

Geometry · Right Triangle Trigonometry · Lesson

Common Core G-SRT.C.6, G-SRT.C.7, G-SRT.C.8 · mathovore.fr

Trigonometric ratios compare two sides of a right triangle. For an acute angle A, sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent. Right triangles with equal acute angles are similar, so each ratio depends only on the angle and can find missing sides and angles.

Geometry Common Core: G-SRT.C.6 · G-SRT.C.7 · G-SRT.C.8

Key facts:

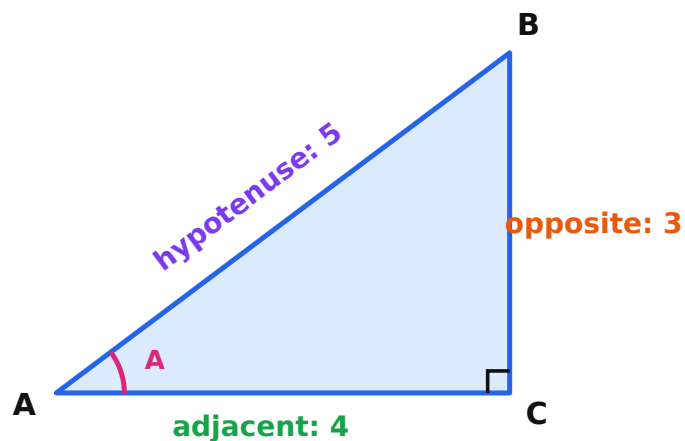
- Standards G-SRT.C.6, G-SRT.C.7 and G-SRT.C.8 cover ratios from similarity, complementary angles and solving right triangles.
- SOH CAH TOA: sine, cosine and tangent are opposite/hypotenuse, adjacent/hypotenuse and opposite/adjacent.
- If $A + B = 90$ degrees, then $\sin A = \cos B$.
- 45-45-90 sides are $s, s, s\sqrt{2}$; 30-60-90 sides are $x, x\sqrt{3}, 2x$.
- Angles of elevation and depression are measured from the horizontal and are equal for the same line of sight.

What are sine, cosine and tangent?

In a right triangle the longest side, across from the right angle, is the **hypotenuse**. For a chosen acute angle, the leg across from it is the **opposite** side and the leg touching it is the **adjacent** side. The three trigonometric ratios compare these sides.

SOH CAH TOA

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}, \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

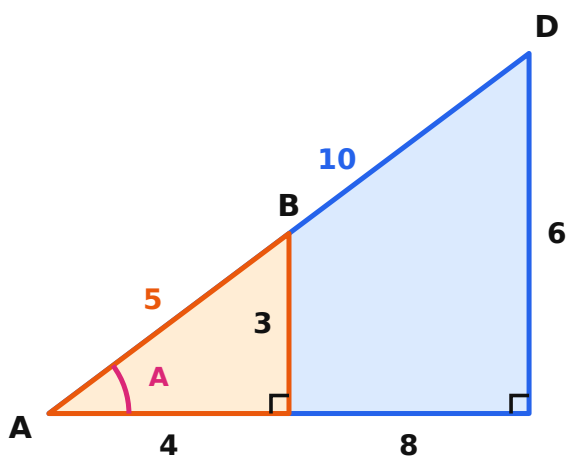


For angle A in this 3-4-5 triangle, the opposite side is 3, the adjacent side is 4 and the hypotenuse is 5.

Here $\sin A = \frac{3}{5} = 0.6$, $\cos A = \frac{4}{5} = 0.8$ and $\tan A = \frac{3}{4} = 0.75$.

Why do the ratios depend only on the angle?

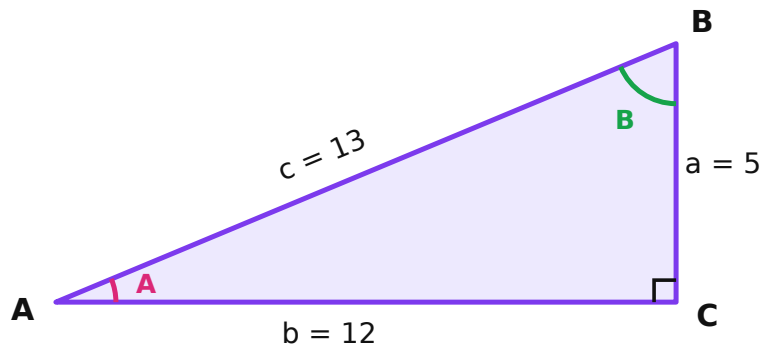
Right triangles with the same acute angle are similar by AA, so their sides are proportional and every ratio of two sides matches. The small triangle gives $\frac{3}{5}$ and the large one gives $\frac{6}{10}$: both equal 0.6. That is why one stored value of $\sin 36.9^\circ$ works for every triangle with that angle.



