

Precalculus · Conic Sections: Parabolas, Ellipses and Hyperbolas · Lesson

Common Core G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · mathovore.fr

This precalculus conic sections lesson explains conic sections: parabolas, ellipses and hyperbolas in one printable page, with definitions, a method and a worked example.

Conic sections are curves defined by distances. A circle keeps a constant distance from its center, a parabola is equally far from a focus and a directrix, an ellipse has a constant sum of distances to two foci, and a hyperbola has a constant difference. Completing the square converts a general equation to standard form to identify each one.

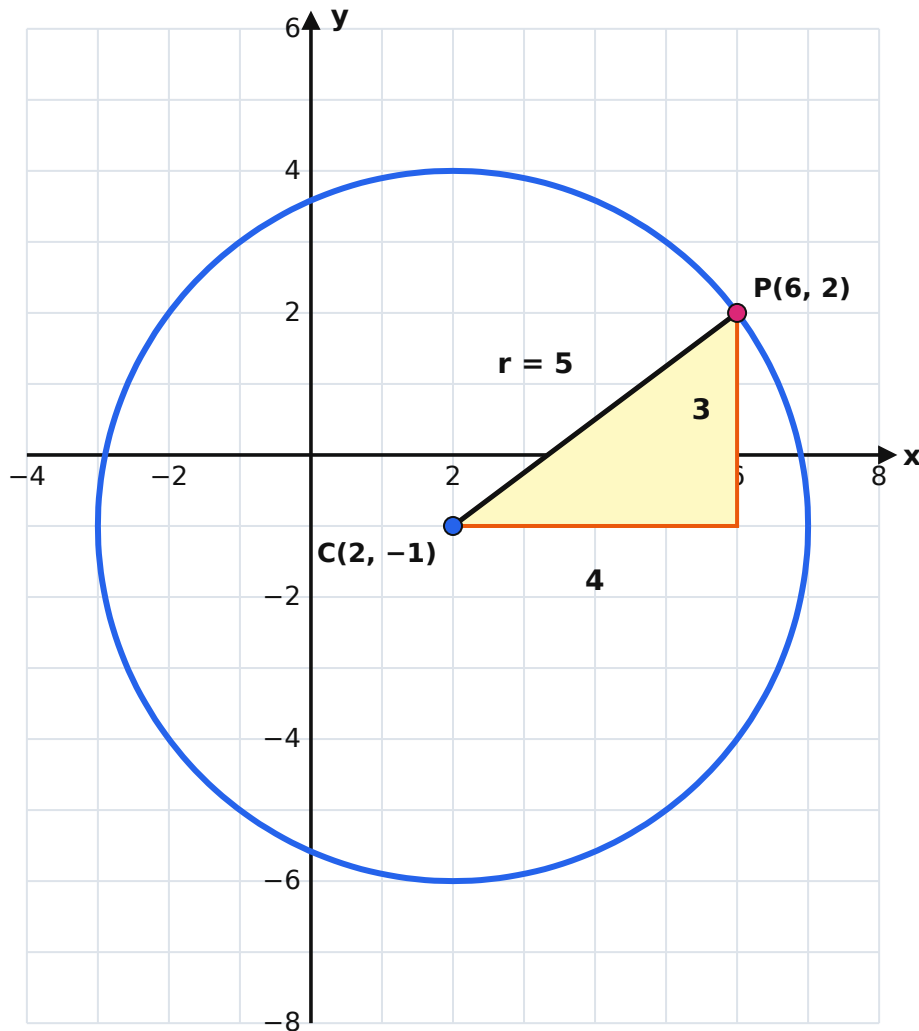
Precalculus Common Core: G-GPE.A.1 · G-GPE.A.2 · G-GPE.A.3

Precalculus Conic Sections Lesson: key facts

- Circle: $(x-h)^2 + (y-k)^2 = r^2$, from the distance formula.
- Parabola: $x^2 = 4py$ has focus $(0, p)$ and directrix $y = -p$.
- Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $c^2 = a^2 - b^2$; the distance sum is $2a$.
- Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with $c^2 = a^2 + b^2$ and asymptotes $y = \pm \frac{b}{a}x$.
- Complete the square in x and y to rewrite a general conic equation in standard form.

What is the equation of a circle?

