

Precalculus · Complex Factorization and Volume of a Sphere · Test

Common Core N-CN.C.8, G-GMD.A.2 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This test has 8 questions worth 20 points in total and takes about 45 minutes. Give exact answers unless a decimal is requested, and show each step of a factorization. A calculator is allowed for decimal approximations.

Test information

- **Grade:** Precalculus
- **Chapter:** Complex Factorization and Volume of a Sphere
- **Common Core:** N-CN.C.8, G-GMD.A.2
- **Time:** 45 minutes
- **Total:** 20 points (Question 1: 2 pts; Question 2: 2 pts; Question 3: 2 pts; Question 4: 3 pts; Question 5: 3 pts; Question 6: 3 pts; Question 7: 2 pts; Question 8: 3 pts)

Question 1: Powers of i (2 points)

Simplify.

- a. i^{38} (1 point)
- b. i^{55} (1 point)

Question 2: Factor a sum of squares (2 points)

Factor $25x^2+4$ over the complex numbers. (1 point) Then expand your answer to verify it. (1 point)

Question 3: Solve over the complex numbers (2 points)

Solve.

- a. $x^2+64=0$ (1 point)
- b. $x^2+18=0$ (1 point)

Question 4: Fourth-degree factorization (3 points)

Factor $x^4 - 256$ completely over the complex numbers.

Question 5: Quadratic with no real zeros (3 points)

The graph of $y = x^2 - 6x + 25$ is shown.

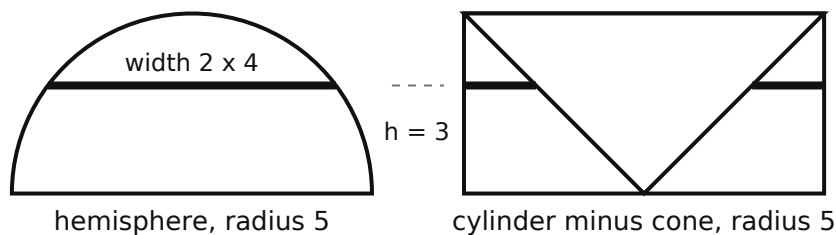


Graph of the quadratic.

- Explain from the graph why the quadratic has no real zeros. (1 point)
- Factor $x^2 - 6x + 25$ over the complex numbers. (2 points)

Question 6: Matching cross-sections (3 points)

A hemisphere of radius 5 ft and a cylinder of radius 5 ft and height 5 ft with a cone removed are each cut by a plane 3 ft above their base.



Both solids cut at height 3 ft.

- Find the area of the cross-section of the hemisphere. (1 point)
- Find the area of the cross-section of the cylinder with the cone removed. (1 point)
- State what Cavalieri's principle concludes if this holds at every height. (1 point)

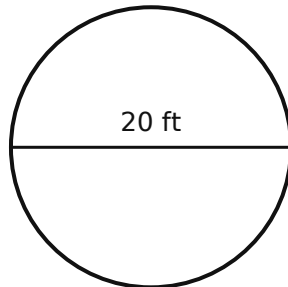
Question 7: Volume from a cylinder and a cone (2 points)

A cylinder and a cone each have radius 6 in. and height 6 in.

- Find the volume of the cylinder with the cone removed. (1 point)
- Use that result to find the volume of a sphere of radius 6 in. (1 point)

Question 8: Spherical tank (3 points)

A spherical storage tank has a diameter of 20 ft.



water storage sphere
The tank.

- Find its exact volume. (1 point)
- Find its volume to the nearest tenth of a cubic foot. (1 point)
- Find the capacity in gallons to the nearest gallon, using 1 cubic foot = 7.48 gallons. (1 point)