

Geometry · Triangle Theorems and Points of Concurrency · Worksheet Answer Key

Common Core G-CO.C.10, G-C.A.3 · mathovore.fr

Each answer shows the theorem used, the algebra, and a check. When a problem asks for a center, compare your result with the figure: the centroid, incenter, and acute-triangle orthocenter always fall inside the triangle.

Geometry Common Core: G-CO.C.10 · G-C.A.3

Problem 1: Find the third angle

The angles add up to 180° : $m\angle C = 180 - 52 - 67 = 61^\circ$. Check: $52 + 67 + 61 = 180$. Answer: 61° .

Problem 2: Acute angles of a right triangle

$m\angle B = 180 - 90 - 35 = 55^\circ$. The acute angles of a right triangle add up to 90° : $35 + 55 = 90$. Answer: 55° .

Problem 3: Base angles of an isosceles triangle

The base angles are congruent. They share $180 - 40 = 140^\circ$, so each is $140 \div 2 = 70^\circ$. Check: $40 + 70 + 70 = 180$. Answer: 70° each.

Problem 4: Angle sum with variables

Set the sum equal to 180: $(x+10) + (2x-5) + (3x+7) = 180$. Combine: $6x+12 = 180$. Subtract 12: $6x = 168$. Divide: $x = 28$.

The angles are $28+10 = 38^\circ$, $2(28)-5 = 51^\circ$, and $3(28)+7 = 91^\circ$. Check: $38+51+91 = 180$. Answer: $x = 28$; 38° , 51° , 91° .

Problem 5: Use the exterior angle

The exterior angle equals the sum of the remote interior angles: $41 + 67 = 108^\circ$.

Check: the interior angle at C is $180 - 41 - 67 = 72^\circ$, and $180 - 72 = 108$. Answer: 108° .

Problem 6: Missing remote angle

The exterior angle equals the sum of the remote angles: $75+x=128$, so $x=128-75=53^\circ$.
Check: the adjacent interior angle is $180-128=52^\circ$, and $75+53+52=180$. Answer: 53° .

Problem 7: Exterior angle with variables

Exterior angle = sum of remote angles: $6x-4=(2x+10)+(3x+6)$. Simplify: $6x-4=5x+16$.
Subtract $5x$: $x-4=16$. Add 4: $x=20$.

Exterior angle: $6(20)-4=116^\circ$. Remote angles: 50° and 66° , and $50+66=116$. Answer: $x=20$; 116° .

Problem 8: Isosceles triangle with variables

Base angles are congruent: $3x+4=5x-16$. Subtract $3x$: $4=2x-16$. Add 16: $20=2x$, so $x=10$.

Each base angle is $3(10)+4=34^\circ$. Then $m\angle A=180-34-34=112^\circ$. Answer: $x=10$; **base angles** 34° ; $m\angle A=112^\circ$.

Problem 9: Equal angles, equal sides

By the converse of the Isosceles Triangle Theorem, the sides opposite equal angles are congruent. $\overline{QR}$ is opposite $\angle P$ and $\overline{PR}$ is opposite $\angle Q$, so $PR=QR$.

$2x+1=4x-13$ gives $14=2x$, so $x=7$. $PR=2(7)+1=15$ cm and $QR=4(7)-13=15$ cm.
 $m\angle R=180-58-58=64^\circ$. Answer: $x=7$, $PR=15$ cm, $m\angle R=64^\circ$.

Problem 10: Midsegment length

The midsegment is half the length of the parallel side: $DE=\frac{1}{2}(26)=13$. Answer: $DE=13$.

Problem 11: Midsegment with variables

$BC=2\cdot DE$, so $7x+2=2(4x-3)$. Distribute: $7x+2=8x-6$. Subtract $7x$: $2=x-6$. Add 6: $x=8$.

$DE=4(8)-3=29$ in and $BC=7(8)+2=58$ in. Check: $2\times 29=58$. Answer: $x=8$, $DE=29$ in, $BC=58$ in.

Problem 12: Midsegment on a grid

$$D=\left(\frac{2+(-2)}{2}, \frac{8+0}{2}\right)=(0,4) \text{ and } E=\left(\frac{2+10}{2}, \frac{8+0}{2}\right)=(6,4).$$

$\overline{DE}$ is horizontal (both y -coordinates are 4) and $\overline{BC}$ lies on the x -axis, so they are parallel.
 $DE=6-0=6$ and $BC=10-(-2)=12$, so $DE=\frac{1}{2}BC$. Answer: $D(0,4)$, $E(6,4)$; **parallel**; $6=\frac{1}{2}(12)$.

Problem 13: Segments of a median

The centroid is $\frac{2}{3}$ of the way from the vertex: $AG = \frac{2}{3}(21) = 14$. The rest is $GD = 21 - 14 = 7$.
Check: $AG = 2 \cdot GD$. Answer: $AG = 14$, $GD = 7$.

