

Grade 8 · The Pythagorean Theorem · Lesson

Common Core 8.G.B.6, 8.G.B.7 · mathovore.fr

The Pythagorean Theorem says that in a right triangle the square of the hypotenuse equals the sum of the squares of the legs: $a^2 + b^2 = c^2$. Use it to find a missing side, to test with its converse whether a triangle has a right angle, and to measure diagonals in two and three dimensions.

Grade 8 Common Core: 8.G.B.6 · 8.G.B.7

Key facts:

- Standards 8.G.B.6 and 8.G.B.7: explain a proof of the theorem and its converse, then apply it in two and three dimensions.
- The hypotenuse is opposite the right angle and is always the longest side.
- To find a missing side, substitute into $a^2 + b^2 = c^2$, then take a positive square root.
- Converse: if $a^2 + b^2 = c^2$ holds for the sides of a triangle, it is a right triangle.
- The diagonal of a box satisfies $d^2 = l^2 + w^2 + h^2$.

What is the Pythagorean Theorem?

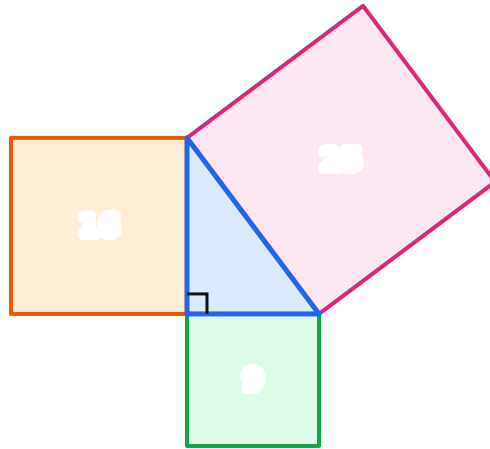
In a right triangle, the two sides that form the right angle are the **legs** (a and b). The side across from the right angle is the **hypotenuse** (c), always the longest side. The theorem states $a^2 + b^2 = c^2$.

Vocabulary

Hypotenuse: the longest side of a right triangle, opposite the right angle.

Converse: the reverse statement. If $a^2 + b^2 = c^2$ for the three sides of a triangle, then it is a right triangle.

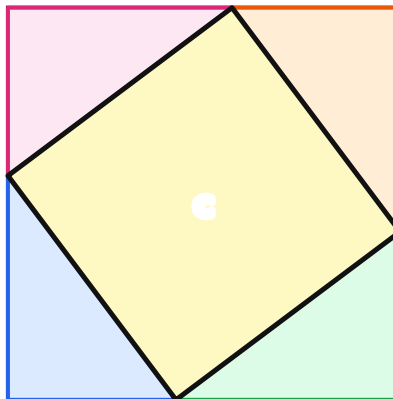
Perfect square: the square of a whole number, such as 25 or 169.



The squares on the legs have areas 9 and 16. Their sum, 25, is the area of the square on the hypotenuse.

Why is the theorem true?

Place four copies of a right triangle inside a square of side $a+b$, as in the figure. The big square has area $(a+b)^2 = a^2 + 2ab + b^2$. It also equals four triangles, $4 \cdot \frac{1}{2}ab = 2ab$, plus the tilted square c^2 . Setting the two expressions equal and subtracting $2ab$ leaves $a^2 + b^2 = c^2$.

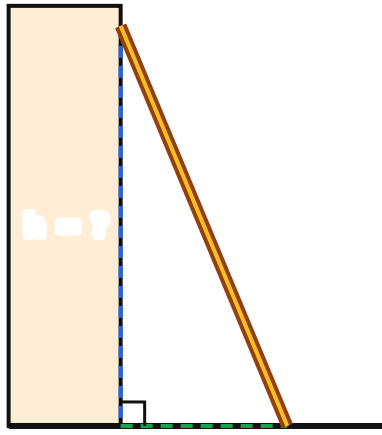


One big square of side $a+b$ holds four copies of the triangle and a tilted square of side c .

How do you find a missing side?

1. Name the hypotenuse c . It sits opposite the right angle.
2. Write $a^2 + b^2 = c^2$ and substitute the known sides.
3. Solve for the unknown square, then take the positive square root.
4. Round if needed and include the unit.

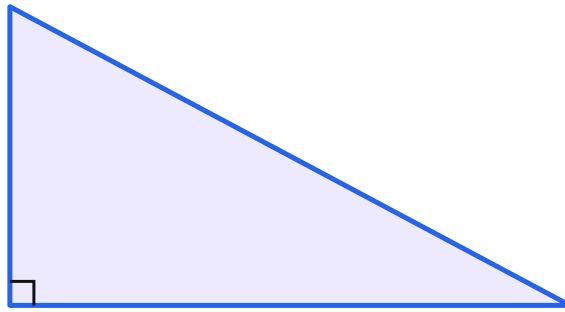
Worked example. A 13-ft ladder has its base 5 ft from a wall. Then $5^2 + h^2 = 13^2$, so $h^2 = 169 - 25 = 144$ and $h = 12$ ft.



A ladder is the hypotenuse. The wall and the ground are the legs.

How does the converse show a right triangle?

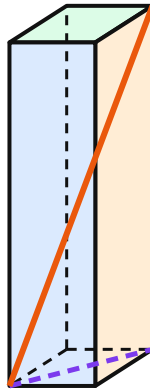
Square the two shorter sides and add them. If the sum equals the square of the longest side, the triangle is a right triangle. For sides 8, 15 and 17, $64 + 225 = 289 = 17^2$, so it is. For sides 6, 7 and 10, $36 + 49 = 85$ but $10^2 = 100$, so it is not.



Sides 8, 15 and 17: $8^2 + 15^2 = 64 + 225 = 289 = 17^2$, so the triangle has a right angle.

How do you use the theorem in three dimensions?

Apply it twice. In a box with length l , width w and height h , the base diagonal satisfies $x^2 = l^2 + w^2$. The space diagonal d then satisfies $d^2 = x^2 + h^2$, so $d^2 = l^2 + w^2 + h^2$. For a 3 by 4 by 12 box, $d^2 = 9 + 16 + 144 = 169$, so $d = 13$.



The base diagonal is 5, because $3^2 + 4^2 = 5^2$. The space diagonal d is the hypotenuse of a triangle with legs 5 and 12.

The Pythagorean Theorem: frequently asked questions

What is the Pythagorean Theorem in simple words?

In any right triangle, if you build a square on each side, the two smaller squares have a total area equal to the largest square. In symbols, $a^2 + b^2 = c^2$, where c is the hypotenuse.

How do you know which side is the hypotenuse?

The hypotenuse is the side opposite the right angle. It is also the longest side, so it is the one that stands alone on the right side of $a^2 + b^2 = c^2$.

What is the converse of the Pythagorean Theorem?

The converse says that if the squares of the two shorter sides add up to the square of the longest side, the triangle must be a right triangle. It lets you check a corner with only three measurements.

Does the theorem work for triangles without a right angle?

No. For a triangle that is not a right triangle, $a^2 + b^2$ is not equal to c^2 . That is exactly why the converse can be used as a test.

How do you find the diagonal of a box?

Find the diagonal of the base first with $x^2 = l^2 + w^2$. Then use that diagonal and the height as the legs of a second right triangle: $d^2 = x^2 + h^2$.