

Grade 8 · Rational and Irrational Numbers · Lesson

Common Core 8.NS.A.1, 8.NS.A.2 · mathovore.fr

A rational number can be written as a fraction of two integers, so its decimal stops or repeats. An irrational number, such as the square root of 2 or pi, has a decimal that never stops and never repeats. Estimate irrational numbers by squaring decimals, and write repeating decimals as fractions using subtraction.

Grade 8 Common Core: 8.NS.A.1 · 8.NS.A.2

Key facts:

- Standards 8.NS.A.1 and 8.NS.A.2: tell rational from irrational numbers and approximate irrational numbers.
- Every repeating decimal equals a fraction, such as $0.\bar{3} = 1/3$.
- The square root of a whole number that is not a perfect square is irrational.
- Bracket a square root between two decimals whose squares surround the number.
- To compare two numbers, use decimals or square both positive numbers.

What are rational and irrational numbers?

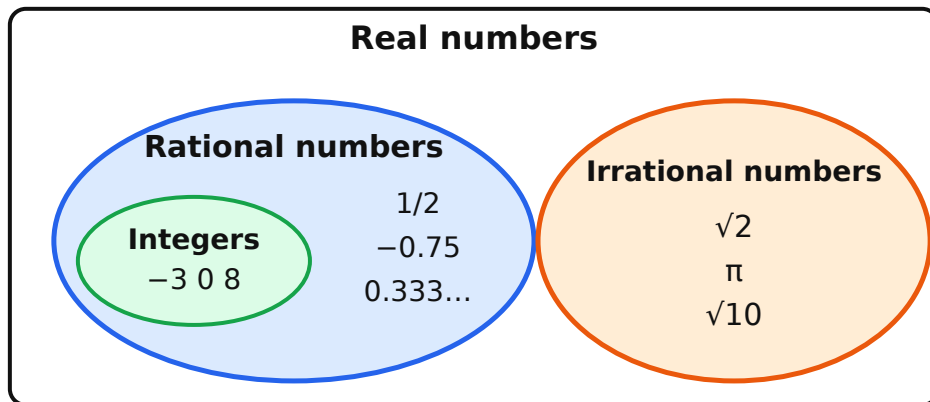
Every number on the number line is either rational or irrational. A **rational number** can be written as a fraction of two integers, so its decimal either stops or repeats a block of digits forever. An **irrational number** cannot be written that way. Its decimal never stops and never repeats.

Vocabulary

Terminating decimal: a decimal that ends, such as 0.375.

Repeating decimal: a decimal with a block of digits that repeats forever, such as $0.\overline{27}$.

Perfect square: the square of an integer, such as 25 or 49. Its square root is an integer, so it is rational.



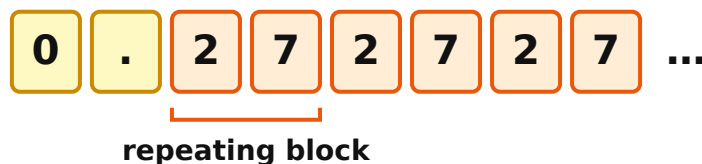
Every real number is either rational or irrational, never both.

The square root of any whole number that is not a perfect square is irrational. So $\sqrt{2}$, $\sqrt{10}$ and π are irrational, while $\sqrt{49} = 7$ is rational.

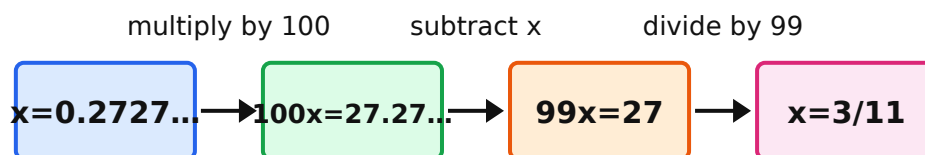
How do you write a repeating decimal as a fraction?

1. Let x equal the repeating decimal.
2. Multiply both sides by 10, 100 or 1,000 so one full block moves left of the decimal point.
3. Subtract the original equation to cancel the repeating part.
4. Divide to solve for x and simplify.

Worked example. Write $0.\overline{27}$ as a fraction. Two digits repeat, so multiply by 100:
 $100x = 27.2727$. Subtract $x = 0.2727$ to get $99x = 27$, so $x = \frac{27}{99} = \frac{3}{11}$.



$\frac{3}{11} = 0.\overline{27}$: the block 27 repeats forever.

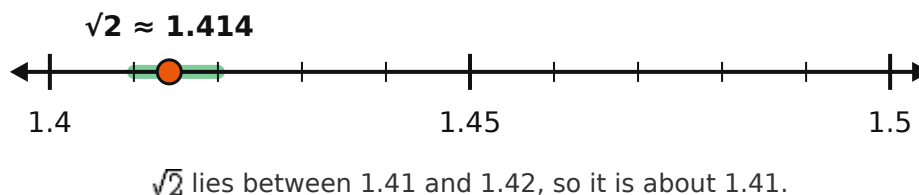


Multiply by a power of 10 so the repeating blocks line up, then subtract.

How do you approximate a square root?

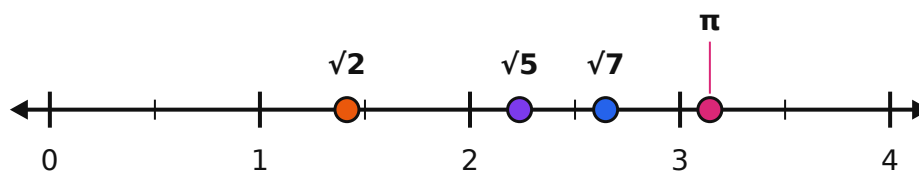
Find the two perfect squares that surround the number, then test decimals. Since $1^2 = 1$ and $2^2 = 4$, the number $\sqrt{2}$ is between 1 and 2. Testing tenths shows $1.4^2 = 1.96$ and $1.5^2 = 2.25$, so it is between 1.4 and 1.5. One more digit gives $1.41^2 = 1.9881$ and $1.42^2 = 2.0164$.

Test	Square	Compared with 2
1.4	1.96	too small
1.41	1.9881	too small
1.42	2.0164	too big
1.5	2.25	too big



How do you compare irrational numbers?

Use decimal approximations, or square both numbers when both are positive. To compare $\sqrt{7}$ and 2.6, note $2.6^2 = 6.76$, which is less than 7. So $\sqrt{7} > 2.6$. On the number line, the number farther right is greater.



Irrational numbers sit between the whole numbers on the number line.

Rational and Irrational Numbers: frequently asked questions

Is zero a rational number?

Yes. Zero can be written as $\frac{0}{1}$, a fraction of two integers, so it is rational. Every integer is rational because it can be written with a denominator of 1.

Is pi a rational number?

No. Pi is irrational because its decimal never ends and never repeats. Fractions such as $\frac{22}{7}$ and decimals such as 3.14 are only close approximations.

How can I tell if a square root is irrational?

Check whether the number under the radical is a perfect square. If it is, the root is an integer. If it is not, such as $\sqrt{20}$, the root is irrational.

Why does a repeating decimal equal a fraction?

Multiplying by a power of 10 shifts the repeating block, and subtracting cancels the endless part. That leaves a whole number equal to a multiple of x , so x is a fraction.

Does a decimal with a pattern have to be rational?

No. A decimal is rational only when one block of digits repeats forever. A pattern such as 0.101001000100001 with a growing gap never repeats a block, so it is irrational.