

Geometry · Triangle Theorems and Points of Concurrency · Worksheet

Common Core G-CO.C.10, G-C.A.3 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

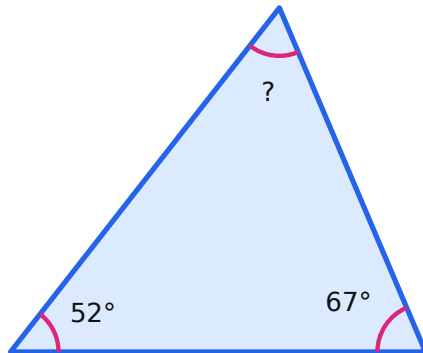
These 26 problems cover Common Core standards G-CO.C.10 and G-C.A.3: the Triangle Angle Sum and Exterior Angle theorems, the Isosceles Triangle Theorem, midsegments, medians and the centroid, and the circumcenter, incenter, and orthocenter with their circles.

The first problems use angle facts and algebra. Later ones use coordinates and circles, and the last six are word problems and multi-step tasks, so show every step.

GeometryCommon Core: G-CO.C.10 · G-C.A.3

Problem 1: Find the third angle

In $\triangle ABC$, $m\angle A = 52^\circ$ and $m\angle B = 67^\circ$. Find $m\angle C$.



Find the unknown angle C.

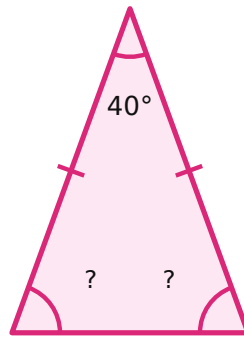
Problem 2: Acute angles of a right triangle

In right triangle ABC, $m\angle C = 90^\circ$ and $m\angle A = 35^\circ$. Find $m\angle B$.

Problem 3: Base angles of an isosceles triangle

Triangle ABC is isosceles with $\overline{AB} \cong \overline{AC}$ and a vertex angle of $m\angle A = 40^\circ$. Find the

measure of each base angle.



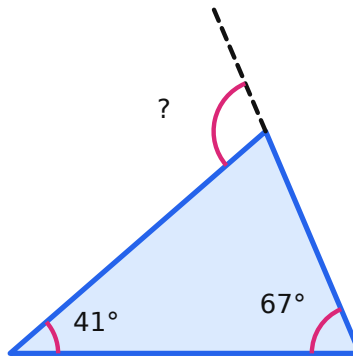
Isosceles triangle with a 40 degree vertex angle.

Problem 4: Angle sum with variables

The angles of a triangle measure $(x+10)^\circ$, $(2x-5)^\circ$, and $(3x+7)^\circ$. Find x and all three angle measures.

Problem 5: Use the exterior angle

In the figure, side $\overline{BC}$ is extended past C. Find the measure of the exterior angle at C.



Exterior angle at C.

Problem 6: Missing remote angle

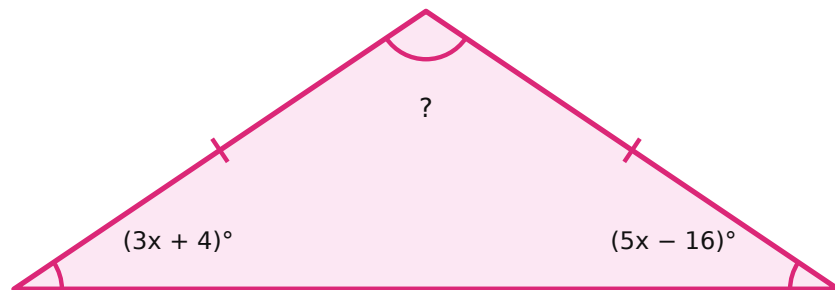
An exterior angle of a triangle measures 128° . One remote interior angle measures 75° . Find the other remote interior angle.

Problem 7: Exterior angle with variables

An exterior angle measures $(6x-4)^\circ$. The remote interior angles measure $(2x+10)^\circ$ and $(3x+6)^\circ$. Find x and the exterior angle.

