

Precalculus · Complex Numbers in Polar Form · Worksheet

Common Core N-CN.B.4, N-CN.B.5, N-CN.B.6, N-CN.A.3 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This precalculus complex polar worksheet gives 22 problems on complex numbers in polar form, ordered from easy to hard, with a separate answer key.

These 22 problems practice complex numbers in the plane and in polar form at the Precalculus level: modulus and argument, conjugates and division, converting between rectangular and polar form, products and quotients, De Moivre's theorem, and distance and midpoint. Standards N-CN.A.3 and N-CN.B.4 through B.6 are covered.

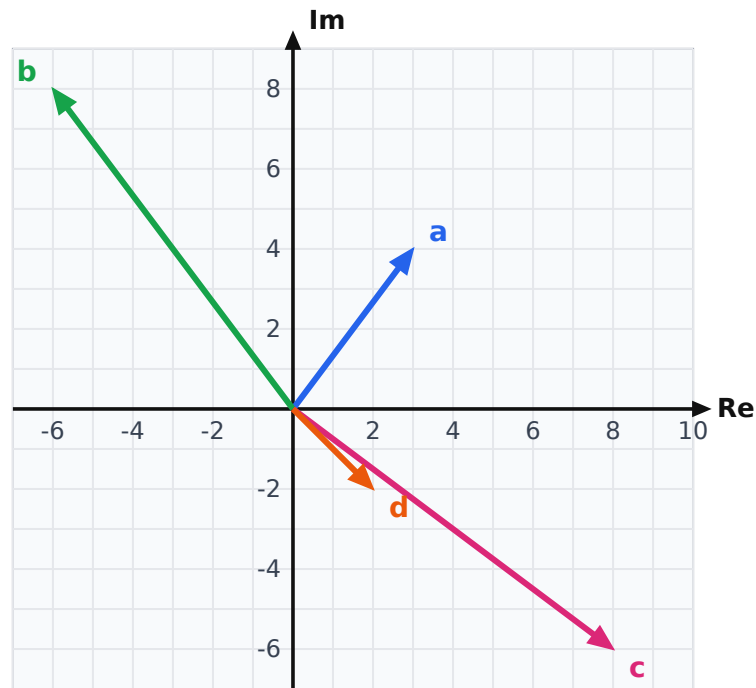
The set moves from short skill checks to multi-step problems about rotation, circuits and geometry in the complex plane. Sketch the point whenever a figure is not provided.

Precalculus Common Core: N-CN.B.4 · N-CN.B.5 · N-CN.B.6 · N-CN.A.3

Precalculus Complex Polar Worksheet: problems 1-5

Problem 1: Plot and find the modulus

The figure shows four complex numbers. Write each one in the form $a+bi$ and find its modulus.



Four points in the complex plane.

Problem 2: Conjugates and products

Find the conjugate of each number and the product of the number with its conjugate.

- a. $3-2i$
- b. $5+i$
- c. $-4i$
- d. $-2-6i$

Problem 3: Dividing complex numbers

Write each quotient in the form $a+bi$.

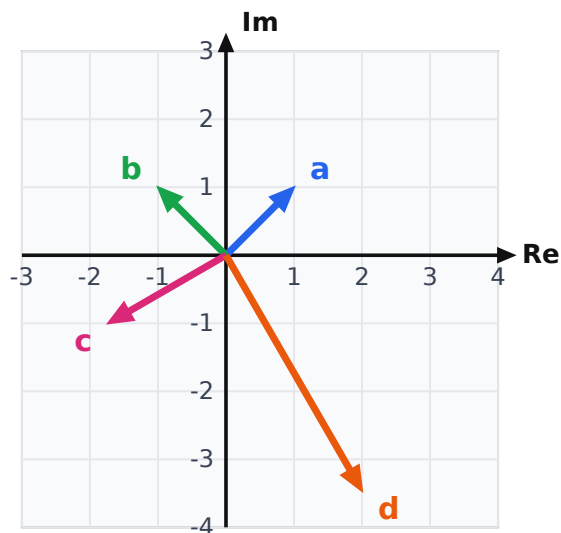
- a. $\frac{2+3i}{1-i}$
- b. $\frac{2+i}{3-i}$
- c. $\frac{1+2i}{4+2i}$
- d. $\frac{4+2i}{-i}$

Problem 4: Finding the argument

Find the argument, between 0° and 360° , of each number shown.

