

Grade 7 · Proportional Relationships · Lesson

Common Core 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2d · mathovore.fr

Two quantities are proportional when the ratio $\frac{y}{x}$ is the same for every pair of values. That constant ratio is the constant of proportionality k , also called the unit rate. On a graph, a proportional relationship is a straight line through the origin $(0,0)$ that contains the point $(1,k)$.

Grade 7 Common Core: 7.RP.A.2a · 7.RP.A.2b · 7.RP.A.2d

Key facts:

- A relationship is proportional when $\frac{y}{x}$ is the same constant k for every pair.
- The constant of proportionality k equals the unit rate: the value of y when $x=1$.
- A graph is proportional only if it is a straight line through $(0,0)$.
- The point $(1,k)$ on a proportional graph shows the unit rate.
- A table that does not give a constant ratio, or a line that misses the origin, is not proportional.

What is a proportional relationship?

Two quantities are in a **proportional relationship** when one is always the same multiple of the other. If Mei earns \$12 for each hour she works, her pay y and her hours x always have the same ratio: $\frac{y}{x}=12$.

Constant of proportionality

In a proportional relationship the ratio $k=\frac{y}{x}$ is the same for every pair of values. This constant k is the **constant of proportionality**. It is also the unit rate: the amount of y for 1 unit of x .

Hours x	1	2	3	4
Pay y (dollars)	12	24	36	48
Ratio y/x	12	12	12	12

Every ratio is 12, so the relationship is proportional.

How do you decide from a table?

1. Divide y by x for every pair in the table.
2. If all the ratios are equal, the relationship is proportional and the common ratio is k .
3. If even one ratio is different, the relationship is not proportional.

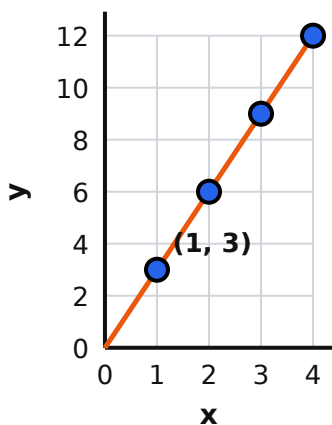
Example. A table has $x=1,2,3,4$ and $y=5,8,11,14$. The ratios are $\frac{5}{1}=5$, $\frac{8}{2}=4$ and $\frac{11}{3}\approx 3.67$. They are not equal, so the relationship is not proportional.

x	1	2	3	4
y	5	8	11	14
y/x	5	4	3.67	3.5

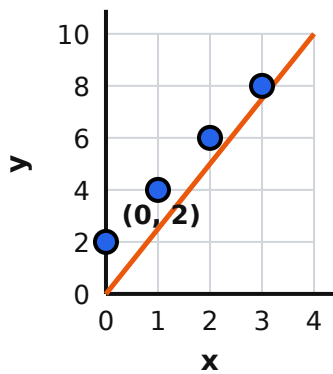
The ratios change, so this is not proportional.

How do you decide from a graph?

The graph of a proportional relationship is a **straight line that passes through the origin** $(0,0)$. If the line is crooked, or if it crosses the y -axis above or below zero, the relationship is not proportional.



A line through $(0, 0)$: proportional with $k = 3$.

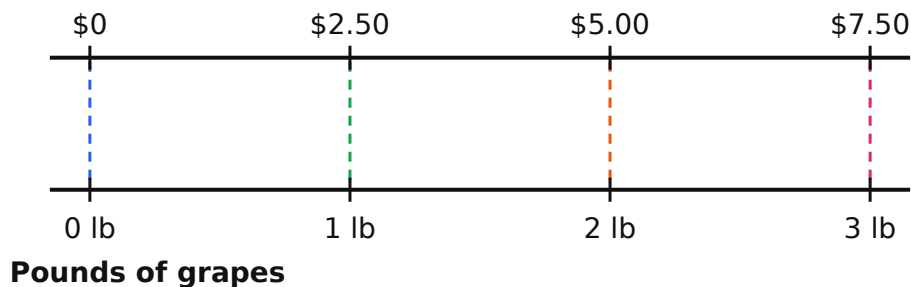


A straight line that misses the origin: not proportional.

What do the points $(0, 0)$ and $(1, r)$ mean?

The point $(0,0)$ says that 0 of one quantity goes with 0 of the other: 0 pounds of grapes cost \$0. The point $(1,r)$ shows the unit rate r : the value of y when $x=1$. Here r equals k . If grapes cost \$2.50 per pound, the graph contains $(1,2.50)$.

Cost



Each pound adds \$2.50, so $k = 2.50$.

Proportional Relationships: frequently asked questions

How can I tell if a table is proportional?

Divide y by x in every row. If every quotient is the same number, the table is proportional and that number is k . One different quotient means it is not proportional.

Why must a proportional graph pass through the origin?

If $x=0$, then $y=k \cdot 0=0$. So the point $(0,0)$ is always on the graph. A line that crosses the y -axis anywhere else does not represent a proportional relationship.

Is any straight line proportional?

No. The line must also pass through $(0,0)$. A line such as the one through $(0,2)$ and $(1,4)$ is

straight, but it is not proportional because it misses the origin.

What does the point (1, r) tell you?

It tells you the unit rate. When $x=1$, the value of y is r , and r equals the constant of proportionality k . For \$2.50 per pound, (1,2.50) is on the graph.

What words suggest a proportional relationship?

Phrases such as *per*, *for each* and *for every* often describe a constant ratio. A fixed starting fee added to a per-unit charge is not proportional, however.