

Grade 7 · Rational Numbers: Decimals, Fractions and Word Problems · Lesson

Common Core 7.NS.A.2d, 7.NS.A.3 · mathovore.fr

A rational number is any number that can be written as a fraction of two integers, and its decimal form comes from long division of the numerator by the denominator. The decimal either terminates, like $\frac{3}{8} = 0.375$, or repeats forever, like $\frac{1}{3} = 0.\bar{3}$. Add, subtract, multiply and divide rational numbers with the usual sign rules.

Grade 7 Common Core: 7.NS.A.2d · 7.NS.A.3

Key facts:

- Every rational number is a terminating or a repeating decimal.
- Convert $\frac{a}{b}$ by dividing a by b ; a remainder of 0 means the decimal terminates.
- A repeated remainder means a repeating decimal, written with a bar: $\frac{5}{11} = 0.\overline{45}$.
- Subtracting a number is adding its opposite: $a - b = a + (-b)$.
- Multiplying or dividing two numbers with the same sign gives a positive result; with different signs, a negative result.

How do you write a fraction as a decimal?

A **rational number** is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Every rational number has a decimal form, and you find it by dividing the numerator by the denominator with **long division**. Add zeros after the decimal point and keep dividing.

Long division of a fraction

To convert $\frac{3}{8}$, divide 3 by 8: $30 \div 8 = 3$ remainder 6; $60 \div 8 = 7$ remainder 4; $40 \div 8 = 5$ remainder 0. So $\frac{3}{8} = 0.375$. For a negative fraction, convert the positive fraction and keep the negative sign.

$$\begin{array}{r}
 0.375 \\
 8 \overline{) 3.000} \\
 \underline{-24} \\
 60 \\
 \underline{-56} \\
 40 \\
 \underline{-40} \\
 0
 \end{array}
 \begin{array}{l}
 \text{remainder 6} \\
 \text{remainder 4} \\
 \text{remainder 0}
 \end{array}$$

The remainder reaches 0, so $\frac{3}{8} = 0.375$ terminates.

What is the difference between a terminating and a repeating decimal?

If a remainder becomes 0, the decimal **terminates**. If a remainder appears a second time, the digits repeat forever and the decimal is a **repeating decimal**. A bar marks the repeating block: $\frac{5}{11} = 0.\overline{45}$.

$$\begin{array}{r}
 0.4545 \\
 11 \overline{) 5.0000} \\
 \underline{-44} \\
 60 \\
 \underline{-55} \\
 50 \\
 \underline{-44} \\
 60 \\
 \underline{-55} \\
 5
 \end{array}
 \begin{array}{l}
 \text{remainder 6} \\
 \text{remainder 5} \\
 \text{remainder 6} \\
 \text{remainder 5}
 \end{array}$$

The remainders 6 and 5 repeat, so the digits 45 repeat: $\frac{5}{11} = 0.\overline{45}$.

Fraction	Decimal	Type
$\frac{1}{2}$	0.5	terminates
$\frac{3}{8}$	0.375	terminates
$\frac{1}{3}$	0.333...	repeats
$\frac{7}{12}$	0.58333...	repeats

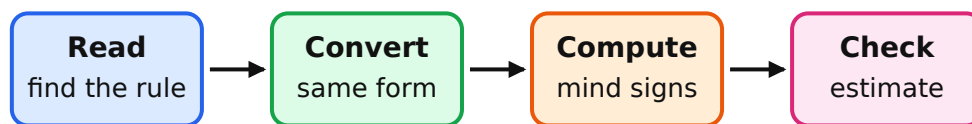
Some fractions terminate and some repeat.

A fraction in lowest terms terminates only when its denominator has no prime factors other than 2 and 5. That is why $\frac{3}{8}$ terminates but $\frac{1}{3}$ and $\frac{7}{12}$ repeat.

How do you solve real-world problems with rational numbers?

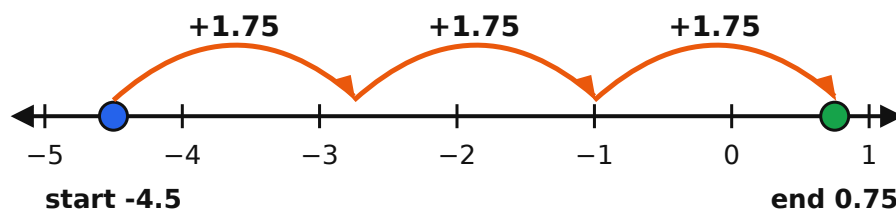
1. Read the problem and decide which operation matches the situation.
2. Write the numbers in one form, all decimals or all fractions.
3. Compute, using the sign rules below.
4. Check the sign and size of the answer with an estimate.

Operation	Sign rule
Add	Same signs: add and keep the sign. Different signs: subtract and take the sign of the larger absolute value.
Subtract	$a - b = a + (-b)$
Multiply or divide	Same signs give a positive result; different signs give a negative result.



A routine for rational-number word problems.

Example. At 6 AM it is -4.5°F . The temperature rises $1\frac{3}{4}^{\circ}\text{F}$ each hour for 3 hours. The total rise is $3 \times 1.75 = 5.25$, so the temperature is $-4.5 + 5.25 = 0.75^{\circ}\text{F}$.



Three rises of 1.75 degrees take the temperature from -4.5 to 0.75.

Rational Numbers: Decimals, Fractions and Word Problems: frequently asked questions

How do I know whether a fraction gives a repeating decimal?

Divide until a remainder is 0 or a remainder repeats. If a fraction is in lowest terms and its denominator has a prime factor other than 2 or 5, the decimal repeats. For example, $\frac{7}{12}$ repeats because 12 has the factor 3.

What does the bar over repeating digits mean?

The bar covers the block of digits that repeats forever. In $0.8\bar{3}$ only the 3 repeats, so the number is 0.8333 with the 3s going on forever, written compactly as $0.8\bar{3}$.

How do I convert a negative fraction to a decimal?

Convert the positive fraction by long division, then put the negative sign in front. Since $\frac{7}{8} = 0.875$, we get $-\frac{7}{8} = -0.875$.

Should I use fractions or decimals in a word problem?

Use whichever form makes the arithmetic easier, but do not mix them in one step. Money and temperatures usually suit decimals, while measurements such as cups or feet often suit fractions.

How do I subtract a negative number?

Change the subtraction to addition of the opposite. For example, $\frac{3}{4} - \left(-\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$.