- a. $a=1+i$
- b. $b=-1+i$
- c. $c=-\sqrt{3}-i$

d. $d = 2 - 2\sqrt{3}i$



Four numbers in different quadrants.

Problem 5: Rectangular to polar form

Write each number in polar form $r(\cos\theta + i\sin\theta)$ with $0^\circ \leq \theta < 360^\circ$.

- a. $3\sqrt{3} + 3i$
- b. $-2 + 2i$
- c. $-5i$
- d. $1 - \sqrt{3}i$

Precalculus Complex Polar Worksheet: problems 6-10

Problem 6: Polar to rectangular form

Write each number in the form $a + bi$. Give exact values.

- a. $4(\cos 60^\circ + i\sin 60^\circ)$
- b. $10(\cos 180^\circ + i\sin 180^\circ)$
- c. $6(\cos 225^\circ + i\sin 225^\circ)$
- d. $2\sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$

Problem 7: Radian measure

Write each number in the form $a + bi$.

- a. $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
- b. $5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$
- c. $8\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

d. $6\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

Problem 8: Conjugate in polar form

Let $z = 6(\cos 50^\circ + i\sin 50^\circ)$.

- Write $\bar{z}$ in polar form with an angle between 0° and 360° .
- Find $z\bar{z}$ using polar form.

Problem 9: Multiplying in polar form

Find each product. Write the result in polar form, then in the form $a+bi$.

- $2(\cos 25^\circ + i\sin 25^\circ) \cdot 7(\cos 35^\circ + i\sin 35^\circ)$
- $3(\cos 110^\circ + i\sin 110^\circ) \cdot 4(\cos 70^\circ + i\sin 70^\circ)$
- $\sqrt{2}(\cos 200^\circ + i\sin 200^\circ) \cdot \sqrt{2}(\cos 160^\circ + i\sin 160^\circ)$

Problem 10: Dividing in polar form

Find each quotient. Write the result in polar form, then in the form $a+bi$.

- $\frac{12(\cos 150^\circ + i\sin 150^\circ)}{3(\cos 30^\circ + i\sin 30^\circ)}$
- $\frac{10(\cos 80^\circ + i\sin 80^\circ)}{5(\cos 125^\circ + i\sin 125^\circ)}$
- $\frac{6(\cos \pi + i\sin \pi)}{2(\cos \frac{\pi}{2} + i\sin \frac{\pi}{2})}$

