, and one pays \$10. Each probability is a count of sections divided by 10, so the distribution is below and the probabilities add to 1.

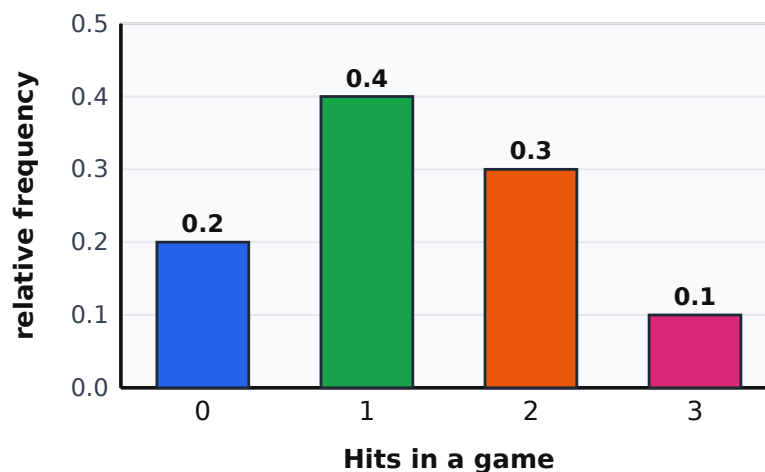


Theoretical distribution for the carnival wheel: the bar heights add to 1.

Where do the probabilities come from?

Theoretical probabilities come from counting equally likely outcomes, as with the wheel.

Empirical probabilities come from data: divide each count by the total. A Little League coach recorded the hits for one player in 50 games: 10 games with no hits, 20 with one, 15 with two, and 5 with three. The relative frequencies $\frac{10}{50}, \frac{20}{50}, \frac{15}{50}, \frac{5}{50}$ give 0.2, 0.4, 0.3, 0.1.



Empirical distribution built from 50 games.

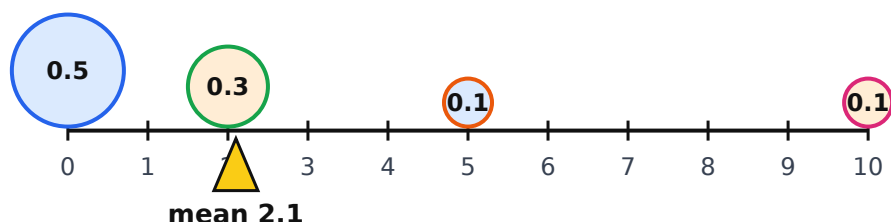
How do you find an expected value?

Expected value

The **expected value** of X is the long-run average. Multiply each value by its probability, then add all the products: $E(X) = x_1p_1 + x_2p_2 + x_3p_3$ for a variable with three values, and so on for more.

1. List each value x and its probability $P(x)$.
2. Check that the probabilities add to 1.
3. Multiply each x by $P(x)$.
4. Add the products.

For the wheel: $E(X) = 0(0.5) + 2(0.3) + 5(0.1) + 10(0.1) = 0 + 0.6 + 0.5 + 1.0 = 2.1$, so the average prize is \$2.10. For the player with the hits table, $E = 0(0.2) + 1(0.4) + 2(0.3) + 3(0.1) = 1.3$ hits per game, even though no game has exactly 1.3 hits.

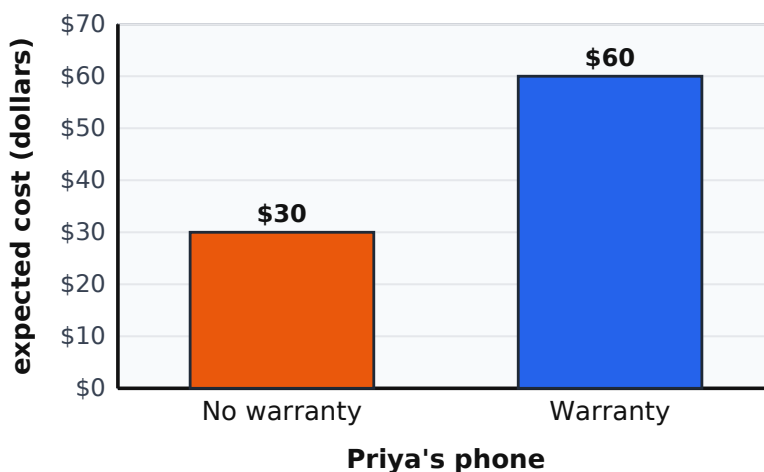


The expected value is the balance point of the distribution.

How do you use expected value to decide?

Compare the expected values of the choices. If the wheel costs \$3 to spin, the expected net result is $2.1 - 3 = -0.9$, a loss of \$0.90 per spin, or about \$90 over 100 spins. A game is **fair** when the expected net result is 0.

Priya can buy a \$60 warranty for a phone. Without it, there is a 15% chance of a \$200 repair, so the expected repair cost is $0.15(200) = 30$ dollars. Expected value favors skipping the warranty, though a warranty also removes the risk of one big bill.



Expected cost of each choice for the phone repair.

Expected Value and Decisions: frequently asked questions

What does expected value mean?

It is the average result you would see after a very large number of repetitions. It describes the long run, not what happens in one trial.

Can an expected value be a number that never occurs?

Yes. A fair die has an expected value of 3.5 even though no roll shows 3.5. The expected value is a balance point, not a single outcome.

How is an empirical distribution different from a theoretical one?

A theoretical distribution comes from counting equally likely outcomes. An empirical distribution uses recorded data, so its probabilities are relative frequencies and may change with more data.

How do you know if a game is fair?

Find the expected net result by subtracting the cost to play from the expected prize. If it equals 0, the game is fair. If it is negative, the player loses money on average.

Why do people buy insurance if expected value says it costs more?

Insurance companies must charge more than the expected payout to cover expenses. Many people still buy it to avoid a loss they could not afford, so risk matters along with expected value.