

Precalculus · Complex Factorization and Volume of a Sphere · Quiz

Common Core N-CN.C.8, G-GMD.A.2 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

This precalculus complex identities volume quiz asks 10 multiple-choice questions on complex factorization and volume of a sphere, and each answer comes with a short explanation.

Ten questions on factoring over the complex numbers and on sphere volume with Cavalieri's principle. Choose the one best answer for each. Some questions include a figure, so read the labels carefully before you choose.

Precalculus Common Core: N-CN.C.8 · G-GMD.A.2

Questions

Question 1

Simplify i^{23} .

- A. i
- B. -1
- C. 1
- D. $-i$

Question 2

Factor x^2+16 over the complex numbers.

- A. $(x+4i)(x-4i)$
- B. $(x+4)(x-4)$
- C. $(x+4i)^2$
- D. $(x+16i)(x-16i)$

Question 3

Solve $x^2+49=0$ over the complex numbers.

- A. $x = \pm 7$
- B. $x = \pm 7i$
- C. $x = \pm 49i$
- D. $x = 7i$ only

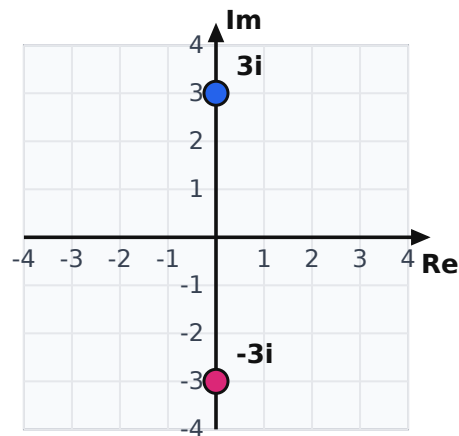
Question 4

Find $(3+4i)(3-4i)$.

- A. -7
- B. 5
- C. 25
- D. $9-16i$

Question 5

The complex plane shows the zeros of a quadratic with leading coefficient 1. Which quadratic is it?



Two zeros.

- A. x^2-9
- B. x^2+9
- C. x^2+3
- D. x^2-3

Question 6

Factor $4x^2+25$ over the complex numbers.

- A. $(4x+25i)(4x-25i)$
- B. $(2x+5)(2x-5)$
- C. $(2x+5i)^2$
- D. $(2x+5i)(2x-5i)$

Question 7

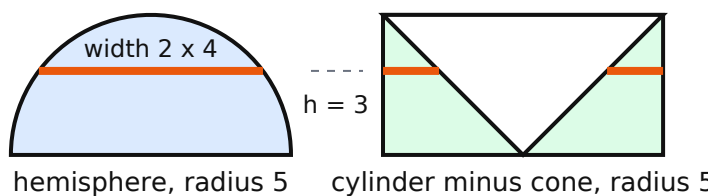
What is the exact volume of a sphere with radius 6?

- A. 288π
- B. 144π

- C. 864π
D. 48π

Question 8

A hemisphere of radius 5 and a cylinder-minus-cone of radius 5 are cut at height 3. What is the area of the hemisphere's cross-section?

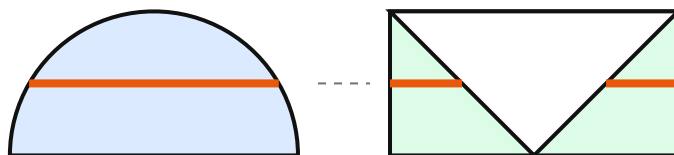


Both solids cut at height 3.

- A. 25π
B. 9π
C. 16π
D. 4π

Question 9

A cylinder and a cone each have radius 4 and height 4. The cone is removed from the cylinder. What is the volume that remains?



The remaining solid matches a hemisphere.

- A. $\frac{128\pi}{3}$
B. 64π
C. $\frac{64\pi}{3}$
D. $\frac{256\pi}{3}$

Question 10

Factor $x^4 - 16$ completely over the complex numbers.

- A. $(x^2-4)(x^2+4)$
- B. $(x-2)^2(x+2)^2$
- C. $(x-2i)^2(x+2i)^2$
- D. $(x-2)(x+2)(x-2i)(x+2i)$

Answer key

1. **D** - $23 = 20 + 3$, so $i^{23} = i^3 = -i$. Using the remainder 3 incorrectly as i or -1 is the usual slip.
2. **A** - $(x+4i)(x-4i) = x^2 - 16i^2 = x^2 + 16$. The product $(x+4)(x-4)$ gives $x^2 - 16$, and $(x+16i)(x-16i)$ gives $x^2 + 256$.
3. **B** - $x^2 = -49$ gives $x = \pm 7i$. Both signs are solutions, and the square root of 49 is 7, not 49.
4. **C** - $(3+4i)(3-4i) = 9 - 16i^2 = 9 + 16 = 25$. Forgetting that $i^2 = -1$ gives -7 .
5. **B** - $(x-3i)(x+3i) = x^2 - 9i^2 = x^2 + 9$. The constant is 9, not 3, because 3 is squared.
6. **D** - $(2x)^2 + 5^2 = (2x+5i)(2x-5i)$. Both terms must be written as squares first, so the coefficients are 2 and 5, not 4 and 25.
7. **A** - $\frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216) = 288\pi$. The value 144π is a hemisphere, and 864π forgets the factor $\frac{1}{3}$.
8. **C** - The cross-section is a circle of radius $\sqrt{25-9} = 4$, so its area is 16π . The ring in the other solid has area $25\pi - 9\pi = 16\pi$ too.
9. **A** - $64\pi - \frac{64\pi}{3} = \frac{128\pi}{3}$. The cylinder alone is 64π , the cone alone is $\frac{64\pi}{3}$, and $\frac{256\pi}{3}$ is the whole sphere.
10. **D** - $x^4 - 16 = (x^2 - 4)(x^2 + 4)$, and each factor splits further: $(x-2)(x+2)$ and $(x-2i)(x+2i)$. The first wrong option stops early.

How does this quiz work?

Read each question, choose one answer among A, B, C and D, then check it: a short explanation appears after every answer. Your score out of 10 is shown at the end, and you can restart the quiz as often as you like.

What should you review before the quiz?

Reread the key facts of the lesson and the worked example. If a question is hard, go back to the matching part of the lesson, then try the quiz again. Write your work on scrap paper: most mistakes come from skipping a step, not from the idea itself.

Remember:

- $a^2 + b^2 = (a + bi)(a - bi)$; for example $x^2 + 4 = (x + 2i)(x - 2i)$.
- Familiar polynomial identities remain true when the variables are complex numbers.
- Cavalieri's principle: equal cross-sectional areas at every height mean equal volumes.
- A hemisphere matches a cylinder minus a cone, so its volume is $\frac{2}{3}\pi r^3$.
- The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

How to use this precalculus complex identities volume quiz

Retake this precalculus complex identities volume quiz until every answer is right, then move on to the worksheet and the test.

