

Precalculus · Counting, Permutations, Combinations and Probability · Lesson

Common Core S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2 · mathovore.fr

This precalculus counting probability lesson explains counting, permutations, combinations and probability in one printable page, with definitions, a method and a worked example.

The fundamental counting principle multiplies the number of choices at each step. Permutations count ordered arrangements, combinations count unordered selections, and probability is favorable outcomes over total outcomes. The general multiplication rule is $P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$, and expected value is the sum of each value times its probability.

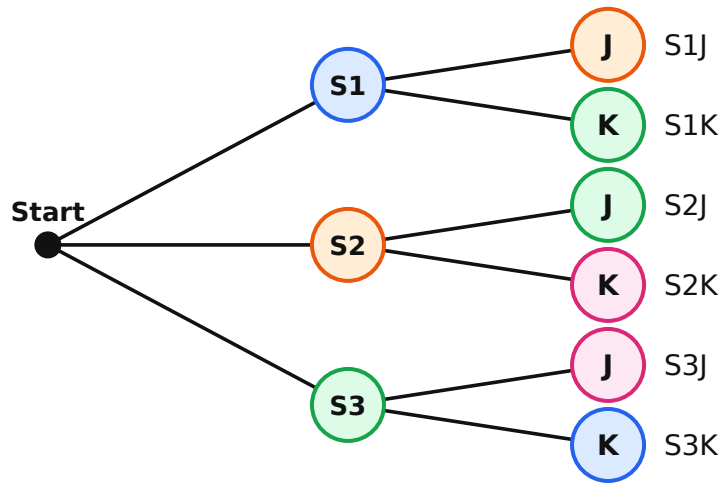
Precalculus Common Core: S-CP.B.8 · S-CP.B.9 · S-MD.A.1 · S-MD.A.2

Precalculus Counting Probability Lesson: key facts

- Standards S-CP.B.8, S-CP.B.9, S-MD.A.1 and S-MD.A.2: use the multiplication rule, permutations and combinations, and expected value.
- Permutations ${}_nP_r = \frac{n!}{(n-r)!}$; combinations ${}_nC_r = \frac{n!}{r!(n-r)!}$.
- General multiplication rule: $P(A \text{ and } B) = P(A) \cdot P(B|A)$.
- Expected value: $E(X) = \sum x \cdot P(X = x)$; a game is fair when the expected net result is \$0.
- Two dice have 36 equally likely outcomes, and a standard deck has 52 cards.

What is the fundamental counting principle?

If one task can be done in m ways and a second task in n ways, the two tasks together can be done in $m \times n$ ways. A tree diagram makes the count visible: Maya has 3 shirts and 2 pairs of pants, so she has $3 \times 2 = 6$ outfits.



3 shirts (S1 to S3) and 2 pairs of pants (J, K) give $3 \times 2 = 6$ outfits.

The same idea works with several decisions. A 4-digit PIN with no repeated digit has $10 \cdot 9 \cdot 8 \cdot 7 = 5040$ possibilities, because every digit used removes one choice.

$$\begin{array}{ccccccc}
 \boxed{10} & \times & \boxed{9} & \times & \boxed{8} & \times & \boxed{7} & = & 5,040 \\
 \text{1st digit} & & \text{2nd digit} & & \text{3rd digit} & & \text{4th digit} & &
 \end{array}$$

One box per decision; multiply the number of choices.

How do permutations and combinations differ?

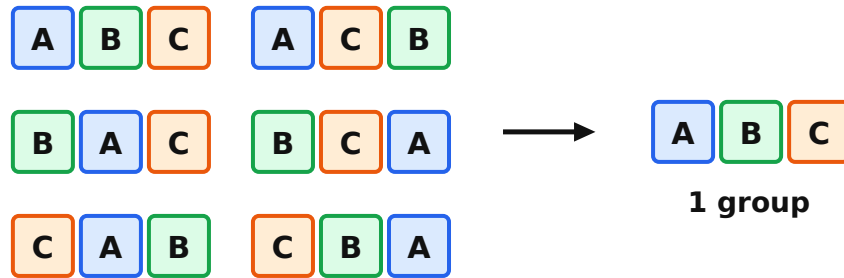
Permutation and combination

A **permutation** is an arrangement where order matters: ${}_nP_r = \frac{n!}{(n-r)!}$. A **combination** is a selection where order does not matter: ${}_nC_r = \frac{n!}{r!(n-r)!}$. Letters repeated a, b times in a word of n letters give $\frac{n!}{a!b!}$ distinct arrangements.

Gold, silver and bronze among 8 runners is a permutation: ${}_8P_3 = 336$. Choosing 3 of the 8 runners for a relay team is a combination: ${}_8C_3 = \frac{336}{6} = 56$, because each team of 3 was counted $3! = 6$ times in the permutations.

