

Precalculus · Conic Sections: Parabolas, Ellipses and Hyperbolas · Worksheet Answer Key

Common Core G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · mathovore.fr

This precalculus conic sections worksheet answer key solves all 26 problems of the conic sections: parabolas, ellipses and hyperbolas worksheet step by step.

Each solution shows the definition used, the values of a , b , c or p , and the final equation or feature list. Graphs confirm the key points. Check your standard forms by substituting a known point.

Precalculus Common Core: G-GPE.A.1 · G-GPE.A.2 · G-GPE.A.3

Precalculus Conic Sections Worksheet Answer Key: problems 1-5

Problem 1: Write the equation of a circle

Use $(x-h)^2 + (y-k)^2 = r^2$.

- a. $x^2 + y^2 = 49$
- b. $(x-3)^2 + (y+2)^2 = 25$
- c. $(x+4)^2 + (y-1)^2 = 7$

(a) $x^2 + y^2 = 49$ **(b)** $(x-3)^2 + (y+2)^2 = 25$ **(c)** $(x+4)^2 + (y-1)^2 = 7$

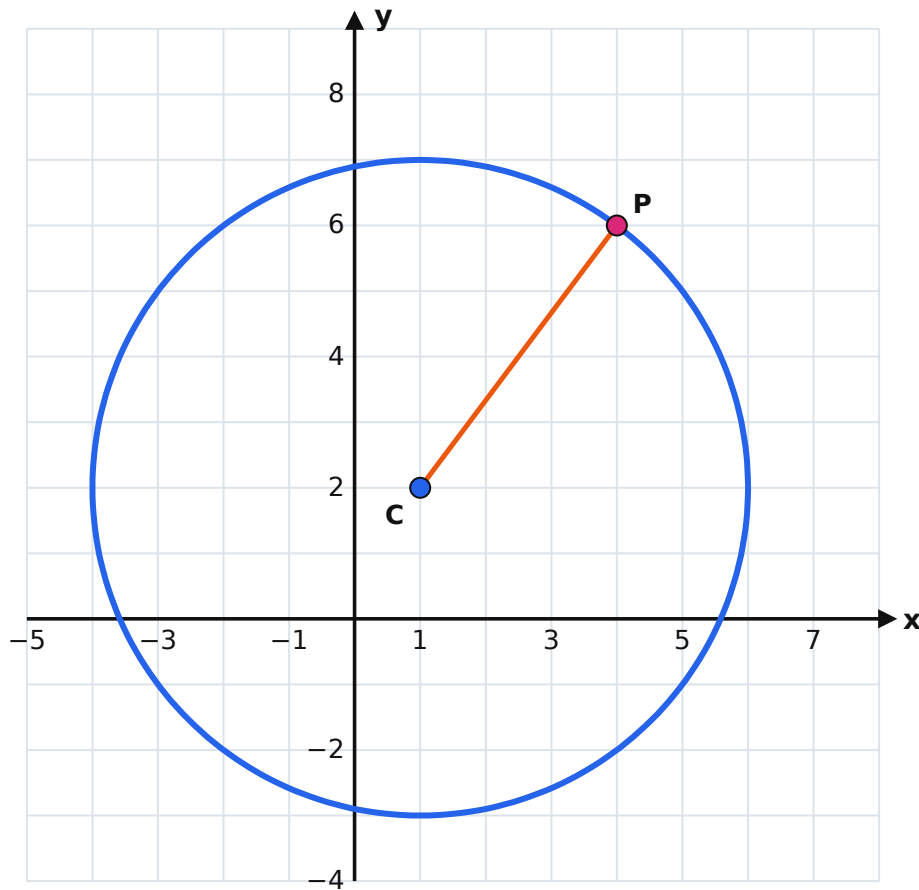
Problem 2: Center and radius from an equation

- a. Center $(-4, 1)$, $r = \sqrt{49} = 7$.
- b. Center $(0, 5)$, $r = 6$.
- c. Center $(2, -3)$, $r = \sqrt{20} = 2\sqrt{5} \approx 4.47$.