Precalculus Complex Polar Worksheet: problems 11-15

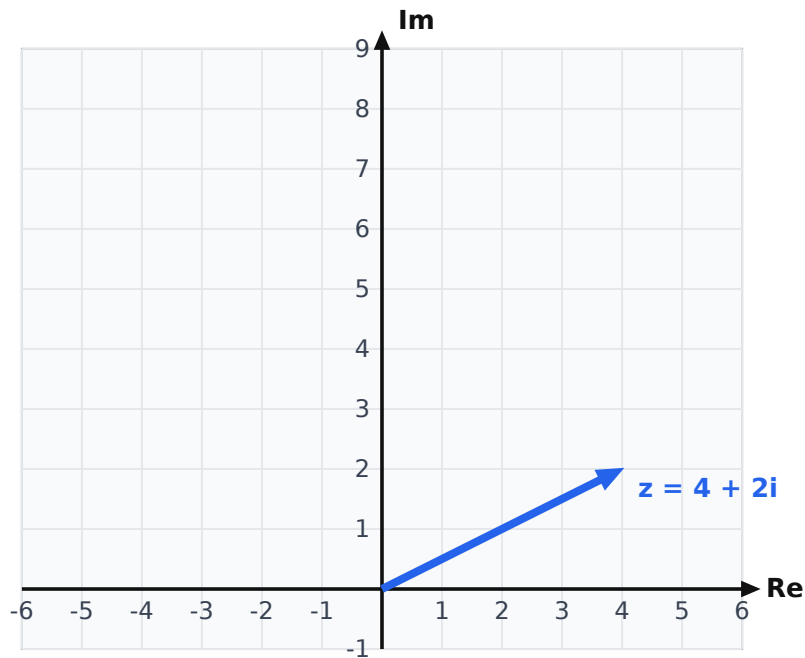
Problem 11: Check a product two ways

Let $z_1 = 1+i$ and $z_2 = \sqrt{3}+i$.

- Find $z_1 z_2$ by multiplying in rectangular form.
- Write z_1 and z_2 in polar form and find the product in polar form.
- Confirm that the two results agree by comparing the real parts to the nearest hundredth.

Problem 12: Rotation and scaling

The point $z = 4+2i$ is shown. Multiply z by $2i$. Describe the effect on the modulus and the argument, and plot the result.



The point $z = 4 + 2i$.

Problem 13: Modulus of products and quotients

Let $z_1 = 3 + 4i$ and $z_2 = 5 - 12i$.

- Find $z_1 z_2$ and its modulus.
- Show that $|z_1 z_2| = |z_1| |z_2|$.
- Find $\left| \frac{z_1}{z_2} \right|$ without dividing the complex numbers.

Problem 14: De Moivre's theorem: basics

Use De Moivre's theorem to simplify. Give each answer in the form $a + bi$.

- $(\cos 20^\circ + i \sin 20^\circ)^9$
- $[2(\cos 15^\circ + i \sin 15^\circ)]^4$
- $(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8})^4$

Problem 15: Powers of complex numbers

Evaluate each power by converting to polar form first.

- $(1 + i)^8$
- $(1 - i)^6$
- $(-1 + \sqrt{3}i)^3$
- $(\sqrt{3} + i)^5$

Precalculus Complex Polar Worksheet: problems 16-20

Problem 16: Distance in the complex plane

Find the distance between each pair of complex numbers.

- a. $1+2i$ and $5+5i$
- b. $-3+4i$ and $2-8i$
- c. $7-i$ and $-1+5i$

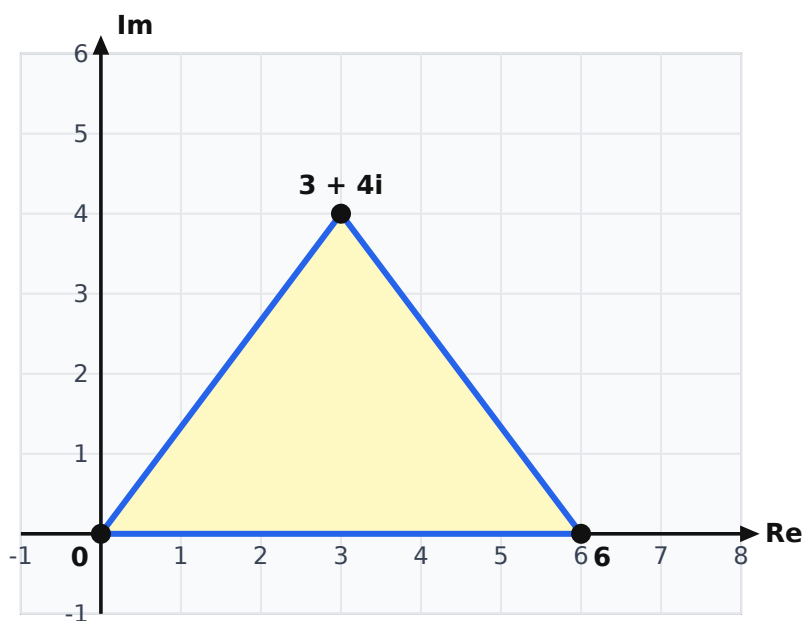
Problem 17: Midpoints

Complete each part.

