

Precalculus · Complex Numbers in Polar Form · Lesson

Common Core N-CN.B.4, N-CN.B.5, N-CN.B.6, N-CN.A.3 · mathovore.fr

This precalculus complex polar lesson explains complex numbers in polar form in one printable page, with definitions, a method and a worked example.

A complex number $a + bi$ is a point in the complex plane with modulus $r = \sqrt{a^2 + b^2}$ and argument θ , giving the polar form $r(\cos \theta + i \sin \theta)$. In polar form, multiplication multiplies moduli and adds arguments, division divides moduli and subtracts arguments, and De Moivre's theorem raises z to the power n .

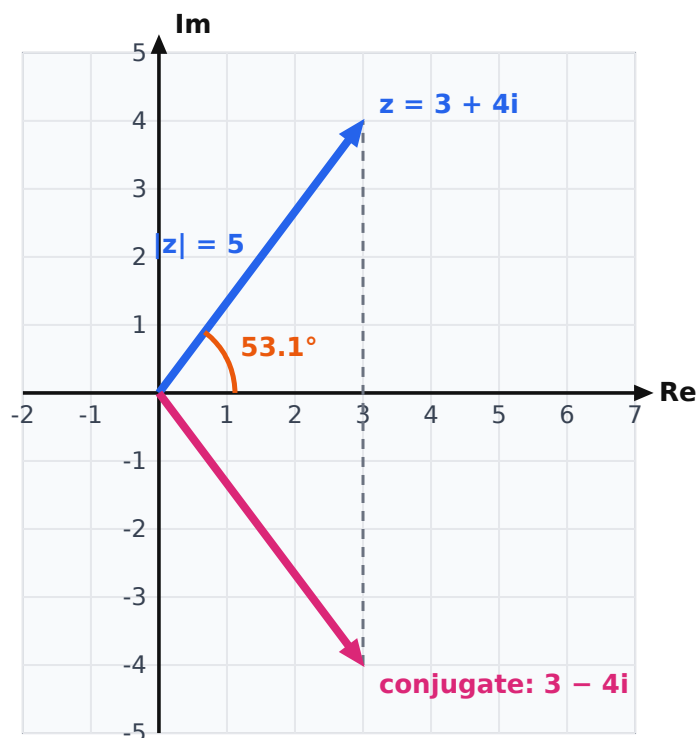
Precalculus Common Core: N-CN.B.4 · N-CN.B.5 · N-CN.B.6 · N-CN.A.3

Precalculus Complex Polar Lesson: key facts

- Modulus $|a + bi| = \sqrt{a^2 + b^2}$; the conjugate $a - bi$ satisfies $z \cdot \bar{z} = |z|^2$.
- Polar form: $z = r(\cos \theta + i \sin \theta)$, with $a = r \cos \theta$ and $b = r \sin \theta$.
- Product: multiply moduli, add arguments. Quotient: divide moduli, subtract arguments.
- De Moivre: $[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$.
- Distance between z_1 and z_2 is $|z_1 - z_2|$; the midpoint is $(z_1 + z_2)/2$.

What are the modulus, argument, and conjugate of a complex number?

Every complex number $z = a + bi$ is a point (a, b) in the **complex plane**, with the real part on the horizontal axis and the imaginary part on the vertical axis. The **modulus** $|z| = \sqrt{a^2 + b^2}$ is the distance from the origin. The **argument** θ is the angle from the positive real axis to the segment from the origin to z , with $\tan \theta = \frac{b}{a}$ adjusted for the quadrant. The **conjugate** $\bar{z} = a - bi$ is the reflection of z across the real axis, and $z\bar{z} = a^2 + b^2 = |z|^2$.

