

Geometry · Logic and Two-Column Proofs · Worksheet Answer Key

Common Core G-CO.C.9 · mathovore.fr

Each solution names the statement, the reason or the counterexample. For proofs, check that every reason is Given, a definition, a postulate, a property or a theorem, and that the last statement is the Prove.

GeometryCommon Core: G-CO.C.9

Problem 1: Hypothesis and conclusion

Hypothesis (after *if*): **it rains on Saturday**. Conclusion (after *then*): **the Little League game is canceled**.

Problem 2: Writing if-then form

If a number is divisible by both 2 and 3, then it is divisible by 6. The hypothesis is “a number is divisible by both 2 and 3.” The words after *if* in the original sentence form the hypothesis.

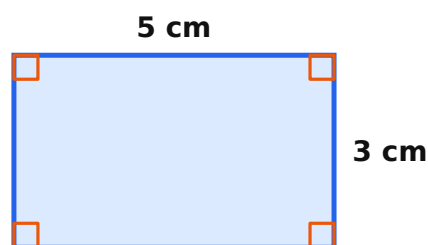
Problem 3: Rewriting a statement

If a figure is a square, then it has four congruent sides.

Problem 4: Writing a converse

Converse: **If a figure is a rectangle, then it is a square.** It is **false**.

Counterexample: a rectangle that measures 5 cm by 3 cm has four right angles but its sides are not equal.



A 5 cm by 3 cm rectangle is not a square.

Problem 5: Inverse and contrapositive

Inverse (negate both parts): **If an angle does not measure 90° , then it is not a right angle.**

Contrapositive (swap and negate): **If an angle is not a right angle, then it does not measure 90° .**

Problem 6: Truth values of related statements

- a. Statement: **true** (every multiple of 4 is even).
- b. Converse: "If a number is even, then it is a multiple of 4." **False**; counterexample 6.
- c. Inverse: "If a number is not a multiple of 4, then it is not even." **False**; counterexample 6.
- d. Contrapositive: "If a number is not even, then it is not a multiple of 4." **True**, matching the statement.

Problem 7: Counterexample with primes

The number **2** is prime, because its only factors are 1 and 2, but it is even. The hypothesis is true and the conclusion is false, so the statement is false.

Problem 8: Counterexample with squares

Let $x = -7$. Then $(-7)^2 = 49$, so the hypothesis is true, but $x \neq 7$. $x = -7$ is a counterexample.

Problem 9: Counterexample with angles

$100 + 80 = 180$, so the angles are supplementary and the hypothesis is true. They do not share a side, so they are not a linear pair and the conclusion is false.

The two angles in the figure are a counterexample.



Supplementary angles that do not form a linear pair.

Problem 10: Inductive pattern of dots

Counts: 2, 6, 12, 20. The differences are 4, 6, 8, so the next differences are 10 and 12.

Pattern 5: $20+10=30$. Pattern 6: $30+12=42$.

Pattern n is a rectangle with n rows and $n + 1$ columns: $n(n+1)$. Check: $5 \times 6 = 30$ and $6 \times 7 = 42$.

Problem 11: Inductive or deductive?

- a. **Inductive**: it generalizes from a pattern of cases.
- b. **Deductive**: it applies a general fact to a specific case.
- c. **Inductive**: three measured cases suggest a rule but do not prove it.
- d. **Deductive**: it applies a known theorem to a specific triangle.

Problem 12: Making a conjecture

Every sum is even. **Conjecture: the sum of two odd numbers is even.** This is inductive reasoning, so it is a conjecture until it is proven.

Problem 13: Disproving a conjecture

Try a negative number: $-3+5=2$ and $2 < 5$. The sum 2 is not greater than 5. **The numbers -3 and 5 are a counterexample.**

Problem 14: Naming properties of equality

- a. **Symmetric Property.**
- b. **Transitive Property.**
- c. **Reflexive Property.**
- d. **Division Property of Equality** (divide both sides by 3).

Problem 15: Supplying reasons

Statements	Reasons
1. $3x+7=22$	1. Given
2. $3x=15$	2. Subtraction Property of Equality (subtract 7 from both sides)
3. $x=5$	3. Division Property of Equality (divide both sides by 3)

Check: $3(5)+7=22$.