Problem 8: Isosceles triangle with variables

In isosceles triangle ABC, $\overline{AB} \cong \overline{AC}$, $m\angle B = (3x+4)^\circ$, and $m\angle C = (5x-16)^\circ$. Find x , the base angles, and $m\angle A$.



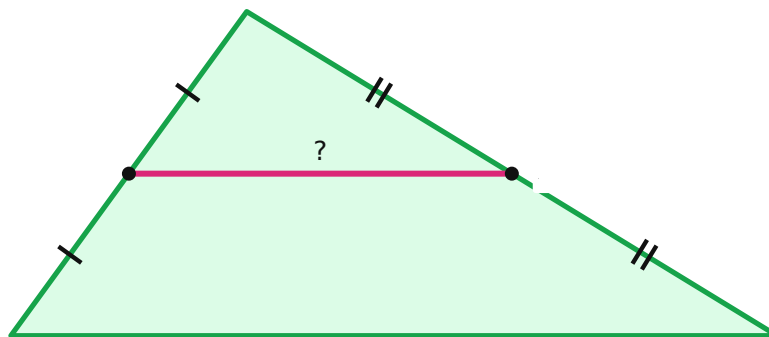
Isosceles triangle ABC.

Problem 9: Equal angles, equal sides

In $\triangle PQR$, $m\angle P = m\angle Q = 58^\circ$, $PR = (2x+1)$ cm, and $QR = (4x-13)$ cm. Find x , PR , and $m\angle R$.

Problem 10: Midsegment length

In $\triangle ABC$, D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, and $BC = 26$. Find DE .



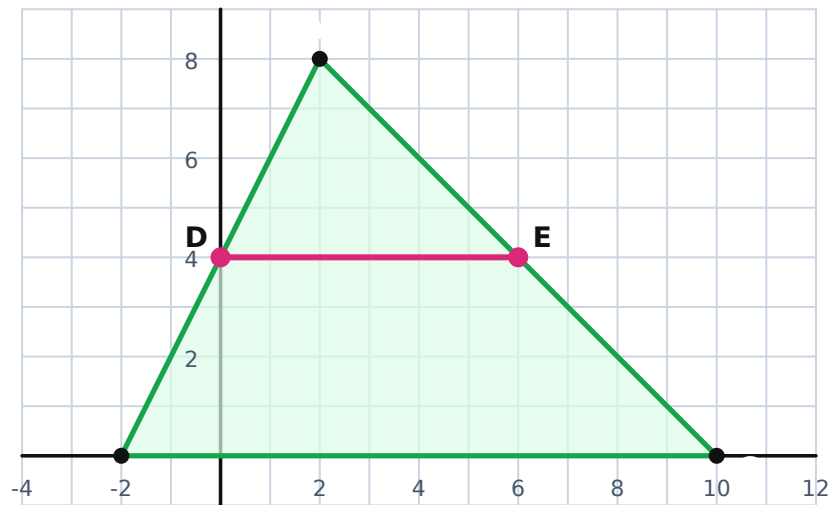
Midsegment DE.

Problem 11: Midsegment with variables

In $\triangle ABC$, $\overline{DE}$ is the midsegment parallel to $\overline{BC}$. $DE = (4x - 3)$ in and $BC = (7x + 2)$ in. Find x , DE , and BC .

Problem 12: Midsegment on a grid

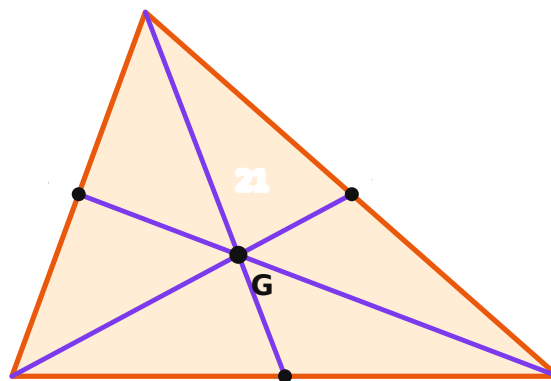
Triangle ABC has $A(2,8)$, $B(-2,0)$, and $C(10,0)$. D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. Find the coordinates of D and E, then show that $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$.