(a) $(-4, 1)$, **7** **(b)** $(0, 5)$, **6** **(c)** $(2, -3)$, $2\sqrt{5}$

Problem 3: Circle through a point

$r^2 = (4-1)^2 + (6-2)^2 = 9 + 16 = 25$, so $r = 5$.



The circle of radius 5.

$$(x-1)^2 + (y-2)^2 = 25$$

Problem 4: Circle from a diameter

The center is the midpoint: $\left(\frac{-2+6}{2}, \frac{3+7}{2}\right) = (2, 5)$. The radius squared is half the diameter squared over two: $AB^2 = 8^2 + 4^2 = 80$, so $r^2 = \frac{80}{4} = 20$.

$$(x-2)^2 + (y-5)^2 = 20$$

Problem 5: Complete the square: circles

a. $(x^2 - 8x + 16) + (y^2 + 6y + 9) = -9 + 16 + 9$, so $(x-4)^2 + (y+3)^2 = 16$: center $(4, -3)$, $r = 4$.

b. $(x^2 + 10x + 25) + (y^2 - 4y + 4) = -13 + 25 + 4$, so $(x+5)^2 + (y-2)^2 = 16$: center $(-5, 2)$, $r = 4$.

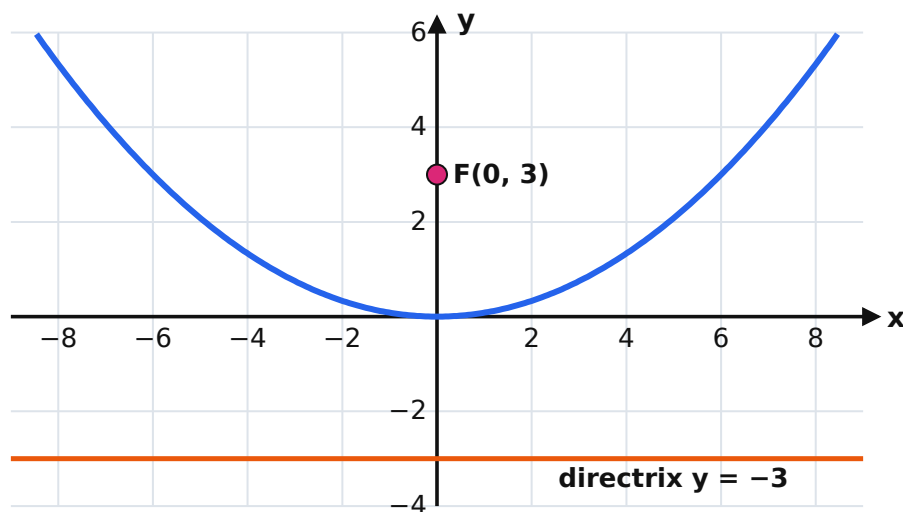
c. $(x^2 + 2x + 1) + (y^2 - 6y + 9) = 15 + 1 + 9$, so $(x+1)^2 + (y-3)^2 = 25$: center $(-1, 3)$, $r = 5$.

(a) $(4, -3)$, **4** **(b)** $(-5, 2)$, **4** **(c)** $(-1, 3)$, **5**

Precalculus Conic Sections Worksheet Answer Key: problems 6-10

Problem 6: Parabola from focus and directrix

The vertex is halfway between focus and directrix: $(0, 0)$. The focus is $p=3$ above the vertex, so $x^2 = 4py = 12y$.



The parabola opens upward.

$$x^2 = 12y$$

Problem 7: Focus and directrix from an equation

- a. $4p=16$, so $p=4$. The parabola opens right: focus $(4, 0)$, directrix $x=-4$.
- b. $4p=-20$, so $p=-5$. It opens down: focus $(0, -5)$, directrix $y=5$.

