

Geometry · Logic and Two-Column Proofs · Lesson

Common Core G-CO.C.9 · mathovore.fr

A conditional statement "If p , then q " has a converse, inverse and contrapositive, and only the contrapositive always has the same truth value. Inductive reasoning finds patterns, deductive reasoning proves them. A two-column proof lists statements with reasons, as in the proof that vertical angles are congruent.

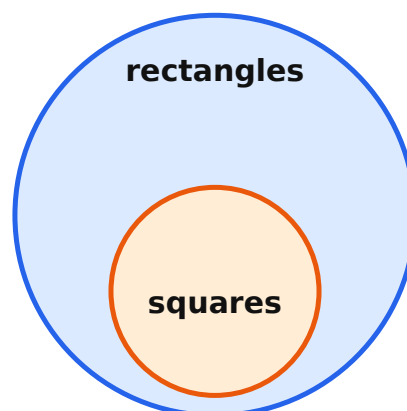
GeometryCommon Core: G-CO.C.9

Key facts:

- A conditional "If p , then q " has a hypothesis (p) and a conclusion (q).
- The converse swaps p and q , the inverse negates both, and the contrapositive does both.
- A statement and its contrapositive always have the same truth value.
- One counterexample disproves a conditional; inductive patterns are not proofs.
- Vertical angles are congruent, proved with linear pairs and the Subtraction Property of Equality.

What is a conditional statement?

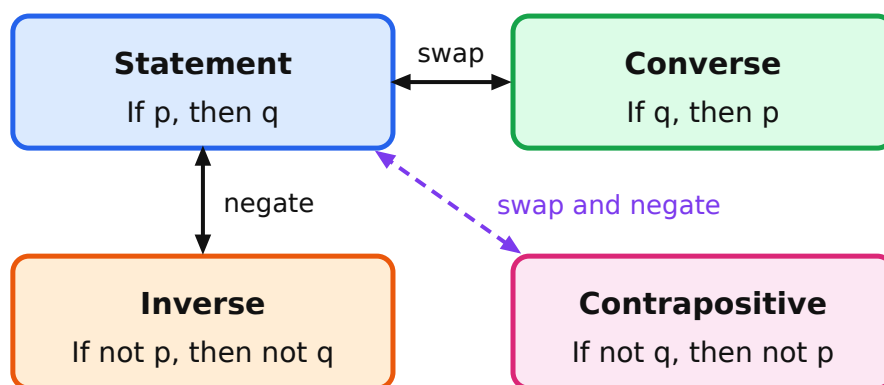
A **conditional statement** has the form "If p , then q ." The *if* part is the **hypothesis** and the *then* part is the **conclusion**. Example: "If a polygon is a square, then it is a rectangle." A conditional is false when one case has a true hypothesis and a false conclusion. That case is a **counterexample**.



Every square lies inside the set of rectangles: If a polygon is a square, then it is a rectangle.

How do you write the converse, inverse and contrapositive?

Statement	Form	Square example
Conditional	If p, then q	If a polygon is a square, then it is a rectangle. (true)
Converse	If q, then p	If a polygon is a rectangle, then it is a square. (false)
Inverse	If not p, then not q	If a polygon is not a square, then it is not a rectangle. (false)
Contrapositive	If not q, then not p	If a polygon is not a rectangle, then it is not a square. (true)

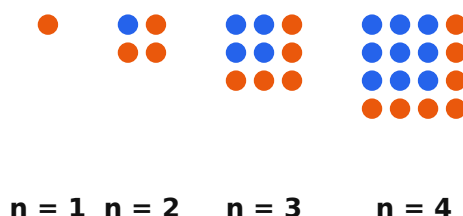


The contrapositive is the only related statement that always matches the truth of the original.

A statement and its contrapositive always have the same truth value. When a conditional and its converse are both true, combine them into a **biconditional** using "if and only if."

What is the difference between inductive and deductive reasoning?

Inductive reasoning forms a conjecture from patterns. The squares below suggest that the sum of the first n odd numbers is n^2 , but patterns can fail, and one counterexample disproves a conjecture. **Deductive reasoning** uses accepted facts (definitions, postulates and proven theorems) in logical steps, so the conclusion must be true.



Each square adds an L-shaped border of the next odd number of dots: 1, 1 + 3, 1 + 3 + 5, 1 + 3 + 5 + 7.

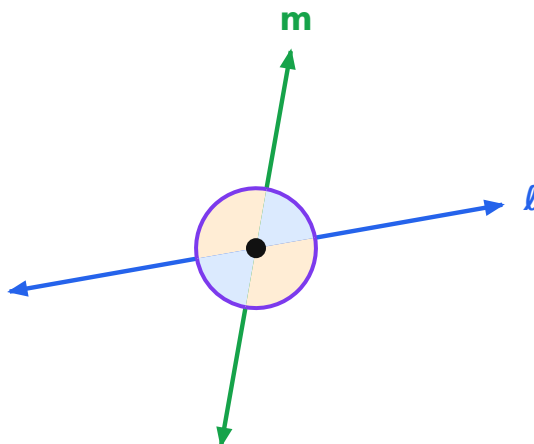
How do you write a two-column proof?

Properties of equality

Reflexive: $a = a$. **Symmetric:** if $a = b$, then $b = a$. **Transitive:** if $a = b$ and $b = c$, then $a = c$.

Addition, Subtraction, Multiplication and Division Properties: doing the same thing to both sides keeps an equation true. **Substitution:** replace a quantity with an equal one.

1. Mark the figure, then write the **Given** and the **Prove**.
2. Write each statement in the left column and its reason in the right column.
3. Every reason is Given, a definition, a postulate, a property or a proven theorem. The last statement is the Prove.



Lines l and m form four angles. Angles 1 and 3 are opposite each other.

Worked example. Given: lines l and m intersect. Prove: $\angle 1 \cong \angle 3$ (vertical angles are congruent).

Statements	Reasons
1. Lines l and m intersect	Given
2. $\angle 1$ and $\angle 2$ are a linear pair; $\angle 2$ and $\angle 3$ are a linear pair	Definition of linear pair
3. $\angle 1$ and $\angle 2$ are supplementary; $\angle 2$ and $\angle 3$ are supplementary	Linear Pair Postulate
4. $m\angle 1 + m\angle 2 = 180^\circ$; $m\angle 2 + m\angle 3 = 180^\circ$	Definition of supplementary angles
5. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Transitive Property of Equality
6. $m\angle 1 = m\angle 3$	Subtraction Property of Equality
7. $\angle 1 \cong \angle 3$	Definition of congruent angles

A **paragraph proof** gives the same steps in sentences.

Logic and Two-Column Proofs: frequently asked questions

How do you find the converse of a conditional statement?

Swap the hypothesis and the conclusion. The converse of "If it is a square, then it is a rectangle" is "If it is a rectangle, then it is a square." A true statement can have a false converse.

Why does the contrapositive matter?

The contrapositive always has the same truth value as the original statement. If a statement is true, its contrapositive is true, which makes it useful in proofs.

What is the difference between a conjecture and a theorem?

A conjecture is a guess based on patterns, found by inductive reasoning. A theorem is a statement that has been proven with deductive reasoning from definitions, postulates and earlier theorems.

How many counterexamples disprove a statement?

Just one. A single case where the hypothesis is true and the conclusion is false shows that a conditional statement is false, even if it works in many other cases.

What reasons can you use in a two-column proof?

Use Given information, definitions, postulates, properties of equality and theorems that were already proven. Never cite a statement you are trying to prove.