Triangle ABC and midsegment DE.

Problem 13: Segments of a median

The medians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ of $\triangle ABC$ meet at the centroid G. If $AD = 21$, find AG and GD .



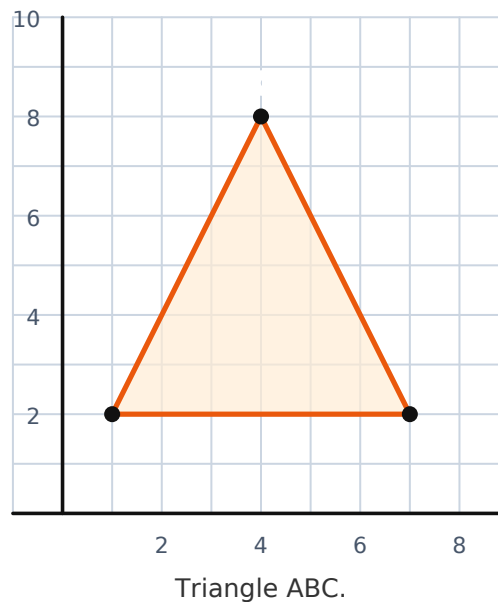
Median AD measures 21.

Problem 14: Centroid with variables

G is the centroid of $\triangle ABC$ and D is the midpoint of $\overline{AC}$, so $\overline{BD}$ is a median. $BG = (5x - 1)$ in and $GD = (2x + 1)$ in. Find x , BG , and BD .

Problem 15: Centroid on a grid

Find the centroid of the triangle with vertices $A(1,2)$, $B(7,2)$, and $C(4,8)$.

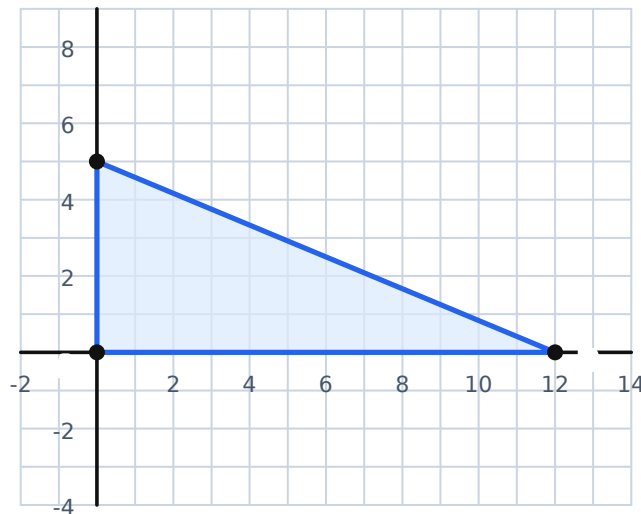


Problem 16: Name the center

Name the point of concurrency described: (a) the intersection of the three medians; (b) the intersection of the perpendicular bisectors of the sides; (c) the intersection of the angle bisectors; (d) the intersection of the lines containing the altitudes.

Problem 17: Circumcenter of a right triangle

Right triangle ABC has $A(0,0)$, $B(12,0)$, and $C(0,5)$. Find the circumcenter and the radius of the circumscribed circle.



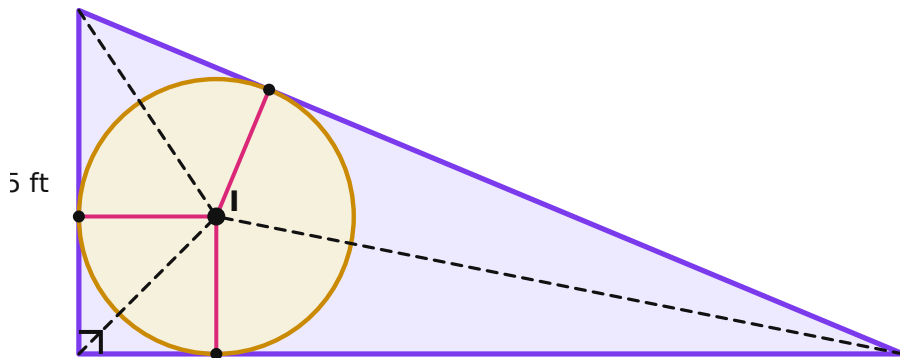
Right triangle ABC.

