

# Grade 8 · Properties of Integer Exponents · Lesson

Common Core 8.EE.A.1 · mathovore.fr

Integer exponents follow rules when the bases match. Add exponents to multiply powers, subtract them to divide, and multiply them for a power of a power. A zero exponent gives 1, and a negative exponent gives a reciprocal, so 3 squared times 3 to the negative 5 equals one twenty-seventh.

## Grade 8 Common Core: 8.EE.A.1

### Key facts:

- Standard 8.EE.A.1: use the properties of integer exponents to write equivalent numerical expressions.
- Product rule:  $a^m \cdot a^n = a^{m+n}$ . Quotient rule:  $\frac{a^m}{a^n} = a^{m-n}$ .
- Power rule:  $(a^m)^n = a^{mn}$ .
- Zero exponent:  $a^0 = 1$  for any nonzero base.
- Negative exponent:  $a^{-n} = \frac{1}{a^n}$ , a reciprocal and not a negative number.

## What is an integer exponent?

An exponent tells how many times a base is used as a factor. In  $3^4$ , the base is 3 and the exponent is 4, so  $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$ . In Grade 8 the exponent can be any integer: positive, zero or negative.

### Vocabulary

**Base:** the number that is multiplied, such as the 3 in  $3^4$ .

**Exponent:** the number of times the base is a factor.

**Equivalent expressions:** expressions that have the same value, such as  $3^2 \cdot 3^{-5}$  and  $\frac{1}{27}$ .

## What are the product, quotient and power rules?

When the bases are the same, add exponents to multiply and subtract exponents to divide.

For example  $3^2 \cdot 3^3$  has five factors of 3 in all, so it equals  $3^5$ .

$$\underbrace{3 \times 3}_{3^2} \times \underbrace{3 \times 3 \times 3}_{3^3}$$

$3^2 \cdot 3^3 = 3^5$ : the factors are simply counted.

In a quotient, matching factors cancel. In  $\frac{5^5}{5^2}$  two factors of 5 cancel, leaving  $5^3$ .

$$\frac{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5}}$$

Cancel matching factors:  $\frac{5^5}{5^2} = 5^3$ .

| Rule               | In symbols                  | Example                         |
|--------------------|-----------------------------|---------------------------------|
| Product            | $a^m \cdot a^n = a^{m+n}$   | $2^3 \cdot 2^4 = 2^7$           |
| Quotient           | $\frac{a^m}{a^n} = a^{m-n}$ | $\frac{5^6}{5^2} = 5^4$         |
| Power of a power   | $(a^m)^n = a^{mn}$          | $(3^2)^4 = 3^8$                 |
| Power of a product | $(ab)^n = a^n b^n$          | $(2 \cdot 3)^2 = 2^2 \cdot 3^2$ |
| Zero exponent      | $a^0 = 1$                   | $7^0 = 1$                       |
| Negative exponent  | $a^{-n} = \frac{1}{a^n}$    | $4^{-2} = \frac{1}{16}$         |

## What do zero and negative exponents mean?

Look at the pattern in the powers of 2. Each step to the right divides by 2, so the value keeps halving. That forces  $2^0 = 1$  and  $2^{-1} = \frac{1}{2}$ . A negative exponent means a reciprocal, not a negative number.

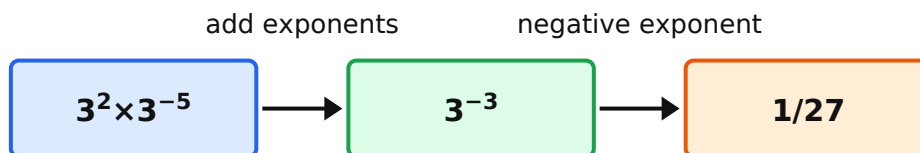
|                      |                      |                      |                      |                       |                       |                       |
|----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|-----------------------|
|                      | ÷ 2                  | ÷ 2                  | ÷ 2                  | ÷ 2                   | ÷ 2                   | ÷ 2                   |
| <b>2<sup>3</sup></b> | <b>2<sup>2</sup></b> | <b>2<sup>1</sup></b> | <b>2<sup>0</sup></b> | <b>2<sup>-1</sup></b> | <b>2<sup>-2</sup></b> | <b>2<sup>-3</sup></b> |
| 8                    | 4                    | 2                    | 1                    | 1/2                   | 1/4                   | 1/8                   |

Dividing by 2 each step explains zero and negative exponents.

## How do you generate equivalent numerical expressions?

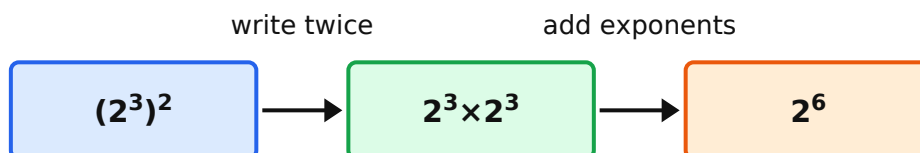
1. Check that the bases match; if not, look for a common base.
2. Apply the product, quotient or power rule to combine exponents.
3. Rewrite a negative exponent as a reciprocal.
4. Evaluate if a number is needed.

**Worked example.** Simplify  $3^2 \cdot 3^{-5}$ . Add exponents:  $2 + (-5) = -3$ , so the product is  $3^{-3}$ . Rewrite as a reciprocal:  $\frac{1}{3^3} = \frac{1}{27}$ .



Two equivalent expressions for the same value.

The power rule works the same way:  $(2^3)^2$  means two copies of  $2^3$ , so multiply the exponents to get  $2^6 = 64$ .



Multiply exponents when a power is raised to a power.

## Properties of Integer Exponents: frequently asked questions

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### Why does any number to the zero power equal 1?

Dividing  $\frac{5^3}{5^3}$  gives 1, and the quotient rule gives  $5^0$ . Both must agree, so  $5^0 = 1$ . This works for every nonzero base.

### Is a negative exponent a negative number?

No. The negative sign in the exponent means take the reciprocal. For example  $2^{-3} = \frac{1}{8}$ , which is positive.

### Can I add exponents when the bases are different?

No. The product rule needs the same base.  $2^3 \cdot 3^2$  is 8 times 9, which is 72, and it is not a power of 6.

### What is the difference between the product rule and the power rule?

The product rule adds exponents for  $a^m \cdot a^n$ . The power rule multiplies exponents for  $(a^m)^n$ . Writing out the factors shows which one applies.

### **How do I write a number such as 0.001 with an exponent?**

Count the place value.  $0.001 = \frac{1}{1000} = \frac{1}{10^3}$ , which is  $10^{-3}$ .