

Grade 8 · Linear Functions and Modeling · Lesson

Common Core 8.F.B.4, 8.F.B.5 · mathovore.fr

A linear function is a function whose graph is a straight line, written $y = mx + b$. The number m is the rate of change, the amount y changes for each 1 step in x , and b is the initial value, the output when x is 0. Two points or a table are enough to build the equation.

Grade 8 Common Core: 8.F.B.4 · 8.F.B.5

Key facts:

- A linear function has a constant rate of change, so its graph is a straight line.
- Rate of change: $m = (y_2 - y_1) / (x_2 - x_1)$. Initial value: b is the output when $x = 0$.
- In a story, m is the change per unit (dollars per hour) and b is the starting amount.
- A graph can be increasing, decreasing or constant, and linear or nonlinear.
- To sketch a graph from a story, steep means fast change and flat means no change.

What is a linear function?

A **linear function** has a graph that is a straight line. Its rule can be written $y = mx + b$, where m is the rate of change and b is the initial value.

Vocabulary

Rate of change: the change in y divided by the change in x between two points. On a line it is the same everywhere. It is also called the slope.

Initial value: the value of y when $x = 0$. It is where the graph crosses the y -axis.

Function: a rule that gives exactly one output for each input.

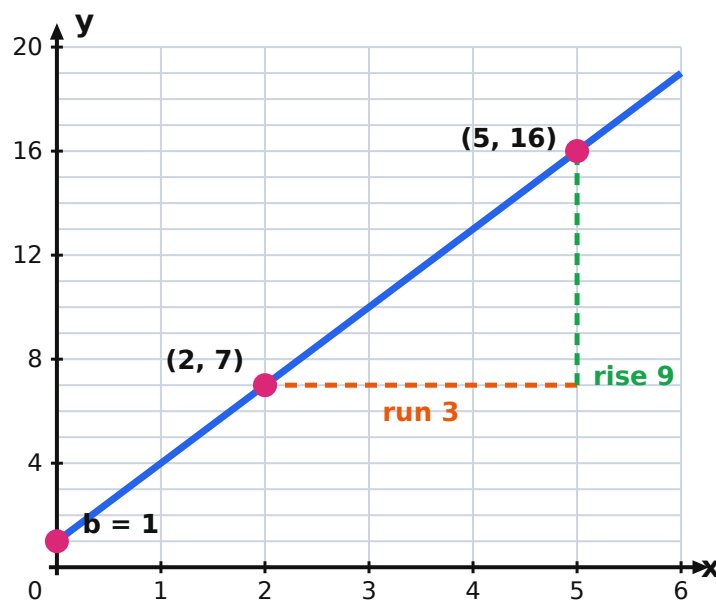


A plant starts 3 inches tall and grows 2 inches per week: $y = 2x + 3$. From the point $(1, 5)$ to $(3, 9)$: run 2, rise 4, rate $\frac{4}{2} = 2$.

How do you write the equation from two points or a table?

1. Find the rate of change: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Put m and one point into $y = mx + b$ and solve for b .
3. Write the equation and check it with the other point.

Worked example. The line passes through $(2, 7)$ and $(5, 16)$. Then $m = \frac{16 - 7}{5 - 2} = 3$. Substitute $(2, 7)$: $7 = 3(2) + b$, so $b = 1$. The function is $y = 3x + 1$. Check: $3(5) + 1 = 16$.



The line through $(2, 7)$ and $(5, 16)$ has rate $\frac{9}{3} = 3$ and crosses the y -axis at 1.