Problem 18: Circumcenter by perpendicular bisectors

Triangle ABC has $A(0,0)$, $B(6,0)$, and $C(2,6)$. Find the circumcenter by writing the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$.

Problem 19: Incenter and inscribed circle

Right triangle ABC has legs of 5 ft and 12 ft and a hypotenuse of 13 ft. The incenter I is the center of the inscribed circle. (a) Find the area and the semiperimeter s . (b) Use $\text{area} = r \cdot s$ to find the radius r of the inscribed circle. (c) How far is I from each side?

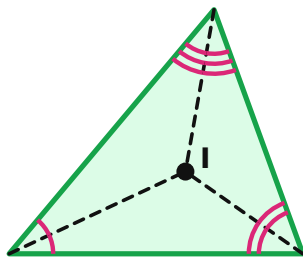


Inscribed circle of the 5-12-13 triangle.

Problem 20: Angles at the incenter

In $\triangle ABC$, $m\angle A = 50^\circ$ and $m\angle B = 70^\circ$. I is the incenter. Find $m\angle BAI$, $m\angle ABI$, and

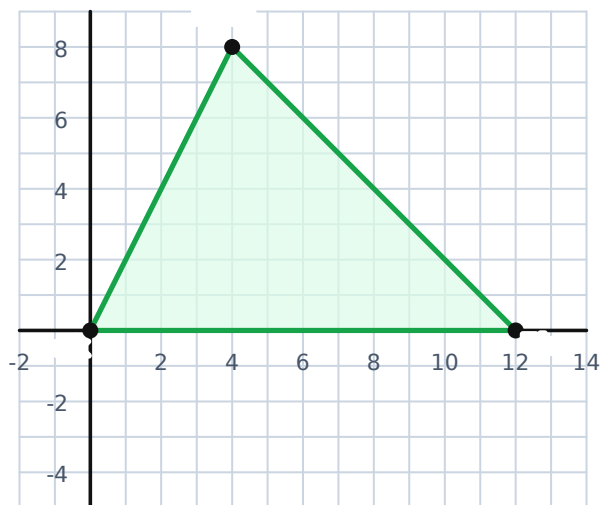
$$m \angle AIB.$$



The angle bisectors meet at I.

Problem 21: Where to put the fountain

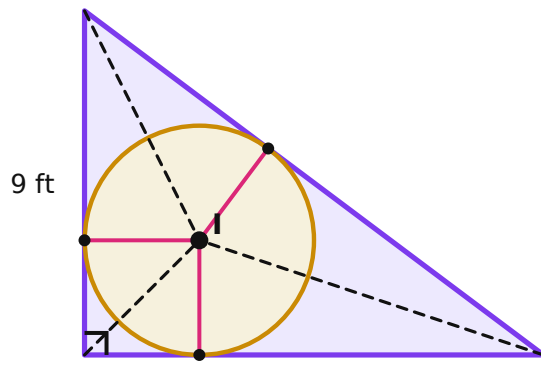
A park designer puts a fountain exactly the same distance from three trees. On a grid in yards, the Oak is at $(0,0)$, the Elm is at $(12,0)$, and the Pine is at $(4,8)$. Find the fountain's location and its distance from each tree to the nearest tenth of a yard.



Three trees, units in yards.

Problem 22: Largest circle in a triangle

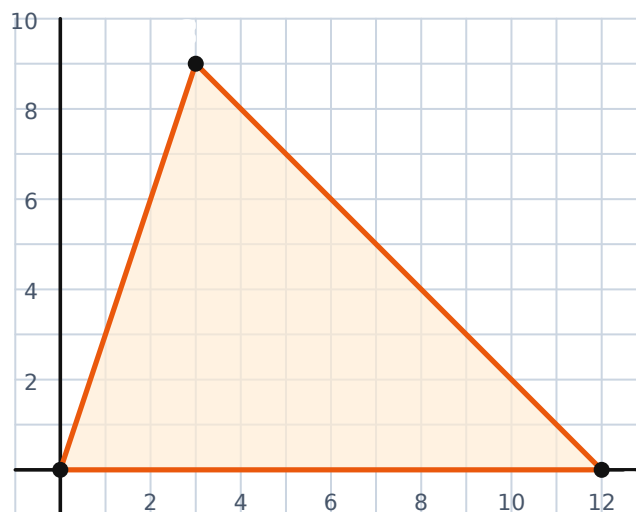
Diego cuts the largest possible circle from a triangular piece of plywood with sides of 9 ft, 12 ft, and 15 ft. What is the diameter of the circle?