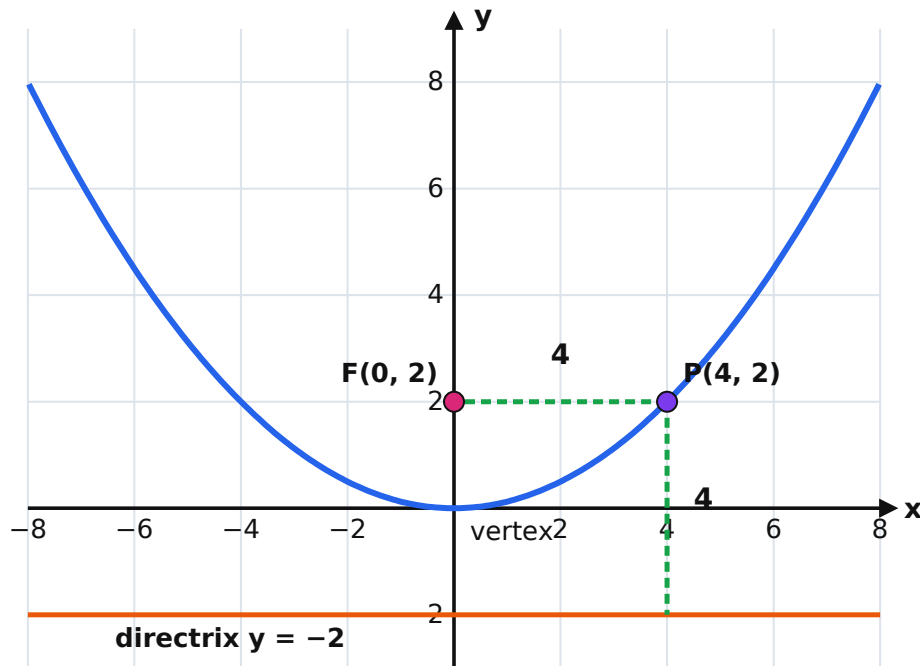
A **circle** is the set of points at a fixed distance r from a center (h, k) . The distance formula, which comes from the Pythagorean theorem, gives $(x-h)^2 + (y-k)^2 = r^2$. The circle below has center $(2, -1)$ and passes through $(6, 2)$, so $r = \sqrt{4^2 + 3^2} = 5$ and its equation is $(x-2)^2 + (y+1)^2 = 25$.



The legs 4 and 3 make a 3-4-5 right triangle, so the radius is 5.

How is a parabola defined by a focus and a directrix?

A **parabola** is the set of points equally far from a fixed point, the **focus**, and a fixed line, the **directrix**. With vertex at the origin and focus $(0, p)$, the directrix is $y = -p$ and the equation is $x^2 = 4py$. For focus $(0, 2)$, $p = 2$, so $x^2 = 8y$. The point $(4, 2)$ is 4 units from the focus and $2 - (-2) = 4$ units from the directrix.



Every point of the parabola is as far from F as from the dashed line.

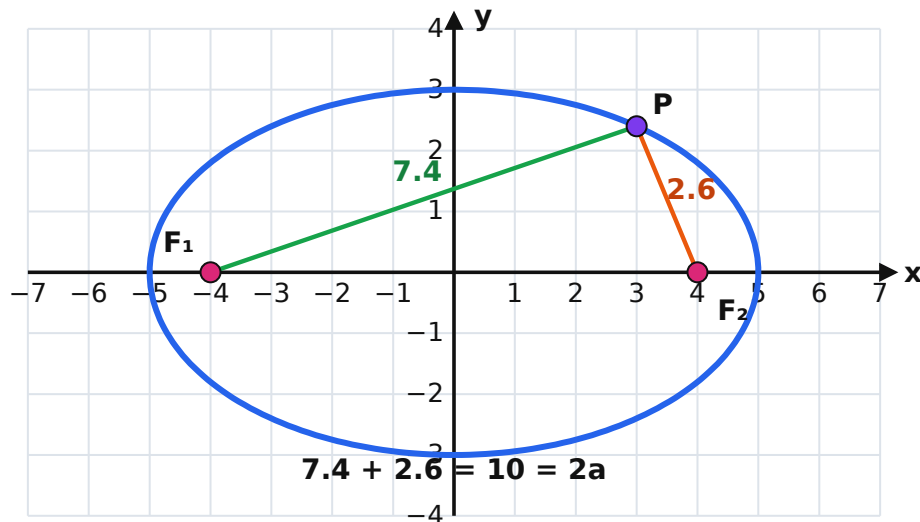
What defines an ellipse and a hyperbola?

Foci definitions

Ellipse: the sum of the distances to two foci is constant, $2a$. Standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$ and $c^2 = a^2 - b^2$; the foci are $(\pm c, 0)$.

