

Precalculus · Complex Factorization and Volume of a Sphere · Test Answer Key

Common Core N-CN.C.8, G-GMD.A.2 · mathovore.fr

The answer key gives a full solution and a scoring guide for each question. Award the stated points for correct setup, correct use of $i^2 = -1$, and a clear conclusion. Accept equivalent exact forms. Total: 20 points.

Precalculus Common Core: N-CN.C.8 · G-GMD.A.2

Question 1: Powers of i (2 points)

- a. $38 = 36 + 2$, so $i^{38} = i^2 = -1$.
- b. $55 = 52 + 3$, so $i^{55} = i^3 = -i$.

Scoring: 1 point per part.

Question 2: Factor a sum of squares (2 points)

$25x^2 + 4 = (5x)^2 + 2^2 = (5x + 2i)(5x - 2i)$. Check: $(5x + 2i)(5x - 2i) = 25x^2 - 4i^2 = 25x^2 + 4$.

Scoring: 1 point for the factorization; 1 point for the expansion with $i^2 = -1$.

Question 3: Solve over the complex numbers (2 points)

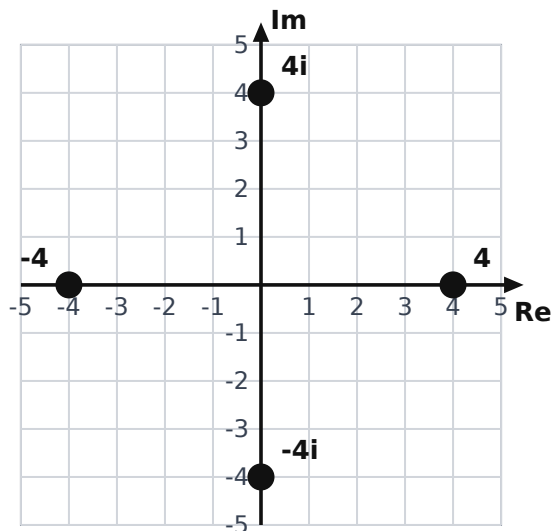
- a. $x^2 = -64$, so $x = \pm 8i$.
- b. $x^2 = -18$, so $x = \pm i\sqrt{18} = \pm 3i\sqrt{2}$.

Scoring: 1 point per part (accept $\pm i\sqrt{18}$ for part b).

Question 4: Fourth-degree factorization (3 points)

$x^4 - 256 = (x^2 - 16)(x^2 + 16)$. Then $x^2 - 16 = (x - 4)(x + 4)$ and $x^2 + 16 = (x + 4i)(x - 4i)$.
 $(x - 4)(x + 4)(x + 4i)(x - 4i)$

Scoring: 1 point for the difference of squares; 1 point for the real factors; 1 point for the complex factors.

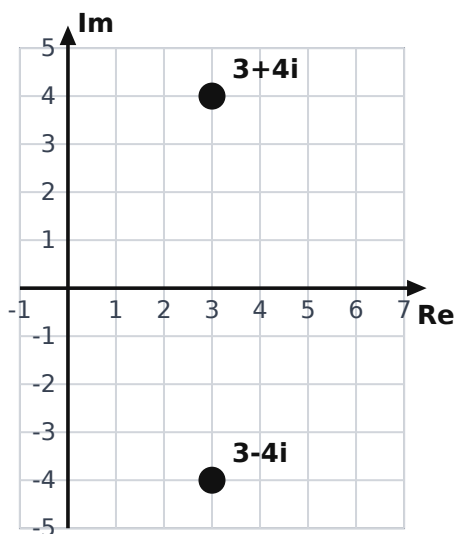


The four zeros of the polynomial.

Question 5: Quadratic with no real zeros (3 points)

- The vertex is $(3,16)$, and the parabola opens upward, so it never reaches the x-axis. There is no real solution of $x^2 - 6x + 25 = 0$.
- $x^2 - 6x + 25 = (x^2 - 6x + 9) + 16 = (x - 3)^2 + 4^2 = (x - 3 + 4i)(x - 3 - 4i)$. The zeros are $3 \pm 4i$.

Scoring: part (a) 1 point; part (b) 1 point for completing the square, 1 point for the factors.



The complex zeros.

Question 6: Matching cross-sections (3 points)

- The radius is $\sqrt{25 - 9} = 4$, so the area is 16π square ft.
- The ring has area $25\pi - 9\pi = 16\pi$ square ft.

c. The two solids have equal volumes.

Scoring: 1 point per part.

Question 7: Volume from a cylinder and a cone (2 points)

a. $\pi(36)(6) - \frac{1}{3}\pi(36)(6) = 216\pi - 72\pi = 144\pi$ cubic in.

b. The hemisphere has the same volume, 144π , so the sphere has $2(144\pi) = 288\pi$ cubic in.

Scoring: 1 point per part.

Question 8: Spherical tank (3 points)

a. The radius is 10. $V = \frac{4}{3}\pi(1000) = \frac{4000\pi}{3}$ cubic ft.

b. $\frac{4000\pi}{3} \approx 4188.8$ cubic ft.

c. $4188.79 \times 7.48 \approx 31332$ gallons.

Scoring: 1 point per part (accept 31,331 or 31,332).

Total
20 points