

Geometry · Logic and Two-Column Proofs · Test Answer Key

Common Core G-CO.C.9 · mathovore.fr

Each answer gives a model solution and a scoring guide. Award credit for correct reasoning even when the wording differs. In proofs, every statement needs a valid reason, and steps may appear in any logical order that works.

GeometryCommon Core: G-CO.C.9

Question 1: Parts of a conditional (2 points)

- a. Hypothesis: **a quadrilateral has four right angles**. Conclusion: **it is a rectangle**.
- b. Converse: **If a quadrilateral is a rectangle, then it has four right angles**. This is **true** by the definition of a rectangle.

Scoring: 1 point for both parts identified; 1 point for the correct converse with the correct truth value.

Question 2: Related statements (3 points)

- a. Converse: **If a figure is a parallelogram, then it is a rhombus**. False. A rectangle that is 5 cm by 3 cm is a parallelogram but not a rhombus.
- b. Inverse: **If a figure is not a rhombus, then it is not a parallelogram**. False. The same 5 cm by 3 cm rectangle is not a rhombus but is a parallelogram.
- c. Contrapositive: **If a figure is not a parallelogram, then it is not a rhombus**. True, because the original statement is true.

Scoring: 1 point per part for the correct statement, correct truth value and, for false statements, a valid counterexample.

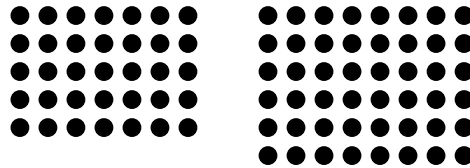
Question 3: Reasoning types and counterexample (2 points)

- a. **Inductive**: the conclusion comes from a pattern of observed cases.
- b. Let $x = -2$. Then $x^2 = 4 > -2$, so the hypothesis is true, but $-2 > 1$ is false. $x = -2$ is a counterexample (other negative numbers and numbers between 0 and 1 that fit the hypothesis also work).

Scoring: 1 point for each part. In part (b), the value must make the hypothesis true and the conclusion false.

Question 4: Inductive pattern (2 points)

- a. The counts are 3, 8, 15, 24, so the differences are 5, 7, 9 and the next are 11 and 13:
 $24+11=35$ and $35+13=48$. **35 dots and 48 dots.**
- b. Pattern n has n rows of $n + 2$ dots: $n(n+2)$. Check: $5 \times 7 = 35$.



$n = 5$

$n = 6$

Pattern 5 has 35 dots and Pattern 6 has 48 dots.

Scoring: 1 point for both counts; 1 point for a correct rule written with n .

Question 5: Properties of equality (2 points)

- a. **Division Property of Equality.**
- b. **Transitive Property of Equality.**

Scoring: 1 point for each correct property.

Question 6: Completing a proof (3 points)

- a. Definition of supplementary angles.
- b. $72^\circ + m \angle B = 180^\circ$.
- c. Subtraction Property of Equality (subtract 72 from both sides).

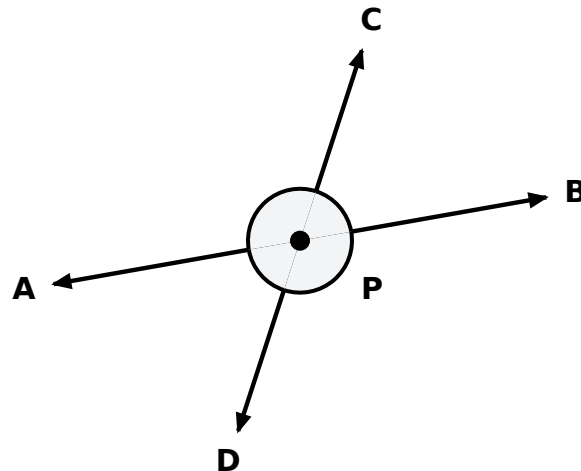
Check: $72+108=180$.

Scoring: 1 point for each correct entry.

Question 7: Vertical angles proof (4 points)

Statements	Reasons
1. Lines AB and CD intersect at P	1. Given
2. $\angle APD$ and $\angle DPB$ are a linear pair; $\angle DPB$ and $\angle BPC$ are a linear pair	2. Definition of linear pair
3. $\angle APD$ and $\angle DPB$ are supplementary; $\angle DPB$ and $\angle BPC$ are supplementary	3. Linear Pair Postulate

Statements	Reasons
4. $m\angle APD + m\angle DPB = 180^\circ$; $m\angle DPB + m\angle BPC = 180^\circ$	4. Definition of supplementary angles
5. $m\angle APD + m\angle DPB = m\angle DPB + m\angle BPC$	5. Transitive Property of Equality
6. $m\angle APD = m\angle BPC$	6. Subtraction Property of Equality
7. $\angle APD \cong \angle BPC$	7. Definition of congruent angles

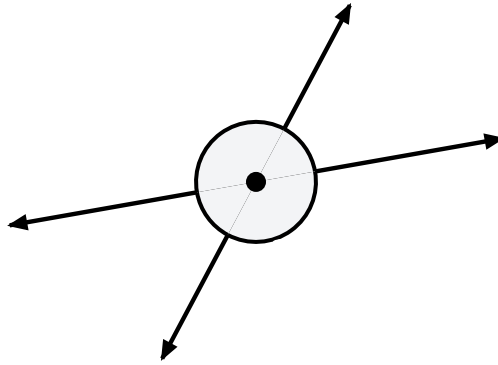


Both angles are supplementary to angle DPB, so they are equal.

Scoring: 1 point for naming the linear pairs with a reason (steps 2-3); 1 point for the two sums equal to 180 (step 4); 1 point for combining them and subtracting (steps 5-6); 1 point for the conclusion with a reason (step 7). Accept an equivalent proof with every statement justified.

Question 8: Vertical angles with algebra (2 points)

- Vertical angles are congruent (Vertical Angles Theorem): $7x+3=10x-18$, so $21=3x$ and $x=7$.
- $m\angle 1=7(7)+3=52^\circ$. Angles 1 and 2 are a linear pair, so they are supplementary: $m\angle 2=180-52=128^\circ$. **128°** .



Angles 1 and 3 measure 52° ; angles 2 and 4 measure 128° .

Scoring: 1 point for $x = 7$ with the theorem named; 1 point for 128° with the linear pair reason.

Total
20 points