

Precalculus · Complex Factorization and Volume of a Sphere · Worksheet Answer Key

Common Core N-CN.C.8, G-GMD.A.2 · mathovore.fr

This precalculus complex identities worksheet answer key solves all 28 problems of the complex factorization and volume of a sphere worksheet step by step.

Each solution shows the identity used, the substitution, and the final factored form or volume in bold. Check every factorization by expanding it, and check every volume against the cylinder-minus-cone comparison where it applies.

Precalculus Common Core: N-CN.C.8 · G-GMD.A.2

Precalculus Complex Identities Worksheet Answer Key: problems 1-5

Problem 1: Powers of i

The powers of i repeat every 4 steps: $i, -1, -i, 1$. Divide the exponent by 4 and use the remainder.

- a. $5 = 4 + 1$, so $i^5 = i \cdot i$
- b. $10 = 8 + 2$, so $i^{10} = i^2 = -1$
- c. $27 = 24 + 3$, so $i^{27} = i^3 = -i$
- d. $100 = 4(25)$, so $i^{100} = 1$

Problem 2: Expand a conjugate product

$$(x+3i)(x-3i) = x^2 - 3ix + 3ix - 9i^2 = x^2 - 9(-1) = x^2 + 9$$

Problem 3: Factor sums of squares

- a. $x^2 + 5^2 = (x+5i)(x-5i)$
- b. $x^2 + 7^2 = (x+7i)(x-7i)$
- c. $y^2 + 1^2 = (y+i)(y-i)$

Problem 4: Leading coefficients

- a. $(2x)^2 + 1^2 = (2x+i)(2x-i)$
- b. $(3x)^2 + 4^2 = (3x+4i)(3x-4i)$
- c. $(7y)^2 + 2^2 = (7y+2i)(7y-2i)$

Problem 5: Irrational constant

$$x^2+7 = x^2+(\sqrt{7})^2 = (x+i\sqrt{7})(x-i\sqrt{7}). \quad (x+i\sqrt{7})(x-i\sqrt{7})$$

Precalculus Complex Identities Worksheet Answer Key: problems 6-10

Problem 6: Factor out a common factor

- a. $2(x^2+9) = 2(x+3i)(x-3i). \quad 2(x+3i)(x-3i)$
- b. $5(x^2+9) = 5(x+3i)(x-3i). \quad 5(x+3i)(x-3i)$
- c. $3(x^2+25) = 3(x+5i)(x-5i). \quad 3(x+5i)(x-5i)$

Problem 7: Solve over the complex numbers

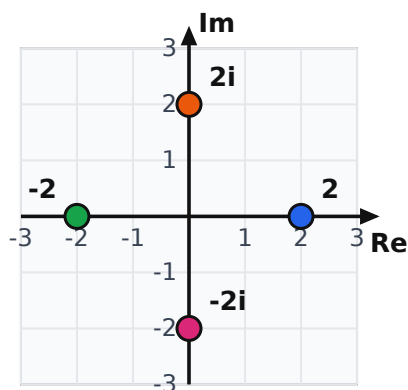
- a. $x^2 = -36$, so $x = \pm 6i$. $x = \pm 6i$
- b. $x^2 = -12$, so $x = \pm i\sqrt{12} = \pm 2i\sqrt{3}$. $x = \pm 2i\sqrt{3}$
- c. $x^2 = -\frac{9}{4}$, so $x = \pm \frac{3}{2}i$. $x = \pm \frac{3}{2}i$

Problem 8: Write a polynomial from its zeros

The factors are $(x-3i)$ and $(x+3i)$. $(x-3i)(x+3i) = x^2 - 9i^2 = x^2 + 9$. $x^2 + 9$

Problem 9: Fourth-degree difference

$x^4 - 16 = (x^2 - 4)(x^2 + 4)$. Then $x^2 - 4 = (x-2)(x+2)$ and $x^2 + 4 = (x+2i)(x-2i)$.
 $(x-2)(x+2)(x+2i)(x-2i)$



The four zeros of $x^4 - 16$ lie on a circle of radius 2.

Problem 10: Another fourth-degree difference

$$x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x-3)(x+3)(x+3i)(x-3i). \quad (x-3)(x+3)(x+3i)(x-3i)$$

Precalculus Complex Identities Worksheet Answer Key: problems 11-15

Problem 11: Cubic with a common factor

- a. $x(x^2+4) = x(x+2i)(x-2i)$. $x(x+2i)(x-2i)$
- b. $x(x^2+1) = x(x+i)(x-i)$. $x(x+i)(x-i)$

Problem 12: Factor by grouping

Group: $x^2(x-2)+4(x-2) = (x-2)(x^2+4) = (x-2)(x+2i)(x-2i)$. $(x-2)(x+2i)(x-2i)$, with zeros $2, 2i, -2i$.

Problem 13: Products of conjugates

- a. $9-4i^2 = 9+4 = 13$. 13
- b. $25-i^2 = 26$. 26
- c. $36-49i^2 = 85$. 85

Each product is a^2+b^2 , a real number.

Problem 14: Numbers as conjugate products

- a. $41 = 5^2+4^2 = (5+4i)(5-4i)$. $(5+4i)(5-4i)$
- b. $85 = 9^2+2^2 = (9+2i)(9-2i)$. $(9+2i)(9-2i)$
- c. $65 = 8^2+1^2 = (8+i)(8-i)$. $(8+i)(8-i)$

Check: $(5+4i)(5-4i) = 25+16 = 41$, and the others work the same way.

