

Geometry · Similar Triangles and Proportions · Lesson

Common Core G-SRT.B.4, G-SRT.B.5 · mathovore.fr

Triangles are similar when corresponding angles are congruent and corresponding sides are proportional, which AA, SAS or SSS similarity proves. A line parallel to one side of a triangle splits the other two sides proportionally, and the altitude to the hypotenuse of a right triangle creates similar triangles and geometric-mean relationships.

GeometryCommon Core: G-SRT.B.4 · G-SRT.B.5

Key facts:

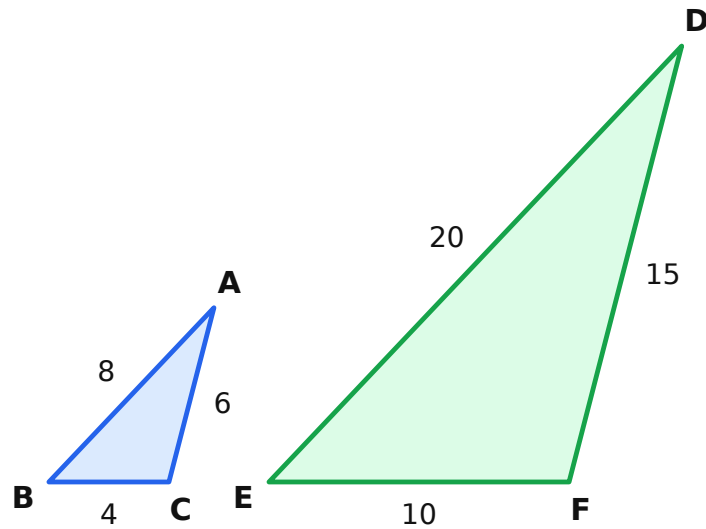
- AA, SAS and SSS similarity criteria prove triangles similar (G-SRT.B.4, B.5).
- Side-splitter theorem: a line parallel to one side divides the other two sides proportionally, $AD/DB = AE/EC$; its converse proves lines parallel.
- In a right triangle with altitude CD to the hypotenuse, CD squared equals AD times DB .
- Each leg is the geometric mean of the hypotenuse and the segment next to that leg.
- Similar triangles justify indirect measurement with shadows, mirrors and a proof of the Pythagorean theorem.

How do you prove two triangles are similar?

Triangles are **similar** when their corresponding angles are congruent and their corresponding sides are proportional. You never need all six facts, because one of three criteria is enough.

Criterion	What you must show
AA	Two pairs of congruent angles
SAS	Two pairs of sides in the same ratio, with congruent included angles
SSS	Three pairs of sides in the same ratio

Example: the sides 4, 6, 8 and 10, 15, 20 satisfy $\frac{10}{4} = \frac{15}{6} = \frac{20}{8} = 2.5$, so the triangles are similar by SSS with scale factor 2.5.



Every side of triangle DEF is 2.5 times the matching side of triangle ABC: SSS similarity.

What does the triangle proportionality theorem say?

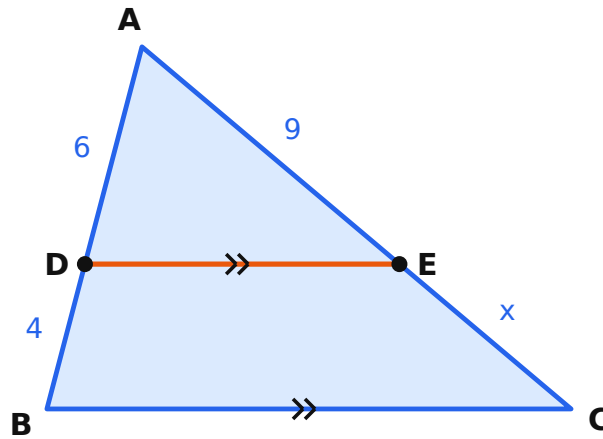
Triangle proportionality (side-splitter) theorem

If a line parallel to one side of a triangle meets the other two sides at D and E, it divides them proportionally: $\frac{AD}{DB} = \frac{AE}{EC}$. **Converse:** if D and E divide the two sides proportionally, then DE is parallel to BC.

The reason is similarity: DE parallel to BC gives congruent corresponding angles at D and B, and angle A is shared, so triangle ADE is similar to triangle ABC by AA.

