

Precalculus · Complex Factorization and Volume of a Sphere · Worksheet

Common Core N-CN.C.8, G-GMD.A.2 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This precalculus complex identities volume worksheet gives 28 problems on complex factorization and volume of a sphere, ordered from easy to hard, with a separate answer key.

These problems practice extending polynomial identities to the complex numbers (N-CN.C.8): powers of i , conjugate products, and factoring sums of squares such as $x^2+4=(x+2i)(x-2i)$. They also build an informal argument for the volume of a sphere with Cavalieri's principle (G-GMD.A.2).

The last six problems are word problems with spheres, cylinders, and cross-sections. Give exact answers first, then decimals where asked.

PrecalculusCommon Core: N-CN.C.8 · G-GMD.A.2

Precalculus Complex Identities Volume Worksheet: problems 1-5

Problem 1: Powers of i

Simplify each power.

- a. i^5
- b. i^{10}
- c. i^{27}
- d. i^{100}

Problem 2: Expand a conjugate product

Expand $(x+3i)(x-3i)$ and simplify.

Problem 3: Factor sums of squares

Factor each expression over the complex numbers.

- a. x^2+25
- b. x^2+49

c. y^2+1

Problem 4: Leading coefficients

Factor each expression over the complex numbers.

a. $4x^2+1$

b. $9x^2+16$

c. $49y^2+4$

Problem 5: Irrational constant

Factor x^2+7 over the complex numbers.

Precalculus Complex Identities Volume Worksheet: problems 6-10

Problem 6: Factor out a common factor

Factor completely over the complex numbers.

a. $2x^2+18$

b. $5x^2+45$

c. $3x^2+75$

Problem 7: Solve over the complex numbers

Solve each equation.

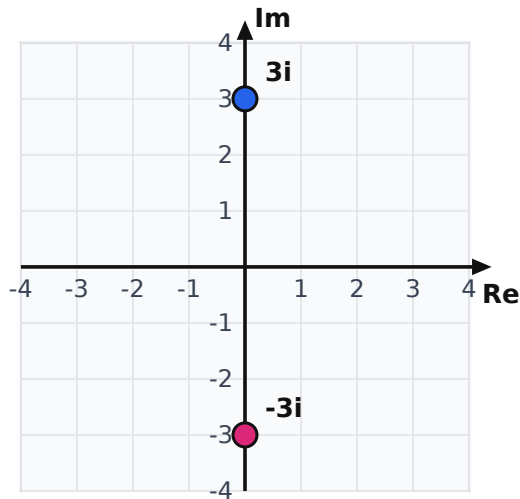
a. $x^2+36=0$

b. $x^2+12=0$

c. $4x^2+9=0$

Problem 8: Write a polynomial from its zeros

The complex plane shows the two zeros of a quadratic with leading coefficient 1.



Two zeros plotted in the complex plane.

Write the quadratic in standard form.

Problem 9: Fourth-degree difference

Factor $x^4 - 16$ completely over the complex numbers.

Problem 10: Another fourth-degree difference

Factor $x^4 - 81$ completely over the complex numbers.

Precalculus Complex Identities Volume Worksheet: problems 11-15

Problem 11: Cubic with a common factor

Factor completely over the complex numbers.

- a. $x^3 + 4x$
- b. $x^3 + x$

Problem 12: Factor by grouping

Factor $x^3 - 2x^2 + 4x - 8$ completely over the complex numbers and list all its zeros.

Problem 13: Products of conjugates

Multiply.

- a. $(3 + 2i)(3 - 2i)$
- b. $(5 + i)(5 - i)$
- c. $(6 + 7i)(6 - 7i)$

What pattern do the answers follow?

Problem 14: Numbers as conjugate products

Write each number as a product of two complex conjugates.

- a. 41 (use $25+16$)
- b. 85 (use $81+4$)
- c. 65 (use $64+1$)

Problem 15: Evaluate at complex values

Let $p(x) = x^2 + 4$. Find $p(2i)$, $p(-2i)$, and $p(1+i)$.

Precalculus Complex Identities Volume Worksheet: problems 16-20

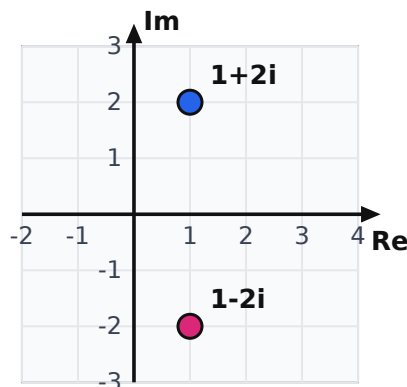
Problem 16: Identities with complex numbers

Use an identity to expand each product.