Problem 15: Evaluate at complex values

$$p(2i) = (2i)^2 + 4 = -4 + 4 = 0. \quad p(-2i) = (-2i)^2 + 4 = -4 + 4 = 0.$$
$$p(1+i) = (1+2i+i^2) + 4 = (1+2i-1) + 4 = 4+2i.$$

$0, 0, 4+2i$. The first two values confirm that $2i$ and $-2i$ are zeros.

Precalculus Complex Identities Worksheet Answer Key: problems 16-20

Problem 16: Identities with complex numbers

- a. $(z+2i)^2 = z^2 + 2z(2i) + (2i)^2 = z^2 + 4iz - 4$. $z^2 + 4iz - 4$
- b. $4+4i+i^2 = 3+4i$. $3+4i$

Problem 17: Complete the square, then factor

$$x^2 - 2x + 5 = (x^2 - 2x + 1) + 4 = (x - 1)^2 + 2^2 = (x - 1 + 2i)(x - 1 - 2i). (x - 1 + 2i)(x - 1 - 2i), \text{ with zeros } 1 \pm 2i.$$

Problem 18: Another quadratic

$$x^2 + 6x + 13 = (x^2 + 6x + 9) + 4 = (x + 3)^2 + 2^2 = (x + 3 + 2i)(x + 3 - 2i). (x + 3 + 2i)(x + 3 - 2i), \text{ zeros } -3 \pm 2i.$$

Problem 19: Volume of a sphere from radius or diameter

- $V = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi \approx 113.10$ cubic cm. 36π
- The radius is 5. $V = \frac{4}{3}\pi(125) = \frac{500\pi}{3} \approx 523.60$ cubic in. $\frac{500\pi}{3}$

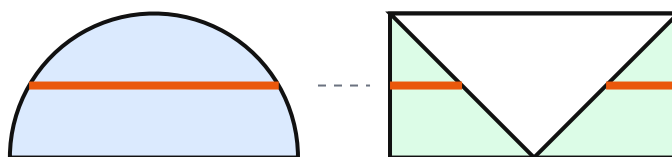
Problem 20: Volume of a hemisphere

$$V = \frac{2}{3}\pi(6)^3 = \frac{2}{3}\pi(216) = 144\pi. 144\pi \text{ cubic feet}$$

Precalculus Complex Identities Worksheet Answer Key: problems 21-25

Problem 21: Cylinder minus cone

Cylinder: $\pi(16)(4) = 64\pi$. Cone: $\frac{1}{3}\pi(16)(4) = \frac{64\pi}{3}$. Difference: $64\pi - \frac{64\pi}{3} = \frac{128\pi}{3}$. Hemisphere: $\frac{2}{3}\pi(64) = \frac{128\pi}{3}$. **Both equal $\frac{128\pi}{3}$ cubic inches.**



The two solids have equal volume.

Problem 22: Matching cross-sections

- The radius is $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$, so the area is 16π square cm.
- The ring has outer radius 5 and inner radius 3, so the area is $25\pi - 9\pi = 16\pi$ square cm.

Both areas are 16π , as Cavalieri's principle requires.

Problem 23: Leaning stack of coins

- Height: $30(0.1) = 3$ in. $V = \pi(0.5)^2(3) = 0.75\pi \approx 2.36$ cubic in. 0.75π
- The same, 0.75π cubic in. Every level cuts both stacks in a circle of area 0.25π square in., so by Cavalieri's principle the volumes are equal.

Problem 24: Melted ice cream

Sphere: $\frac{4}{3}\pi(1.5)^3 = \frac{4}{3}\pi(3.375) = 4.5\pi$. Cup: $\pi(1.5)^2h = 2.25\pi h$. Set equal: $2.25\pi h = 4.5\pi$, so $h = 2$. **2 inches deep**

Problem 25: Tennis ball can

The can height is $3(3) = 9$ in. Can volume: $\pi(2.25)(9) = 20.25\pi$. One ball: $\frac{4}{3}\pi(3.375) = 4.5\pi$, so three balls: 13.5π . Empty space: $20.25\pi - 13.5\pi = 6.75\pi \approx 21.21$. **About 21.21 cubic inches**

Precalculus Complex Identities Worksheet Answer Key: problems 26-28

Problem 26: Spherical water tank

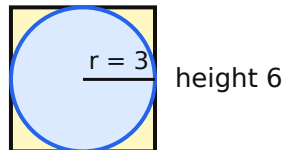
$V = \frac{4}{3}\pi(12)^3 = \frac{4}{3}\pi(1728) = 2304\pi \approx 7238.2$ cubic feet. Gallons: $2304\pi \times 7.48 \approx 54142$. **About 7,238.2 cubic feet, or about 54,142 gallons**

Problem 27: Basketball volume

The radius is 4.75. $V = \frac{4}{3}\pi(4.75)^3 = \frac{4}{3}\pi(107.171875) \approx 448.9$. **About 448.9 cubic inches**

Problem 28: Sphere and cylinder ratio

Cylinder: $\pi r^2(2r) = 2\pi r^3$. Sphere: $\frac{4}{3}\pi r^3$. Ratio: $\frac{4}{3} \div 2 = \frac{2}{3}$. For $r = 3$: cylinder $\pi(9)(6) = 54\pi$, sphere 36π , and $\frac{36}{54} = \frac{2}{3}$. **The ratio is $\frac{2}{3}$**



Sphere 36 pi inside cylinder 54 pi.

How to use this precalculus complex identities worksheet answer key

Read this precalculus complex identities worksheet answer key after trying each problem yourself, and redo any problem you missed.