(a) $p=4$, $(4, 0)$, $x=-4$ **(b)** $p=-5$, $(0, -5)$, $y=5$

Problem 8: Equal distances

Distance to the focus: $\sqrt{(4-0)^2 + (2-2)^2} = 4$. Distance to the directrix: $2 - (-2) = 4$. The distances are equal.

Check with the equation $x^2 = 8y$: $4^2 = 16 = 8(2)$. **Both distances equal 4, so P is on the parabola.**

Problem 9: Parabola with a shifted vertex

The focus is $4-1=3$ above the vertex, so $p=3$ and the parabola opens up. The form is $(x-h)^2 = 4p(y-k)$.

$(x-2)^2 = 12(y-1)$. The directrix is 3 below the vertex: $y = 1-3 = -2$.

$(x-2)^2 = 12(y-1)$; **directrix** $y = -2$

Problem 10: Parabola opening left

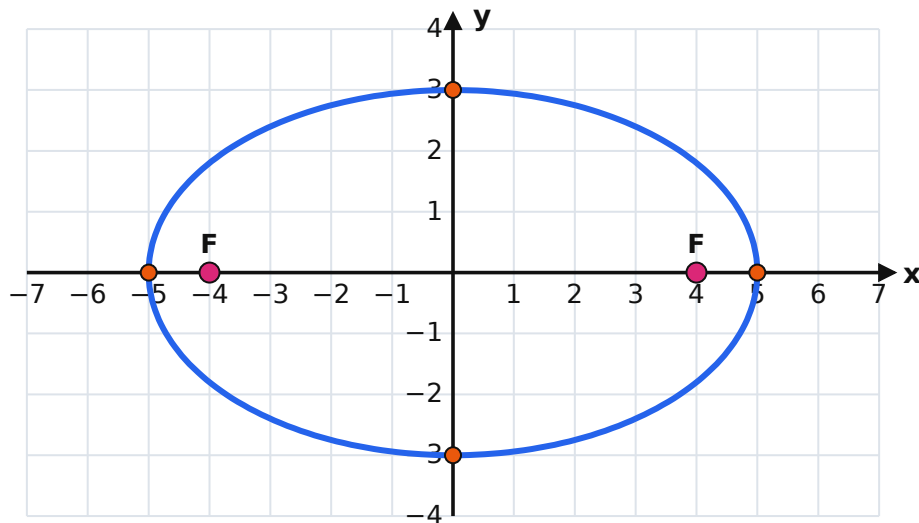
The vertex is $(0, 0)$. The focus is 3 units left, so $p = -3$ and $y^2 = 4px = -12x$.

Check $(-3, 6)$: $36 = -12(-3)$. Distance to focus 6, to directrix $3 - (-3) = 6$. $y^2 = -12x$

Precalculus Conic Sections Worksheet Answer Key: problems 11-15

Problem 11: Ellipse features

$a^2 = 25$ and $b^2 = 9$, so $a = 5$, $b = 3$. Then $c^2 = a^2 - b^2 = 16$ and $c = 4$.



Orange: vertices and co-vertices. Pink: foci.

$a = 5$, $b = 3$, $c = 4$; **vertices** $(\pm 5, 0)$; **co-vertices** $(0, \pm 3)$; **foci** $(\pm 4, 0)$

Problem 12: A vertical ellipse

The larger denominator is under y : $a^2 = 25$, $b^2 = 16$, so $a = 5$, $b = 4$ and the major axis is vertical.
 $c^2 = 25 - 16 = 9$, $c = 3$.