Problem 16: Supplying statements

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary; $m\angle 1 = 35^\circ$	1. Given
2. $m\angle 1 + m\angle 2 = 90^\circ$	2. Definition of complementary angles
3. $35^\circ + m\angle 2 = 90^\circ$	3. Substitution Property of Equality
4. $m\angle 2 = 55^\circ$	4. Subtraction Property of Equality

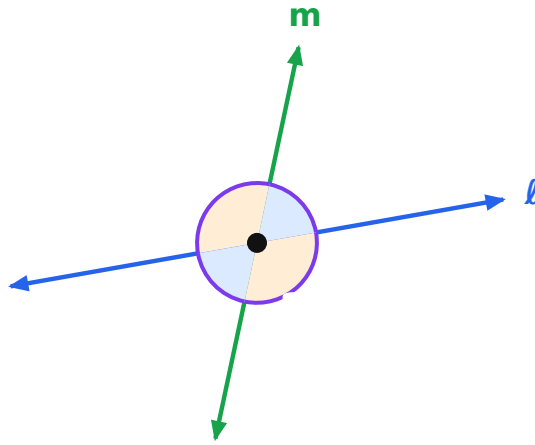
Check: $35 + 55 = 90$.

Problem 17: Midpoint proof

Statements	Reasons
1. M is the midpoint of $\overline{ST}$; $ST = 24$	1. Given
2. $SM = MT$	2. Definition of midpoint
3. $SM + MT = ST$	3. Segment Addition Postulate
4. $SM + SM = 24$	4. Substitution Property of Equality
5. $2(SM) = 24$	5. Combining like terms
6. $SM = 12$	6. Division Property of Equality

Problem 18: Vertical angles proof

Statements	Reasons
1. Lines l and m intersect	1. Given
2. $\angle 2$ and $\angle 3$ are a linear pair; $\angle 3$ and $\angle 4$ are a linear pair	2. Definition of linear pair
3. $\angle 2$ and $\angle 3$ are supplementary; $\angle 3$ and $\angle 4$ are supplementary	3. Linear Pair Postulate
4. $m\angle 2 + m\angle 3 = 180^\circ$; $m\angle 3 + m\angle 4 = 180^\circ$	4. Definition of supplementary angles
5. $m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$	5. Transitive Property of Equality (both sums equal 180)
6. $m\angle 2 = m\angle 4$	6. Subtraction Property of Equality (subtract $m\angle 3$)
7. $\angle 2 \cong \angle 4$	7. Definition of congruent angles



Example: angles 2 and 4 both measure 112° .

Problem 19: Paragraph proof

Because $\angle A$ and $\angle B$ are complementary, $m\angle A + m\angle B = 90^\circ$. Because $\angle C$ and $\angle B$ are complementary, $m\angle C + m\angle B = 90^\circ$. Both sums equal 90° , so $m\angle A + m\angle B = m\angle C + m\angle B$ by the Transitive Property of Equality. Subtracting $m\angle B$ from both sides gives $m\angle A = m\angle C$. Angles with equal measures are congruent, so $\angle A \cong \angle C$.

Problem 20: Contrapositive in context

Contrapositive: "If a student does not have a GPA of at least 3.5, then the student is not on the honor roll." Since $3.2 < 3.5$, Priya's GPA is below 3.5.

Priya is not on the honor roll. The contrapositive is true because the original rule is true.

Problem 21: Chaining conditionals

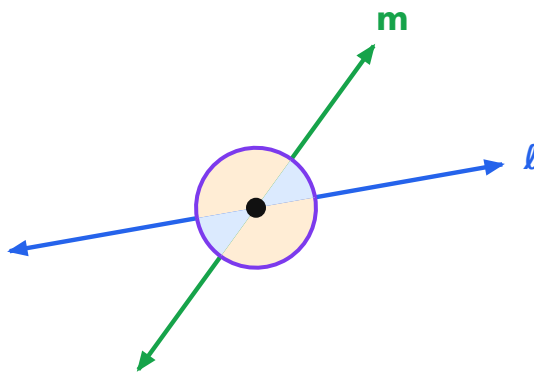
Link the conclusion of the first rule to the hypothesis of the second: **If it snows more than 6 inches, then the bake sale moves to Sunday.**

Eight inches is more than 6 inches, so the hypothesis is true. Therefore **the bake sale moves to Sunday.**

Problem 22: Solve and justify

1. $3x + 14 = 5x - 6$: Vertical Angles Theorem ($\angle 1 \cong \angle 3$).
2. $20 = 2x$: Addition and Subtraction Properties of Equality. 3. $x = 10$: Division Property of Equality.
4. $m\angle 1 = 3(10) + 14 = 44^\circ$: Substitution. 5. $\angle 1$ and $\angle 2$ are a linear pair, so they are

supplementary: $m\angle 2 = 180 - 44 = 136^\circ$. **136°** .



Check: $44^\circ + 136^\circ = 180^\circ$.

Problem 23: Writing a biconditional

Converse: **If the measure of an angle is 90° , then it is a right angle.** It is **true** by the definition of a right angle.

Biconditional: **An angle is a right angle if and only if its measure is 90° .**