Order matters: $3! = 6$

Order ignored



Six permutations of the same three letters form one combination.

How do you find a probability by counting?

When all outcomes are equally likely, $P(E) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$, and counting techniques supply both numbers. Two dice have $6 \times 6 = 36$ outcomes, and six of them add to 7, so $P(\text{sum } 7) = \frac{6}{36} = \frac{1}{6}$. For two cards drawn from a standard deck, $P(\text{both hearts}) = \frac{{}_{13}C_2}{{}_{52}C_2} = \frac{78}{1326} = \frac{1}{17}$.

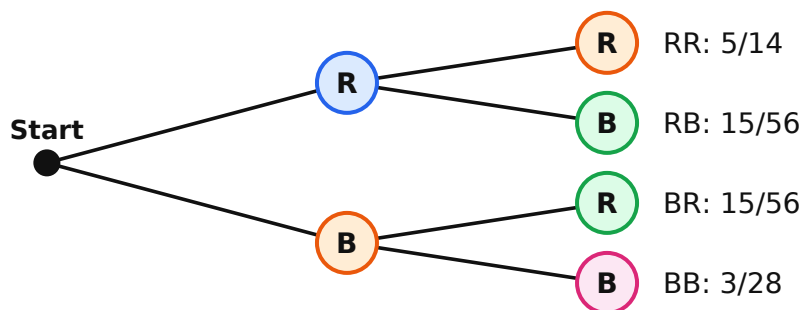
		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Sums equal to 7 are highlighted: 6 of 36 outcomes

Each cell is one equally likely outcome.

What is the general multiplication rule?

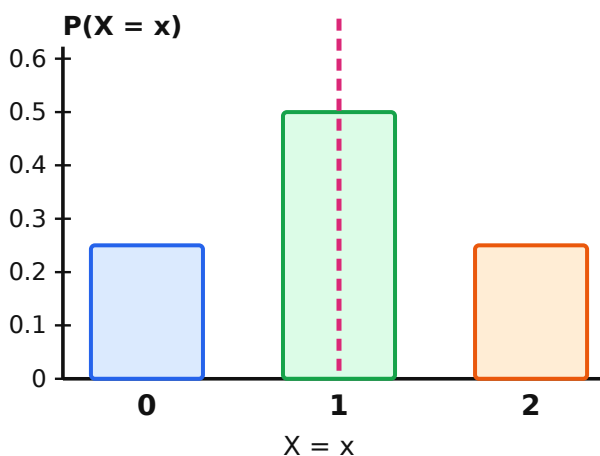
For any two events, $P(A \text{ and } B) = P(A) \cdot P(B|A)$, where $P(B|A)$ is the probability of B given that A has happened. Drawing two marbles without replacement from a bag with 5 red and 3 blue marbles changes the second probability, because the bag has changed.



Multiply along each path: the probability of two reds is $\frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$.

What is a random variable and its expected value?

A **random variable** X assigns a number to each outcome. Its **expected value** is the long-run average, $E(X) = \sum x \cdot P(X = x)$. For the number of heads in two coin flips, $E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1$.



Bar heights are probabilities and add to 1.

Worked example: a game costs \$2. Roll a die and win \$9 on a six or \$3 on a four or five. Then $E = 9 \cdot \frac{1}{6} + 3 \cdot \frac{2}{6} = \frac{15}{6} = 2.5$, so the expected net result is $2.50 - 2.00 = \$0.50$ per play.

Counting, Permutations, Combinations and Probability: frequently asked questions

How do I know whether to use a permutation or a combination?

Ask whether swapping the chosen items gives a different result. Officers of a club (president, treasurer) form a permutation; a committee of the same size is a combination. Use ${}_nP_r$ for the first and ${}_nC_r$ for the second.

Why does the second probability change when drawing without replacement?

Removing an item changes both the count of favorable outcomes and the total. With 5 red and 3 blue marbles, the chance of a second red is $\frac{4}{7}$ after a red was removed, not $\frac{5}{8}$.

What is the general multiplication rule used for?

It finds the probability that two events both happen when they may depend on each other. Multiply the probability of the first event by the probability of the second given the first:
 $P(A \text{ and } B) = P(A) \cdot P(B|A)$.

Does an expected value have to be a possible outcome?

No. Expected value is a long-run average, so it can be a value that never occurs. A die has expected value 3.5 even though no roll gives 3.5.

How do I use combinations to find a probability?

Count the favorable selections with combinations, count all selections with combinations, and divide. For two hearts from a deck, $\frac{{}_{13}C_2}{{}_{52}C_2} = \frac{78}{1326} = \frac{1}{17}$.

How to use this precalculus counting probability lesson

Use this precalculus counting probability lesson before the worksheet, then check what you learned with the quiz.