A table works the same way. If y changes by the same amount for every equal step in x , the

function is linear, and that amount per step is m .

x	0	1	2	3	4
y	1	4	7	10	13
change in y		+3	+3	+3	+3

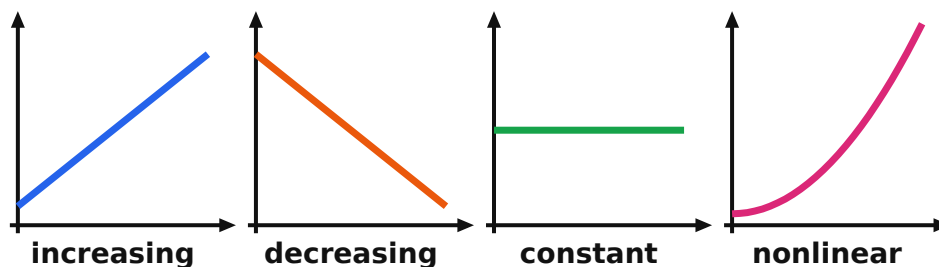
The value of y goes up by 3 for every step of 1 in x : the rate is 3 and the value at $x=0$ is 1, so $y=3x+1$.

What do the rate and initial value mean in a story?

In a story, the rate has units such as dollars per hour or inches per week, and the initial value is the starting amount, such as a flat fee. A taxi that charges a \$3 fee plus \$2 per mile has cost $C=2m+3$: the rate is \$2 per mile and the initial value is \$3. A negative rate means the quantity decreases, like a tank that drains.

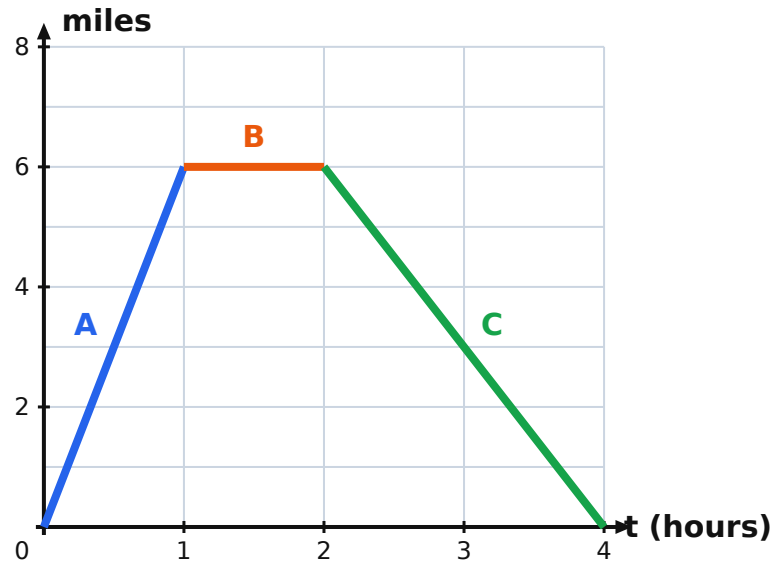
How do you describe a graph and sketch one from a story?

- **Increasing:** the graph rises from left to right.
- **Decreasing:** the graph falls from left to right.
- **Constant:** the graph is horizontal.
- **Linear or nonlinear:** a straight line is linear, while a curve is nonlinear.



Three straight-line graphs and one curve. The curve is nonlinear because its steepness keeps changing.

To sketch a graph from a story, read it in order. A steep part means a fast change, a flat part means no change, and a part that goes down means the quantity is shrinking. Jamal rides 6 miles away in 1 hour, rests for 1 hour, then rides home in 2 hours.



Jamal's bike trip to the farmers market: A increasing, B constant, C decreasing.

Linear Functions and Modeling: frequently asked questions

How do you find the rate of change from two points?

Subtract the y-values and divide by the difference of the x-values: $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep the subtraction in the same order on top and bottom. For (2,7) and (5,16) the rate is $\frac{9}{3} = 3$.

What is the initial value of a linear function?

The initial value is the output when the input is 0. In $y = mx + b$ it is b . On a graph it is the point where the line crosses the y -axis.

How can you tell from a table that a function is linear?

Check the change in y for each equal step in x . If every change is the same, the function is linear. If the changes grow or shrink, it is nonlinear.

Can the rate of change be negative?

Yes. A negative rate means the output decreases as the input increases, so the line slants down from left to right. A tank draining 20 gallons per hour has rate -20 .

What does a flat part of a graph mean in a story?

A flat (constant) part means the quantity is not changing. In a distance graph it means the person is staying in one place, so the rate of change there is 0.