Problem 14: Centroid with variables

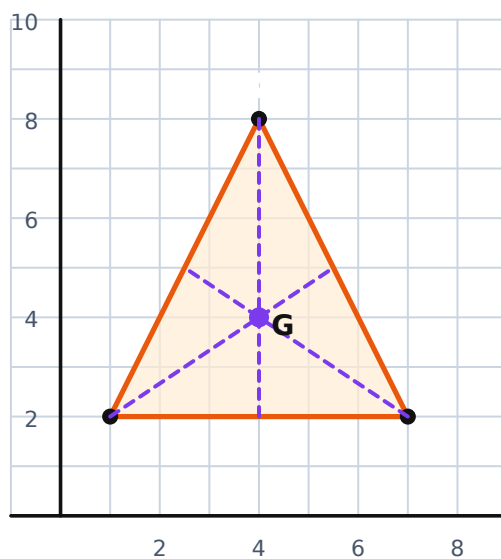
$BG = 2 \cdot GD$, so $5x - 1 = 2(2x + 1)$. Distribute: $5x - 1 = 4x + 2$. Subtract $4x$: $x - 1 = 2$. Add 1: $x = 3$.

$BG = 5(3) - 1 = 14$ and $GD = 2(3) + 1 = 7$, so $BD = 14 + 7 = 21$ in. Answer: $x = 3$, $BG = 14$ in, $BD = 21$ in.

Problem 15: Centroid on a grid

The centroid has the average of the vertex coordinates.

$x = \frac{1+7+4}{3} = \frac{12}{3} = 4$ and $y = \frac{2+2+8}{3} = \frac{12}{3} = 4$. Answer: (4,4).



The medians meet at G(4, 4).

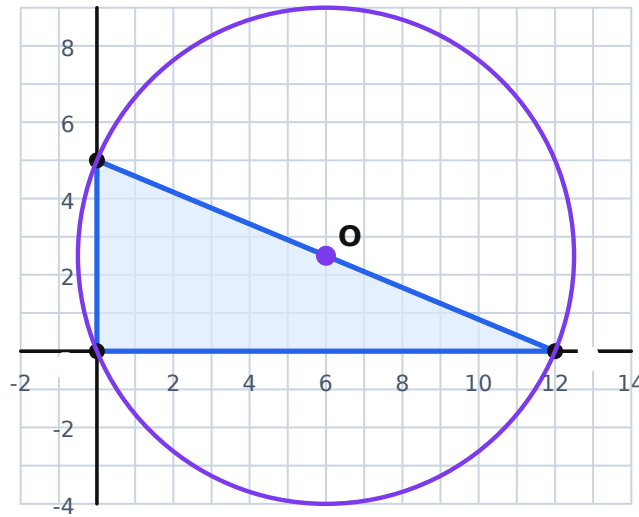
Problem 16: Name the center

- a. **Centroid**.
- b. **Circumcenter**, the center of the circumscribed circle.
- c. **Incenter**, the center of the inscribed circle.
- d. **Orthocenter**.

Problem 17: Circumcenter of a right triangle

In a right triangle the circumcenter is the midpoint of the hypotenuse $\overline{BC} : \left(\frac{12+0}{2}, \frac{0+5}{2}\right) = (6, 2.5)$.

The hypotenuse is $\sqrt{12^2+5^2}=13$, and the radius is half of it, 6.5. Check: $\sqrt{6^2+2.5^2}=\sqrt{42.25}=6.5$.
 . Answer: **circumcenter** (6,2.5) , **radius** 6.5.



The circumcenter is the midpoint of the hypotenuse.

Problem 18: Circumcenter by perpendicular bisectors

$\overline{AB}$ is horizontal with midpoint (3,0), so its perpendicular bisector is $x=3$.

$\overline{AC}$ has midpoint (1,3) and slope $\frac{6}{2}=3$, so the perpendicular slope is $-\frac{1}{3}$: $y-3=-\frac{1}{3}(x-1)$.

At $x=3$: $y=3-\frac{1}{3}(2)=\frac{7}{3}$. Check: the squared distance to A is $3^2+(\frac{7}{3})^2=\frac{130}{9}$, and to C is $1^2+(\frac{11}{3})^2=\frac{130}{9}$. Answer: $(3,\frac{7}{3})$.

Problem 19: Incenter and inscribed circle

- Area $=\frac{1}{2}(5)(12)=30$ ft². Semiperimeter $s=\frac{5+12+13}{2}=15$ ft.
- $30=r\cdot 15$, so $r=2$ ft.
- The incenter is equidistant from all three sides, so each distance is **2 ft**.

Problem 20: Angles at the incenter

I lies on the angle bisectors of $\angle A$ and $\angle B$. So $m\angle BAI=\frac{50}{2}=25^\circ$ and $m\angle ABI=\frac{70}{2}=35^\circ$.