Triangles ABC and ADE share angle A, so the ratio of opposite side to hypotenuse is 0.6 in both.

How are sine and cosine of complementary angles related?

The acute angles of a right triangle add to 90° . The side opposite one acute angle is adjacent to the other, so $\sin A = \cos B$ whenever $A + B = 90^\circ$. In the figure, $\sin A = \frac{5}{13} = \cos B$. In general, $\sin x^\circ = \cos(90^\circ - x^\circ)$.

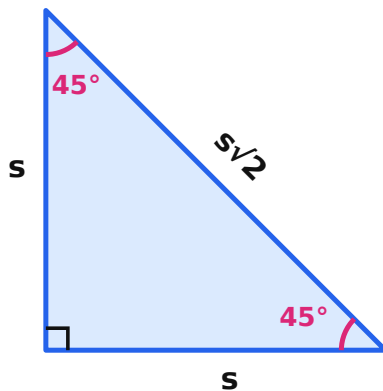


The side of length 5 is opposite angle A and adjacent to angle B, so $\sin A$ and $\cos B$ are both $5/13$.

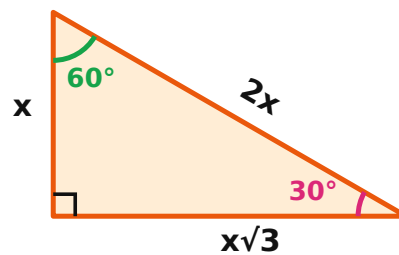
What are the special right triangles?

Two triangles have exact side ratios, so no calculator is needed.

Triangle	Side lengths
45-45-90	Legs s , hypotenuse $s\sqrt{2}$
30-60-90	Short leg x , long leg $x\sqrt{3}$, hypotenuse $2x$



45-45-90 triangle



30-60-90 triangle

Exact side lengths of the 45-45-90 and 30-60-90 triangles.

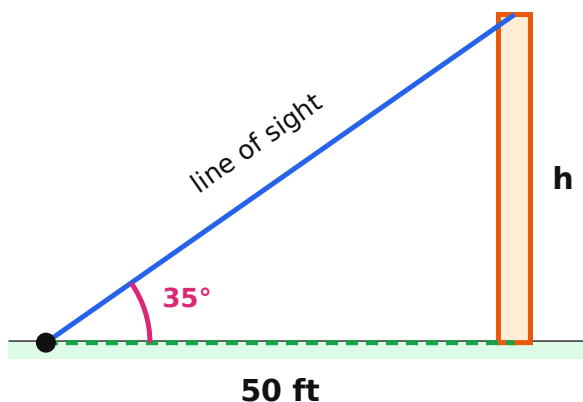
How do you solve a right triangle?

An **angle of elevation** is measured upward from the horizontal. An **angle of depression** is measured downward from the horizontal, and it equals the angle of elevation from the other end of the line of sight.

1. Sketch the triangle and label the known parts.
2. Choose the ratio that links the unknown to two known parts.

3. Write the equation and solve it; use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ to find an angle.
4. Check with the Pythagorean theorem or the 90° sum.

Example. From a point on the ground 50 ft from a tower, the angle of elevation to the top is 35° . Then $\tan 35^\circ = \frac{h}{50}$, so $h = 50 \tan 35^\circ \approx 50(0.7002) \approx 35.0$ ft.



The tower height h is the side opposite the 35 degree angle; the 50 ft distance is adjacent.

Right Triangle Trigonometry: frequently asked questions

What does SOH CAH TOA mean?

It is a memory aid for the three ratios: Sine is Opposite over Hypotenuse, Cosine is Adjacent over Hypotenuse, and Tangent is Opposite over Adjacent. The words opposite and adjacent are always relative to the acute angle you picked.

Why does sine depend only on the angle?

Right triangles with the same acute angle are similar, so their sides are in proportion. The ratio of opposite to hypotenuse is therefore the same in a small triangle and a large one, which makes it a function of the angle alone.

How do you find an angle when you know two sides?

Pick the ratio that uses the two known sides and apply the inverse function. For opposite 7 and adjacent 11, $\tan A = \frac{7}{11}$, so $A = \tan^{-1}\left(\frac{7}{11}\right) \approx 32.5^\circ$.

What is the relationship between sine and cosine of complementary angles?

If two angles add to 90° , the sine of one equals the cosine of the other. For example, $\sin 25^\circ = \cos 65^\circ$.

What is the difference between angle of elevation and angle of depression?

Elevation is measured up from the horizontal to a line of sight, and depression is measured down from the horizontal. The two angles are equal for the same line of sight because they are alternate interior angles.