

Grade 8 · Congruence, Dilations and Similarity

· Lesson

Common Core 8.G.A.2, 8.G.A.3, 8.G.A.4 · mathovore.fr

Congruent figures are related by rigid motions, which are translations, reflections and rotations that keep every length and angle. Similar figures also allow a dilation: multiplying coordinates by a scale factor k changes size but not angle measures. Congruent figures are similar with $k=1$.

Grade 8 Common Core: 8.G.A.2 · 8.G.A.3 · 8.G.A.4

Key facts:

- Standards 8.G.A.2, 8.G.A.3 and 8.G.A.4: congruence, coordinates under transformations, and similarity.
- Rigid motions (translation, reflection, rotation) keep lengths and angle measures.
- Dilation about the origin: $(x,y) \rightarrow (kx,ky)$; lengths scale by k , angles do not change.
- Similar figures: a sequence of rigid motions and dilations maps one onto the other.
- Rotation rules about the origin: 90° is $(-y,x)$, 180° is $(-x,-y)$, 270° is $(y,-x)$.

What are congruent figures?

Two figures are **congruent** when a sequence of rotations, reflections and translations moves one exactly onto the other. These moves are called **rigid motions** because they never change a length or an angle measure.

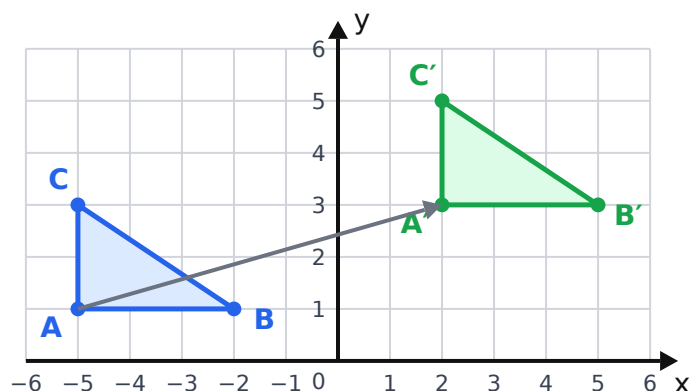
Vocabulary

Translation: slides every point the same distance in the same direction.

Reflection: flips a figure across a line, like a mirror.

Rotation: turns a figure about a point, here the origin, through a given angle.

Dilation: multiplies every distance from a center by a scale factor k .

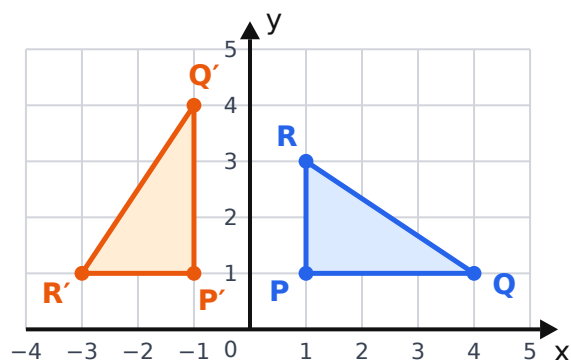


Translation 7 units right and 2 units up: each vertex moves the same way, so $A(-5, 1)$ goes to $A'(2, 3)$.

How do rigid motions change coordinates?

On a coordinate plane, each move follows a rule applied to every point (x, y) . The last column shows the rule on the point $(3, 2)$.

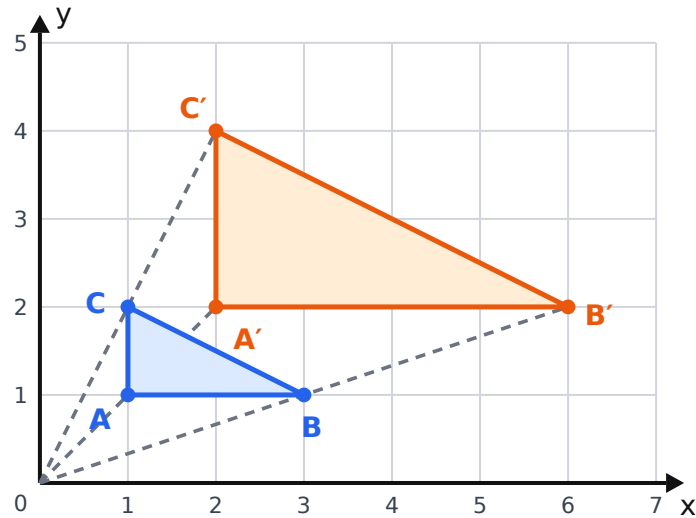
Move	Rule	Image of $(3, 2)$
Translate a right, b up	$(x+a, y+b)$	$(7, 1)$ for $a=4, b=-1$
Reflect across the x -axis	$(x, -y)$	$(3, -2)$
Reflect across the y -axis	$(-x, y)$	$(-3, 2)$
Rotate 90° counterclockwise	$(-y, x)$	$(-2, 3)$
Rotate 180°	$(-x, -y)$	$(-3, -2)$
Rotate 270° counterclockwise	$(y, -x)$	$(2, -3)$
Dilate by k	(kx, ky)	$(6, 4)$ for $k=2$



A 90° counterclockwise rotation about the origin sends (x, y) to $(-y, x)$. Point $Q(4, 1)$ lands at $Q'(-1, 4)$.