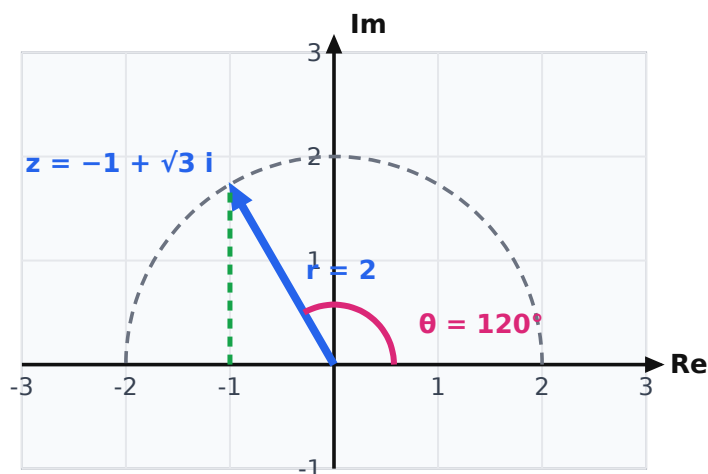


The point $3 + 4i$ has modulus 5; its conjugate is its mirror image across the real axis.

Conjugates also divide complex numbers: multiply the numerator and denominator by the conjugate of the denominator, so the denominator becomes the real number $|z|^2$.

How do you write a complex number in polar form?

Since $a = r \cos \theta$ and $b = r \sin \theta$, every complex number can be written as $z = r(\cos \theta + i \sin \theta)$ with $r = |z|$. For $z = -1 + \sqrt{3}i$: $r = \sqrt{1+3} = 2$, the point is in quadrant II, and $\tan \theta = -\sqrt{3}$ gives $\theta = 120^\circ$. So $z = 2(\cos 120^\circ + i \sin 120^\circ)$.



Polar form records the distance r and the angle θ instead of the horizontal and vertical steps.

Key formulas

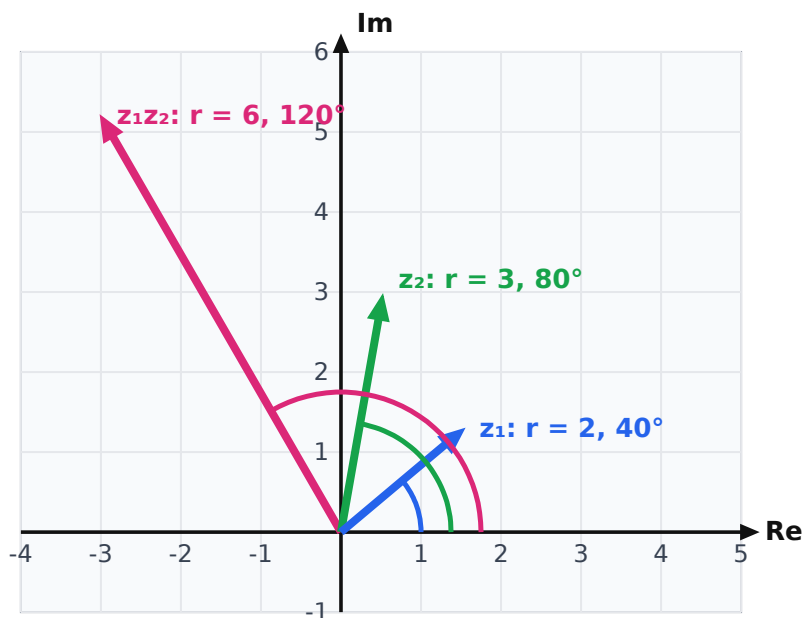
Product: $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$

Quotient: $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$

De Moivre's theorem: $z^n = r^n [\cos(n\theta) + i \sin(n\theta)]$

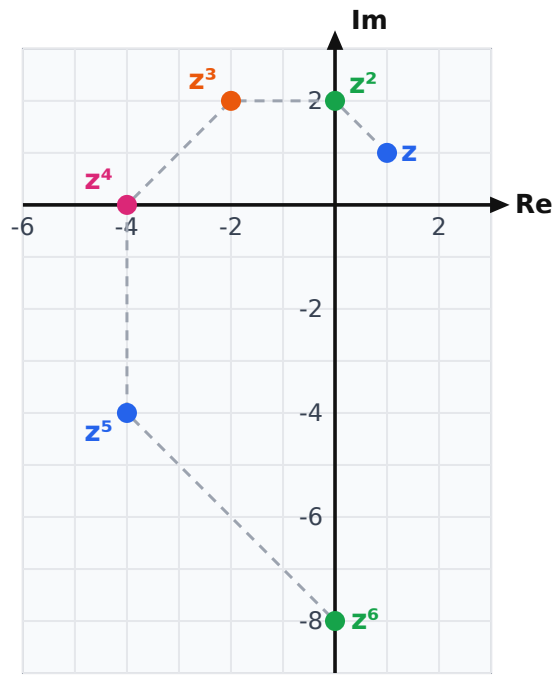
What happens when you multiply, divide, or take powers in polar form?