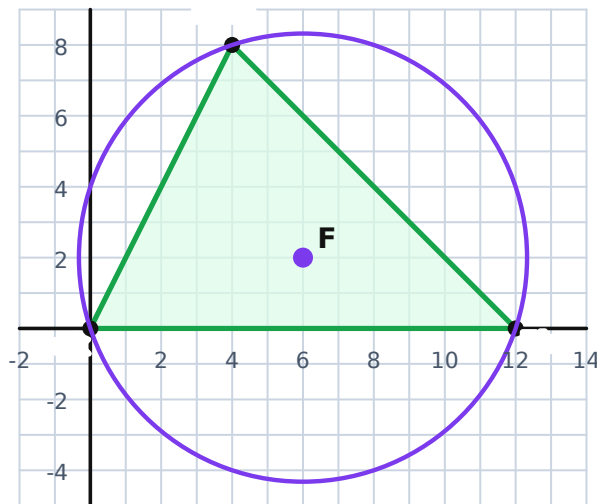
In $\triangle ABI$: $m\angle AIB=180-25-35=120^\circ$. Answer: 25° , 35° , 120° .

Problem 21: Where to put the fountain

The fountain is the circumcenter. The perpendicular bisector of the Oak-Elm segment is $x=6$.

The Oak-Pine segment has midpoint $(2,4)$ and slope 2, so its perpendicular bisector has slope $-\frac{1}{2}$: $y-4=-\frac{1}{2}(x-2)$. At $x=6$: $y=4-2=2$. The fountain is at $(6,2)$.

Distance to the Oak: $\sqrt{6^2+2^2}=\sqrt{40}$, which is about 6.3 yd. Pine: $\sqrt{(4-6)^2+(8-2)^2}=\sqrt{40}$, the same. Answer: $(6,2)$, **about 6.3 yd from each tree**.



The fountain F is the circumcenter.

Problem 22: Largest circle in a triangle

The largest circle is the inscribed circle. Since $9^2+12^2=225=15^2$, the triangle is a right triangle with area $\frac{1}{2}(9)(12)=54$ ft².

$s = \frac{9+12+15}{2} = 18$, and $54 = r \cdot 18$ gives $r = 3$ ft. The diameter is $2 \times 3 = 6$. Answer: **6 ft**.

Problem 23: Balance point of a triangle

A uniform triangle balances at its centroid, the average of the vertices.

$x = \frac{0+12+3}{3} = 5$ and $y = \frac{0+0+9}{3} = 3$. Answer: $(5,3)$, 5 inches right and 3 inches up from the lower left corner.

Problem 24: Roof gable angles

- $180-110=70$, shared by two congruent base angles: $70 \div 2 = 35^\circ$.
- Exterior angle at B: $180-35=145^\circ$. Check: remote angles are 110 and 35, and $110+35=145$. The exterior angle is 145° .

Problem 25: Midsegment triangle perimeter

Each side of $\triangle DEF$ is a midsegment, half as long as the parallel side of $\triangle ABC$:

$DE = \frac{1}{2}(18) = 9$ (parallel to $\overline{CA}$), $EF = \frac{1}{2}(14) = 7$ (parallel to $\overline{AB}$), and $DF = \frac{1}{2}(22) = 11$ (parallel to $\overline{BC}$).

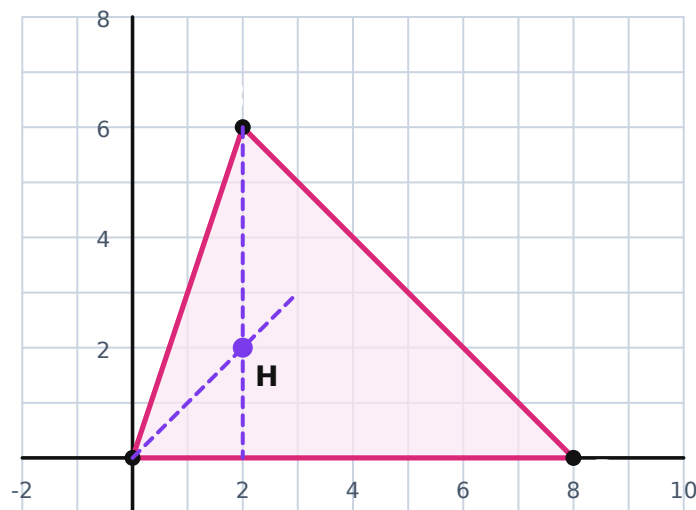
Perimeter of $\triangle DEF$: $9+7+11=27$ in. Perimeter of $\triangle ABC$: $14+22+18=54$ in. Answer: **27 in, exactly half of 54 in.**

Problem 26: Orthocenter on a grid

The altitude from C to the horizontal side $\overline{AB}$ is vertical: $x=2$.

$\overline{BC}$ has slope $\frac{6-0}{2-8} = -1$, so the altitude from A has slope 1: $y=x$. At $x=2$, $y=2$. The orthocenter is $(2,2)$.

Check: the altitude from B should be perpendicular to $\overline{AC}$, which has slope 3. The slope from B to H is $\frac{2-0}{2-8} = -\frac{1}{3}$, and $3 \times (-\frac{1}{3}) = -1$. Answer: $(2,2)$.



Two altitudes meet at $H(2, 2)$.