1. Write the proportion $\frac{AD}{DB} = \frac{AE}{EC}$.
2. Substitute the known lengths.
3. Cross-multiply.
4. Solve for the unknown.

Worked example. With $AD=6$, $DB=4$, $AE=9$: $\frac{6}{4} = \frac{9}{EC}$, so $6 \cdot EC = 36$ and $EC = 6$.



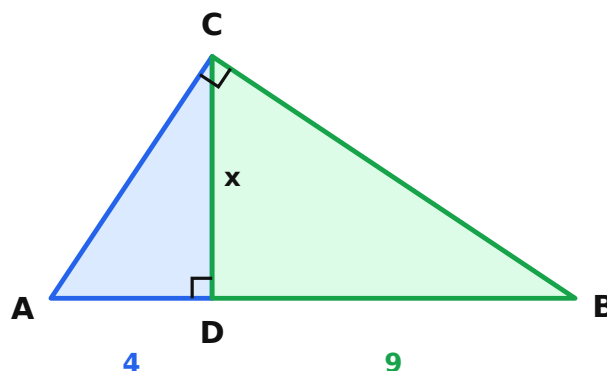
DE is parallel to BC, so $AD/DB = AE/EC$. Here $EC = x$ is the unknown.

How does similarity work in right triangles?

The altitude from the right angle to the hypotenuse creates two smaller right triangles, and each one is similar to the original. The **geometric mean** of positive numbers a and b is $\sqrt{ab}$, the number x with $\frac{a}{x} = \frac{x}{b}$.

Part of the triangle	Relationship
Altitude CD	$CD^2 = AD \cdot DB$
Leg AC	$AC^2 = AD \cdot AB$
Leg BC	$BC^2 = DB \cdot AB$

For $AD=4$ and $DB=9$, $CD=\sqrt{36}=6$ and $AB=13$. Adding the two leg formulas gives $AC^2 + BC^2 = AB(AD + DB) = AB^2$, which proves the Pythagorean theorem.

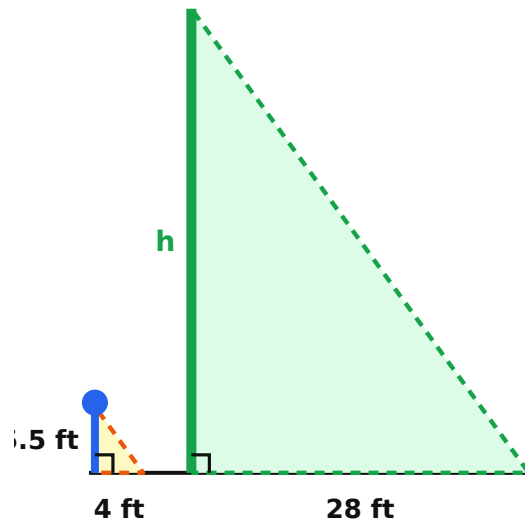


The altitude CD splits the right triangle into two smaller right triangles similar to ABC.

How do you measure a tall object with similar triangles?

Draw the two right triangles that sunlight makes, confirm they share an angle, write a

proportion of heights to shadows, and solve. A 5.5 ft person casts a 4 ft shadow while a tree casts a 28 ft shadow: $\frac{h}{28} = \frac{5.5}{4}$, so $h = 38.5$ ft.



Sunlight makes the same angle with the ground at both places, so the two right triangles are similar (AA).

Similar Triangles and Proportions: frequently asked questions

What is the difference between similar and congruent triangles?

Congruent triangles have the same shape and size. Similar triangles have the same shape, so their angles match and their sides share one ratio, but their sizes can differ. Congruent triangles are the special case with ratio 1.

Why is SSA not a similarity criterion?

Two sides and an angle that is not between them can fit two different triangles, so the remaining sides are not forced into one ratio. SAS works because the included angle fixes the shape.

How do you decide which sides correspond?

Match sides opposite congruent angles, or match the shortest with the shortest and the longest with the longest. Writing the similarity statement in matching order, such as $ABC \sim DEF$, keeps the correspondence straight.

What is a geometric mean and why does it appear in right triangles?

The geometric mean of a and b is the square root of ab . It appears because the altitude to the hypotenuse makes similar triangles whose proportions contain the same middle length twice.

How does the side-splitter theorem differ from its converse?

The theorem starts with parallel lines and concludes that two sides are divided proportionally. The converse starts with equal ratios and concludes that the segment is parallel to the third side.