What does a dilation do?

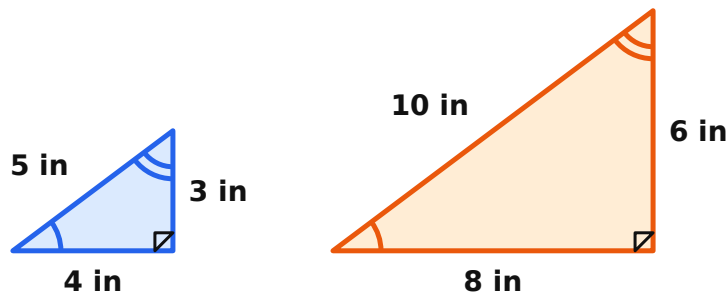
A dilation about the origin with scale factor k sends (x, y) to (kx, ky) . If $k > 1$ the image is an enlargement, and if k is between 0 and 1 it is a reduction. Every length is multiplied by k , but every angle measure stays the same.



A dilation about the origin with scale factor 2 doubles every coordinate. The dashed rays from the origin pass through each vertex and its image.

What are similar figures?

Two figures are **similar** when a sequence of rotations, reflections, translations and dilations maps one onto the other. Corresponding angles are equal, and corresponding sides are proportional with ratio k . Congruent figures are the special case $k=1$.



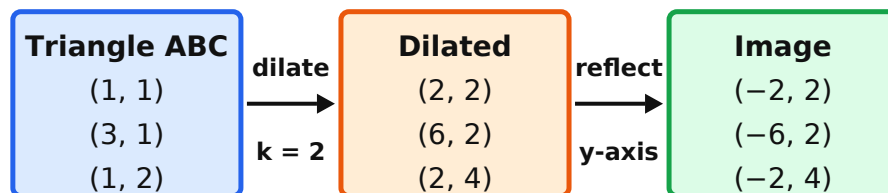
Similar triangles: every side of the large triangle is 2 times the matching side of the small one, and matching angles (same arcs) are equal.

How do you decide whether two figures are similar?

1. Match corresponding vertices.
2. Divide each side of the second figure by the matching side of the first. Equal quotients give the scale factor k .
3. Check that corresponding angles are equal.
4. Name a dilation and the rigid motions that finish the job.

Worked example. Triangle ABC has vertices $(1,1)$, $(3,1)$, $(1,2)$. Its image has vertices $(-2,2)$, $(-6,2)$, $(-2,4)$. Dilating by $k=2$ gives $(2,2)$, $(6,2)$, $(2,4)$. Reflecting across the y -axis gives $(-2,2)$

, $(-6,2)$, $(-2,4)$. The triangles are similar with scale factor 2, but they are not congruent.



Two moves in a row: a dilation with $k = 2$, then a reflection across the y -axis.

Congruence, Dilations and Similarity: frequently asked questions

What is the difference between congruent and similar figures?

Congruent figures have the same size and shape, so a sequence of rigid motions maps one onto the other. Similar figures have the same shape but may differ in size, so a dilation is also needed. Every congruent pair is similar with $k = 1$.

Does a dilation change angle measures?

No. A dilation multiplies lengths by the scale factor k , but corresponding angles stay equal. A 40° angle is still 40° in the enlarged or reduced image.

What is the rule for a dilation about the origin?

Multiply both coordinates by the scale factor: $(x,y) \rightarrow (kx,ky)$. For $k = 3$, the point $(2,-5)$ goes to $(6,-15)$. For $k = \frac{1}{2}$, it goes to $(1,-\frac{5}{2})$.

How do you find the scale factor from coordinates?

Divide an image coordinate by the matching original coordinate. If $A(2,3)$ goes to $A'(10,15)$, then $k = 10 \div 2 = 5$. Check that the other coordinate gives the same quotient: $15 \div 3 = 5$.

What is the rule for a 90° counterclockwise rotation?

The point (x,y) goes to $(-y,x)$ when the center is the origin. For example, $(4,1)$ goes to $(-1,4)$. Do the move three times and you get the 270° rule $(y,-x)$.