Vertices $(0, \pm 5)$, **foci** $(0, \pm 3)$; **the major axis is the y -axis**

Problem 13: Ellipse from its foci

$2a = 10$, so $a = 5$. The foci give $c = 3$. Then $b^2 = a^2 - c^2 = 25 - 9 = 16$.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Problem 14: A shifted ellipse

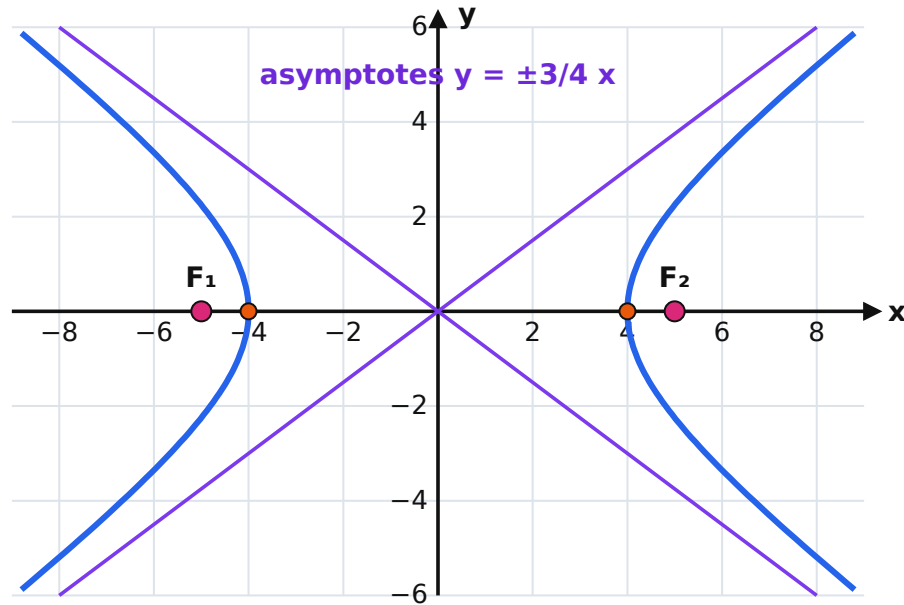
Center $(2, -1)$. $a^2 = 36$, $b^2 = 20$, so $a = 6$ and $c^2 = 36 - 20 = 16$, $c = 4$. The major axis is horizontal.

Vertices: $(2 \pm 6, -1)$, that is $(-4, -1)$ and $(8, -1)$. Foci: $(2 \pm 4, -1)$, that is $(-2, -1)$ and $(6, -1)$. The sum is $2a = 12$.

Center $(2, -1)$; **vertices** $(-4, -1)$, $(8, -1)$; **foci** $(-2, -1)$, $(6, -1)$; **sum** 12

Problem 15: Hyperbola features

$a = 4$, $b = 3$, $c^2 = 16 + 9 = 25$, $c = 5$. The branches open left and right.



Vertices orange, foci pink, asymptotes violet.

Vertices $(\pm 4, 0)$; **foci** $(\pm 5, 0)$; **asymptotes** $y = \pm \frac{3}{4}x$

Precalculus Conic Sections Worksheet Answer Key: problems 16-20

Problem 16: A vertical hyperbola

The positive term is y , so the branches open up and down. $a^2 = 9$, $b^2 = 16$, $a = 3$, $b = 4$, $c^2 = 9 + 16 = 25$, $c = 5$.

The asymptotes have slope $\pm \frac{a}{b} = \pm \frac{3}{4}$.

Vertices $(0, \pm 3)$; **foci** $(0, \pm 5)$; **asymptotes** $y = \pm \frac{3}{4}x$

Problem 17: Hyperbola from its foci

$2a = 8$, so $a = 4$. The foci give $c = 5$, and $b^2 = c^2 - a^2 = 25 - 16 = 9$.

Check $(5, 2.25)$: distances 2.25 and 10.25 differ by 8. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Problem 18: Identify an ellipse

$$4(x^2-4x+4)+9(y^2+2y+1)=11+16+9=36, \text{ so } 4(x-2)^2+9(y+1)^2=36.$$

Divide by 36: $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{4}=1$. It is an ellipse with center $(2, -1)$, $a=3$, $b=2$, $c=\sqrt{9-4}=\sqrt{5}$.

