

Geometry · Rigid Transformations · Lesson

Common Core G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5 · mathovore.fr

A rigid transformation moves a figure without changing any distance or angle, so the image is congruent to the original. Translations slide, reflections flip over a line, and rotations turn about a point. Dilations and stretches are not rigid. Symmetries are rigid transformations that map a figure onto itself.

GeometryCommon Core: G-CO.A.2 · G-CO.A.3 · G-CO.A.4 · G-CO.A.5

Key facts:

- Standards G-CO.A.2 to G-CO.A.5 cover transformations as functions, symmetry, precise definitions and drawing images.
- Translations, reflections and rotations preserve distance and angle measure; dilations and stretches do not.
- Reflection over $y = x$ swaps coordinates: (x, y) becomes (y, x) .
- A 90-degree counterclockwise rotation about the origin sends (x, y) to $(-y, x)$.
- A regular n -sided polygon has n lines of symmetry and rotational symmetry of $360/n$ degrees.

What is a rigid transformation?

A **transformation** is a function that takes every point of the plane (the **pre-image**) and assigns it to a new point (the **image**). A **rigid transformation** keeps all distances and angle measures the same, so the image is congruent to the pre-image. Translations, reflections and rotations are rigid. Dilations and stretches are not.

Three rigid transformations

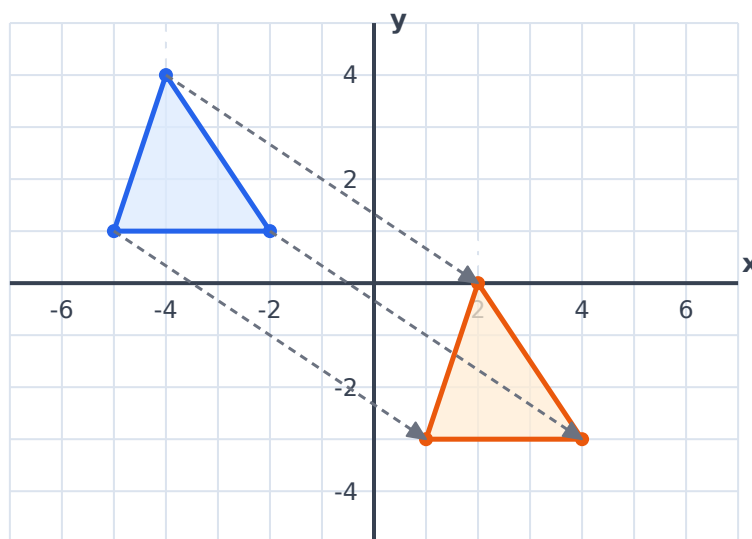
Translation: slides every point the same distance in the same direction, described by a vector.

Reflection: flips the figure over a line so that the line is the perpendicular bisector of each segment joining a point to its image.

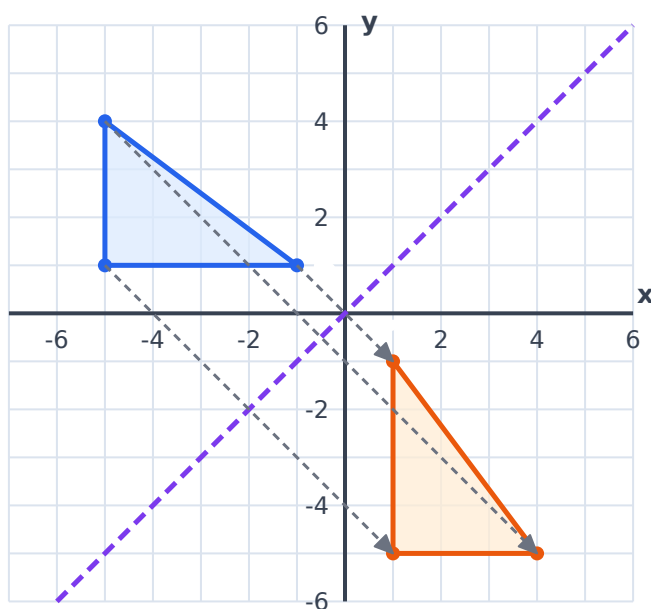
Rotation: turns the figure by a given angle about a fixed center point.

How do you translate, reflect and rotate on the coordinate plane?

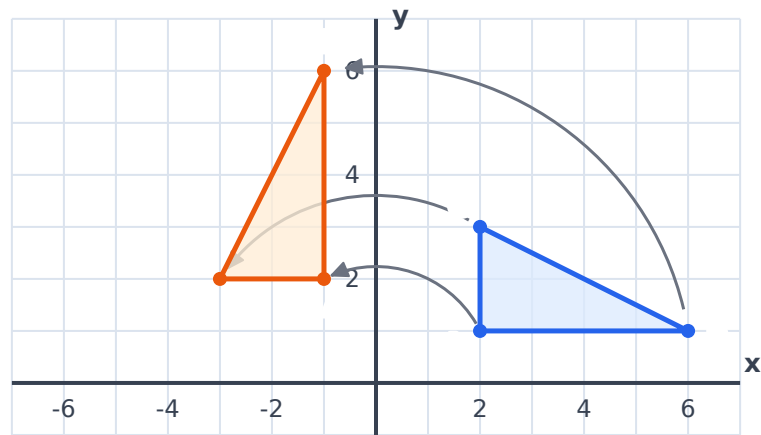
Transformation	Rule for a point (x, y)
Translation by $\langle a, b \rangle$	moves to $(x+a, y+b)$
Reflection over the x-axis	moves to $(x, -y)$
Reflection over the y-axis	moves to $(-x, y)$
Reflection over the line $y = x$	moves to (y, x)
Rotation 90° counterclockwise about the origin	moves to $(-y, x)$
Rotation 180° about the origin	moves to $(-x, -y)$



Translation by $\langle 6, -4 \rangle$: every point slides 6 units right and 4 units down.