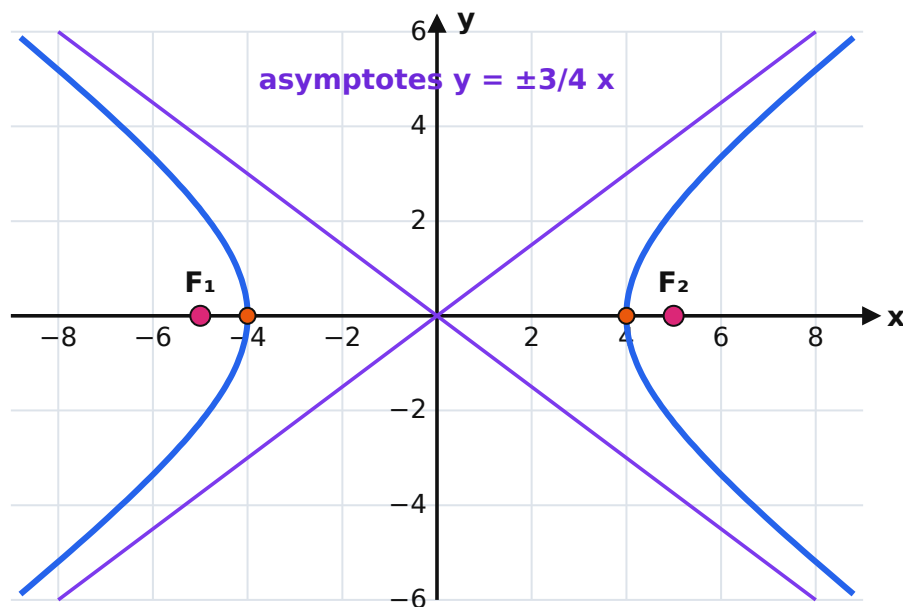
Hyperbola: the absolute difference of the distances is constant, $2a$. Standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with $c^2 = a^2 + b^2$; the foci are $(\pm c, 0)$ and the asymptotes are $y = \pm \frac{b}{a}x$.

For $\frac{x^2}{25} + \frac{y^2}{9} = 1$: $c^2 = 25 - 9 = 16$, so the foci are $(\pm 4, 0)$ and the point $(3, 2.4)$ has distances 7.4 and 2.6, which add to $10 = 2a$.



The two focal distances always add up to 10.

For $\frac{x^2}{16} - \frac{y^2}{9} = 1$: $c^2 = 16 + 9 = 25$, so the foci are $(\pm 5, 0)$. The point $(5, 2.25)$ is 2.25 from one focus and 10.25 from the other, a difference of $8 = 2a$.

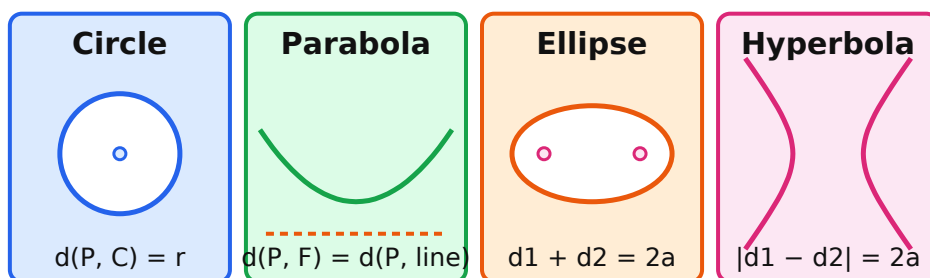


The branches approach the violet asymptotes.

How do you identify a conic by completing the square?

1. Group the x terms and the y terms.
2. Complete the square in each group, adding the same amounts to both sides.
3. Divide so the right side is 1, then read the type: equal positive squares give a circle; a sum with different denominators gives an ellipse; a difference gives a hyperbola; only one squared variable gives a parabola.

Example. $x^2+y^2-6x+4y-12=0$ becomes $(x-3)^2+(y+2)^2=12+9+4=25$: a circle with center $(3, -2)$ and radius 5.



Each conic comes with its own distance rule.

Conic Sections: Parabolas, Ellipses and Hyperbolas: frequently asked questions

What is a conic section?

A conic section is a curve formed by slicing a cone with a plane: a circle, ellipse, parabola or hyperbola. Each can also be defined by a distance rule involving a center, a focus, or two foci.

How do you find the focus of a parabola?

Write the equation as $x^2 = 4py$ or $y^2 = 4px$ and read p . The focus is p units from the vertex inside the curve, and the directrix is p units on the other side.

How can you tell an ellipse from a hyperbola in an equation?

Both have x^2 and y^2 terms. With a plus sign (and different positive denominators) it is an ellipse. With a minus sign it is a hyperbola.

Why is $c^2 = a^2 - b^2$ for an ellipse but $a^2 + b^2$ for a hyperbola?

For an ellipse the foci lie inside the vertices, so $c < a$. For a hyperbola the foci lie beyond the vertices, so $c > a$. The distance definitions give these two relations.

How do you complete the square for a conic?

Group the x terms and the y terms, add the needed constants to both sides to form perfect squares, and divide to make the right side 1 when the conic is an ellipse or hyperbola.

How to use this precalculus conic sections lesson

Use this precalculus conic sections lesson before the worksheet, then check what you learned

with the quiz.