Ellipse; center $(2, -1)$; **foci** $(2\pm\sqrt{5}, -1)$

Problem 19: Identify a hyperbola

$$(x^2+6x+9)-4(y^2-4y+4)=23+9-16=16, \text{ so } (x+3)^2-4(y-2)^2=16.$$

Divide by 16: $\frac{(x+3)^2}{16}-\frac{(y-2)^2}{4}=1$. It is a hyperbola with center $(-3, 2)$, $a=4$, $b=2$, so the asymptotes have slope $\pm\frac{1}{2}$.

Hyperbola; center $(-3, 2)$; **asymptotes** $y-2=\pm\frac{1}{2}(x+3)$

Problem 20: Identify a parabola

$$y^2-6y+9=8x-25+9, \text{ so } (y-3)^2=8x-16=8(x-2).$$

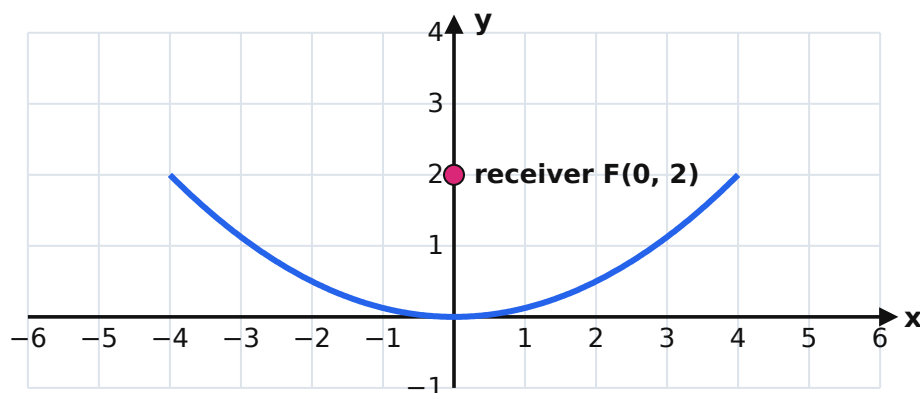
Vertex $(2, 3)$; $4p=8$, so $p=2$ and the parabola opens right. Focus $(2+2, 3)=(4, 3)$; directrix $x=2-2=0$.

$$(y-3)^2=8(x-2); \text{ vertex } (2, 3); \text{ focus } (4, 3); \text{ directrix } x=0$$

Precalculus Conic Sections Worksheet Answer Key: problems 21-25

Problem 21: Satellite dish

The rim point $(4, 2)$ satisfies $x^2=4py$: $16=4p(2)=8p$, so $p=2$. The equation is $x^2=8y$.



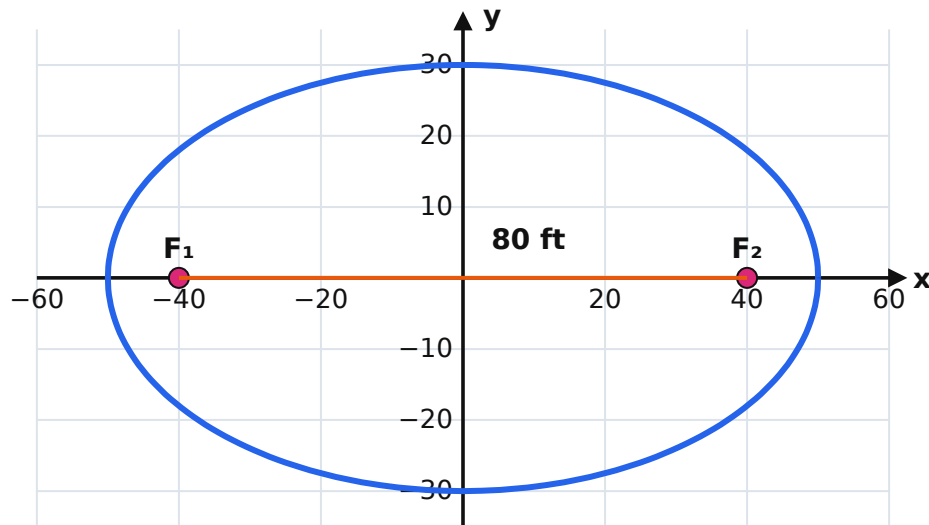
The focus is at height $p = 2$.