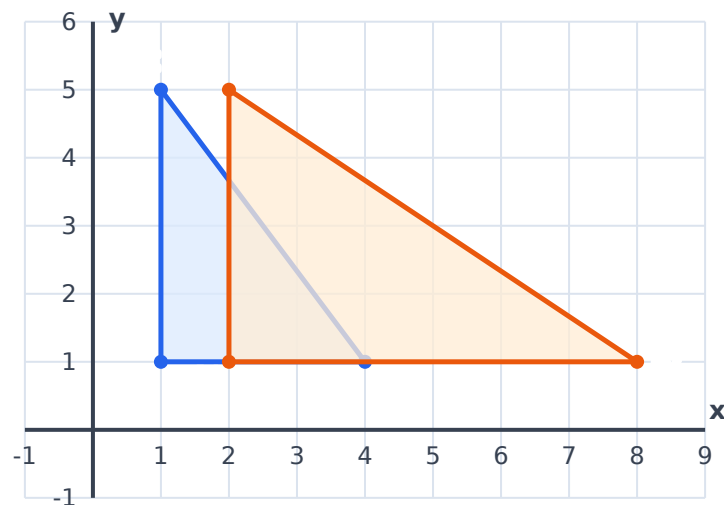
Reflection over the line $y = x$ swaps the coordinates: (x, y) becomes (y, x) .



Rotation of 90° counterclockwise about the origin: (x, y) becomes $(-y, x)$.

How do you tell a rigid transformation from a non-rigid one?

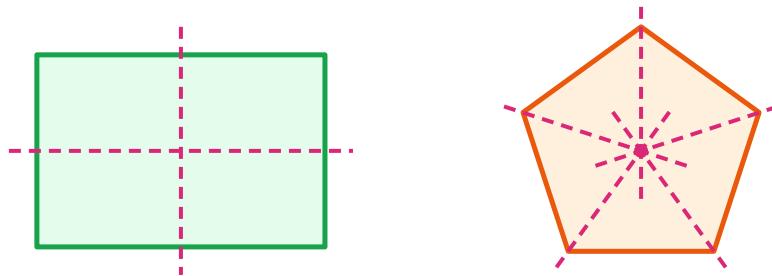
Pick a segment and measure it before and after. A rigid transformation always gives the same length. The stretch below turns a side of length 3 into length 6, so it changes the shape.



The stretch (x, y) becomes $(2x, y)$ changes the leg 3 to 6, so it is **not** rigid.

What is symmetry of a figure?

A figure has **reflection symmetry** if a reflection over some line maps it onto itself, and **rotational symmetry** if a rotation about its center does. A rectangle that is not a square has 2 lines of symmetry and a turn of 180° . A regular polygon with n sides has n lines of symmetry, and its smallest rotation is $\frac{360}{n}$ degrees.



Rectangle: 2 lines, turn 180° Regular pentagon: 5 lines, turn 72°

Dashed lines are lines of symmetry. The pentagon maps onto itself after a turn of 72° about its center.

How do you describe a sequence of transformations?

Apply the steps in the stated order, because the order can change the result.

Worked example. Rotate $(4, -1)$ by 90° counterclockwise about the origin, then reflect it over the x-axis.

1. Rotation: (x, y) becomes $(-y, x)$, so $(4, -1)$ becomes $(1, 4)$.
2. Reflection over the x-axis: (x, y) becomes $(x, -y)$, so $(1, 4)$ becomes $(1, -4)$.
3. The final image is $(1, -4)$.

Rigid Transformations: frequently asked questions

What does it mean for a transformation to be rigid?

A rigid transformation preserves the distance between every pair of points and the measure of every angle. Because of this, the image has exactly the same size and shape as the pre-image.

Does the order of a sequence of transformations matter?

Yes. A translation followed by a reflection usually gives a different image than the same reflection followed by the translation. Always apply the steps in the order given.

How do you find the line of reflection?

Connect any point to its image. The line of reflection is the perpendicular bisector of that segment, so it passes through the midpoint and is perpendicular to the segment.

How many lines of symmetry does a regular polygon have?

A regular polygon with n sides has n lines of symmetry. For example, a regular hexagon has 6 lines and can also turn onto itself after rotations of 60, 120, 180, 240 and 300 degrees.

Is a dilation a rigid transformation?

No. A dilation changes lengths by a scale factor, so the image is similar to the pre-image but not congruent unless the scale factor is 1. Only distance-preserving maps are rigid.