- a. Find the midpoint of $1+2i$ and $5+5i$.
- b. Find the midpoint of $-4+6i$ and $2-2i$.
- c. One endpoint of a segment is $3-i$ and its midpoint is $1+4i$. Find the other endpoint.

Problem 18: Triangle in the complex plane

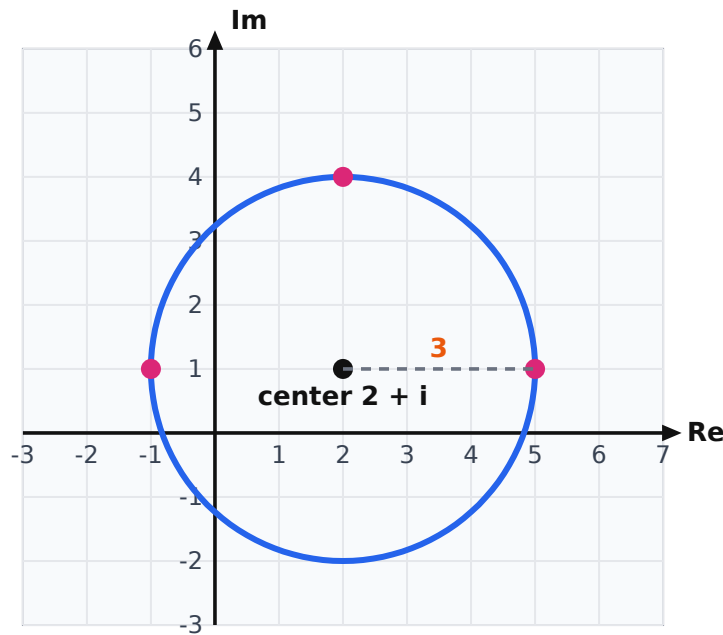
The vertices of a triangle are the complex numbers 0, 6 and $3+4i$. Show that the triangle is isosceles, then find its perimeter and area.



A triangle with vertices 0, 6 and $3 + 4i$.

Problem 19: Circle in the complex plane

The set of points z with $|z-(2+i)|=3$ is a circle. State its center and radius. Then decide whether each point lies on the circle, inside it, or outside it: $5+i$, $-1+i$, $2+4i$, $4+3i$.



The circle with center $2 + i$ and radius 3.

Problem 20: Robot vacuum rotation

A robot vacuum is at the point $z = 2 + 3i$ (distances in feet) on a floor plan whose origin is the charging dock. It rotates about the dock by 90° counterclockwise. Where does it end up? Then it rotates the original position by 30° counterclockwise instead. Find that position to the nearest hundredth.

Precalculus Complex Polar Worksheet: problems 21-22

Problem 21: AC circuit impedance

In an AC circuit, an impedance of $Z = 8 + 6i$ ohms has a voltage of $V = 120$ volts across it. Find the modulus of Z , its phase angle to the nearest hundredth of a degree, and the current $I = \frac{V}{Z}$ in the form $a + bi$. Then find the magnitude of the current.

Problem 22: Powers: multi-step

Complete each part.

- Evaluate $(\sqrt{3} - i)^6$.
- Find the smallest positive integer n for which $(\cos 40^\circ + i \sin 40^\circ)^n = 1$.
- Find the smallest positive integer n for which $(\sqrt{3} + i)^n$ is a purely imaginary number.

How to use this precalculus complex polar worksheet

Print this precalculus complex polar worksheet as a PDF, solve every problem on paper, then

compare with the answer key.