$x^2=8y$; **the receiver is 2 feet above the vertex**

Problem 22: Whispering gallery

$2a = 100$ so $a = 50$; $2b = 60$ so $b = 30$. The equation is $\frac{x^2}{2500} + \frac{y^2}{900} = 1$.

$c^2 = 2500 - 900 = 1600$, so $c = 40$. The foci are $(\pm 40, 0)$.



A whisper at one focus is heard at the other.

$\frac{x^2}{2500} + \frac{y^2}{900} = 1$; the foci are 80 feet apart

Problem 23: Sprinkler coverage

Boundary: $(x-20)^2 + (y-30)^2 = 225$.

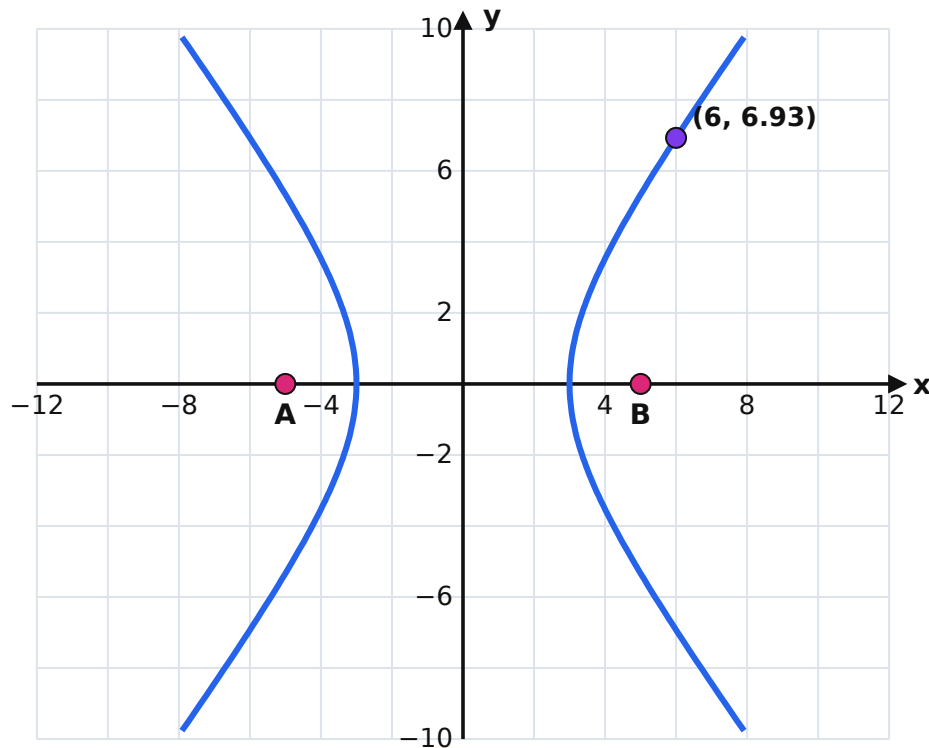
Flower: $(24-20)^2 + (38-30)^2 = 16 + 64 = 80 < 225$, so it is inside. Bench: $(35-20)^2 + (45-30)^2 = 225 + 225 = 450 > 225$, so it is outside.

$(x-20)^2 + (y-30)^2 = 225$; the flower is watered, the bench is not

Problem 24: Navigation stations

$2a = 6$ so $a = 3$; $c = 5$ so $b^2 = 25 - 9 = 16$. The equation is $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

For $x = 6$: $\frac{36}{9} - \frac{y^2}{16} = 1$, so $\frac{y^2}{16} = 3$ and $y^2 = 48$, $y = 4\sqrt{3} \approx 6.93$.