- a. $(z+2i)^2$
- b. $(2+i)^2$

Problem 17: Complete the square, then factor

Factor $x^2 - 2x + 5$ over the complex numbers. The plane shows where its zeros should be.



Zeros of the quadratic.

Problem 18: Another quadratic

Factor $x^2 + 6x + 13$ over the complex numbers and state its zeros.

Problem 19: Volume of a sphere from radius or diameter

Find the exact volume of each sphere and a decimal rounded to the nearest hundredth.

- a. radius 3 cm
- b. diameter 10 in.

Problem 20: Volume of a hemisphere

A hemisphere has radius 6 ft. Find its volume in terms of π .

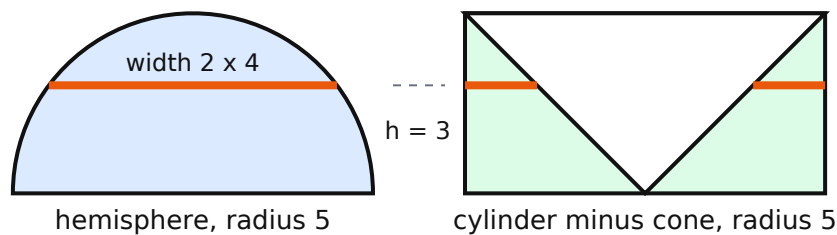
Precalculus Complex Identities Volume Worksheet: problems 21-25

Problem 21: Cylinder minus cone

A cylinder and a cone both have radius 4 in. and height 4 in. Find the volume of the cylinder with the cone removed. Compare it with a hemisphere of radius 4 in.

Problem 22: Matching cross-sections

A sphere of radius 5 cm is cut by a plane 3 cm above its center. The hemisphere on the left of the figure shows this cut, and the cylinder with the cone removed on the right is cut at the same height.

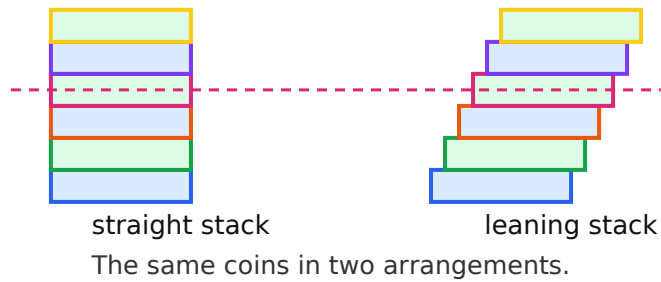


Both solids cut at height 3 cm.

- a. Find the radius and area of the circular cross-section of the sphere.
- b. Find the area of the ring cut from the cylinder with the cone removed.

Problem 23: Leaning stack of coins

Mei stacks 30 coins, each with radius 0.5 in. and thickness 0.1 in., then pushes the stack into a lean without moving any coin out of its level.



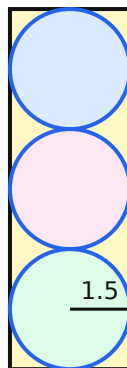
- a. Find the volume of the straight stack.
- b. What is the volume of the leaning stack? Explain.

Problem 24: Melted ice cream

A scoop of ice cream is a sphere of radius 1.5 in. It melts into a cylindrical cup of radius 1.5 in. How deep is the melted ice cream? Assume the volume does not change.

Problem 25: Tennis ball can

A can holds three tennis balls stacked so they just fit: each ball has radius 1.5 in., and the can has the same radius. How much empty space is in the can?



Three balls in a snug can.

Precalculus Complex Identities Volume Worksheet: problems 26-28

Problem 26: Spherical water tank

A town water tank is a sphere with radius 12 ft. Find its volume to the nearest tenth of a cubic foot, and the number of gallons it holds to the nearest gallon (1 cubic foot is about 7.48 gallons).

Problem 27: Basketball volume

A men's basketball has a diameter of about 9.5 in. Find the volume of air in it to the nearest tenth of a cubic inch.

Problem 28: Sphere and cylinder ratio

Show that a sphere of radius r has $\frac{2}{3}$ the volume of the smallest cylinder that holds it (radius r , height $2r$). Check with $r = 3$.

How to use this precalculus complex identities volume worksheet

Print this precalculus complex identities volume worksheet as a PDF, solve every problem on paper, then compare with the answer key.