The largest circle is the inscribed circle.

Problem 23: Balance point of a triangle

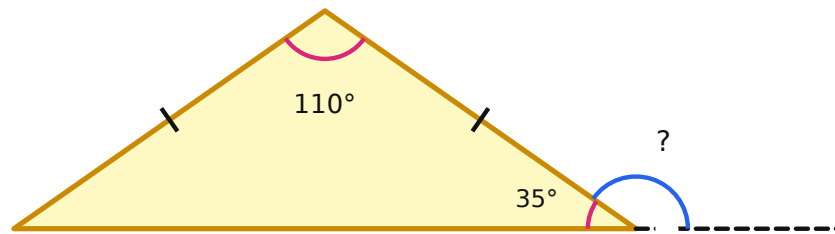
Priya cuts a triangle from uniform cardboard with vertices at $(0,0)$, $(12,0)$, and $(3,9)$ (inches). She wants to balance it on a pencil tip. Where should the tip touch the triangle?



Cardboard triangle, units in inches.

Problem 24: Roof gable angles

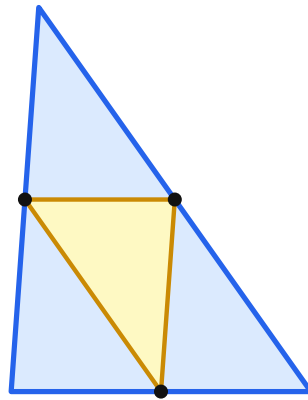
The end of a roof is an isosceles triangle ABC with $\overline{AC} \cong \overline{CB}$ and a peak angle of $m\angle C = 110^\circ$. The base $\overline{AB}$ is extended past B . (a) Find the base angles. (b) Find the exterior angle at B , and check it with the Exterior Angle Theorem.



Roof gable with the base extended.

Problem 25: Midsegment triangle perimeter

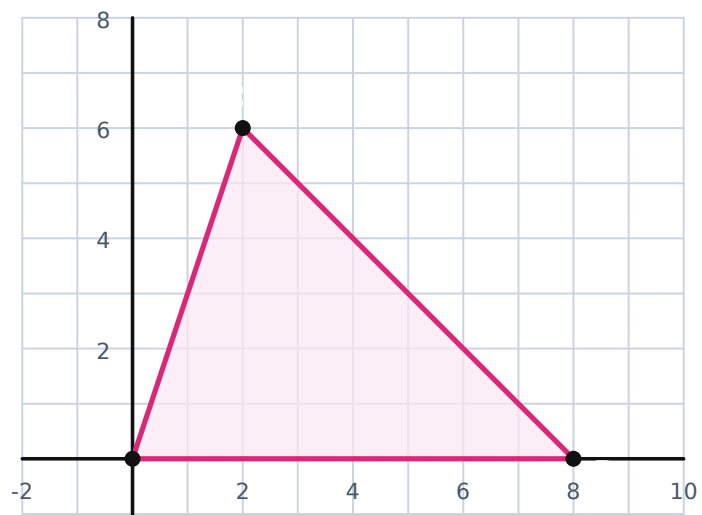
Triangle ABC has sides $AB = 14$ in, $BC = 22$ in, and $CA = 18$ in. D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. Find the three side lengths and the perimeter of $\triangle DEF$, and compare it with the perimeter of $\triangle ABC$.



Triangle DEF joins the midpoints.

Problem 26: Orthocenter on a grid

Find the orthocenter of the triangle with $A(0,0)$, $B(8,0)$, and $C(2,6)$. Check your answer with the third altitude.



Triangle ABC.