Multiplying **multiplies the moduli and adds the arguments**: the point is stretched and rotated. Dividing divides the moduli and subtracts the arguments. For $z_1 = 2(\cos 40^\circ + i \sin 40^\circ)$ and $z_2 = 3(\cos 80^\circ + i \sin 80^\circ)$, the product is $6(\cos 120^\circ + i \sin 120^\circ) = -3 + 3\sqrt{3}i$.



The moduli multiply ($2 \times 3 = 6$) and the angles add ($40^\circ + 80^\circ = 120^\circ$).

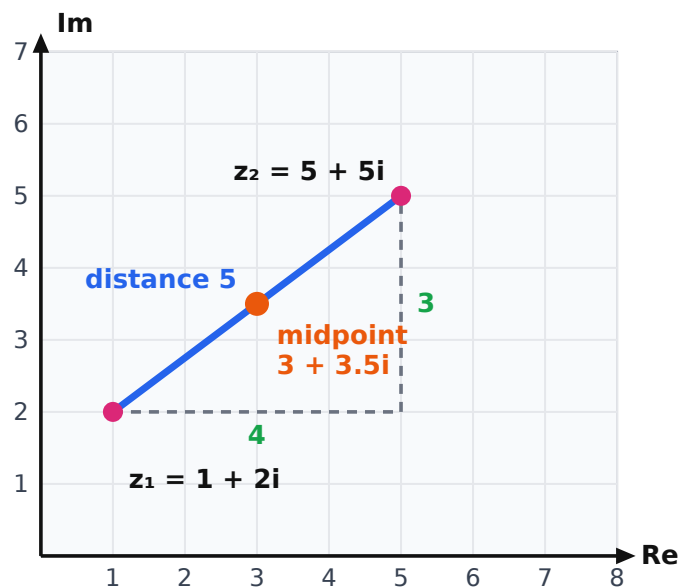
Repeated multiplication gives De Moivre's theorem. For $1+i$ we have $r = \sqrt{2}$ and $\theta = 45^\circ$, so $(1+i)^8 = (\sqrt{2})^8 (\cos 360^\circ + i \sin 360^\circ) = 16$. Each power turns the point another 45° .



Successive powers of $1 + i$ spiral outward, turning 45° each time.

How do you find distance and midpoint in the complex plane?

The distance between z_1 and z_2 is the modulus of their difference, $|z_1 - z_2|$. The midpoint is the average $\frac{z_1 + z_2}{2}$. For $z_1 = 1 + 2i$ and $z_2 = 5 + 5i$: $|z_1 - z_2| = |-4 - 3i| = 5$ and the midpoint is $\frac{6 + 7i}{2} = 3 + 3.5i$.



A 3-4-5 right triangle shows the distance; the midpoint is the average of the two numbers.

Complex Numbers in Polar Form: frequently asked questions

What is the difference between the modulus and the argument?

The modulus is the distance from the origin to the point, a nonnegative number. The argument is the angle, measured counterclockwise from the positive real axis. Together they locate a point just like a and b do.

Why does the argument of a complex number have many correct values?

Adding or subtracting 360° (or 2π radians) returns to the same direction, so 120° and 480° describe the same point. The principal argument is usually chosen between 0° and 360° , or between -180° and 180° .

Why does multiplying by i rotate a point by 90 degrees?

The number i has modulus 1 and argument 90° . Multiplying by it keeps the modulus unchanged and adds 90° to the argument, which is a quarter turn counterclockwise.

How do you divide complex numbers in rectangular form?

Multiply the numerator and denominator by the conjugate of the denominator. The denominator becomes a real number, $a^2 + b^2$, and you can split the result into real and imaginary parts.

When is polar form easier than rectangular form?

Polar form is easier for products, quotients, and powers, because you only multiply or divide moduli and add or subtract angles. Rectangular form is easier for sums and differences.

How to use this precalculus complex polar lesson

Use this precalculus complex polar lesson before the worksheet, then check what you learned with the quiz.