The ship at (6, 6.93).

$$\frac{x^2}{9} - \frac{y^2}{16} = 1; y = 4\sqrt{3} \approx 6.93 \text{ miles}$$

Problem 25: Elliptical garden

$a=12$, $b=9$, so $c^2=144-81=63$ and $c=3\sqrt{7} \approx 7.94$. The stakes are $2c \approx 15.87$ feet apart.

A pencil on the taut string has distances to the stakes adding to $2a=24$ feet. The loop = that sum plus the stake gap: $24+15.87 \approx 39.87$ feet.

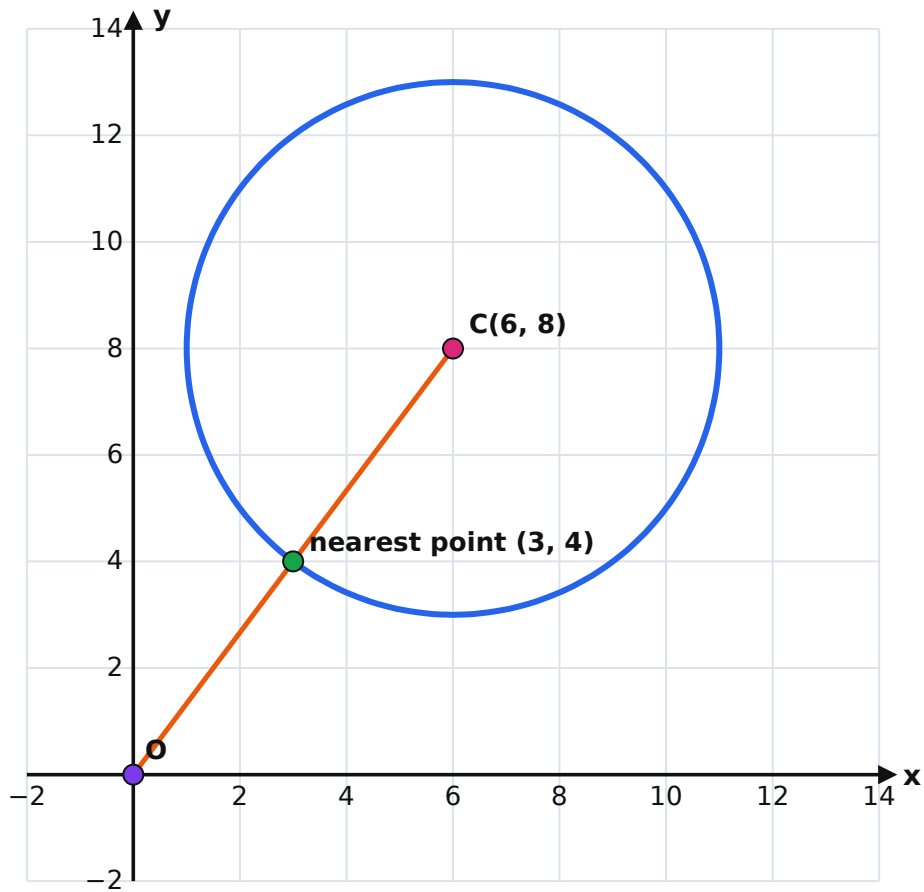
Stakes about 15.87 feet apart; loop about 39.87 feet

Precalculus Conic Sections Worksheet Answer Key: problems 26-26

Problem 26: Circular fountain

$(x^2-12x+36)+(y^2-16y+64)=-75+36+64=25$, so $(x-6)^2+(y-8)^2=25$: center (6, 8), radius 5.

The center is $\sqrt{6^2+8^2}=10$ feet from the entrance, so the nearest edge point is $10-5=5$ feet away.



The nearest point is (3, 4).

Center (6, 8), radius 5 ft; shortest distance 5 ft

How to use this precalculus conic sections worksheet answer key

Read this precalculus conic sections worksheet answer key after trying each problem yourself, and redo any problem you missed.

