

Geometry · Probability: Independence, Conditional Probability and Fair Decisions · Worksheet Answer Key

Common Core S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7, S-CP.B.8, S-CP.B.9, S-MD.B.6, S-MD.B.7 · mathovore.fr

This answer key gives a full worked solution for every probability problem, with each count, product and fraction shown. Compare your method as well as your result, especially for conditional probability, independence checks and the counting problems.

GeometryCommon Core: S-CP.A.1 · S-CP.A.2 · S-CP.A.3 · S-CP.A.4 · S-CP.A.5 · S-CP.B.6 · S-CP.B.7 · S-CP.B.8 · S-CP.B.9 · S-MD.B.6 · S-MD.B.7

Problem 1: Sample space of a coin and a die

(a) $2 \times 6 = 12$ outcomes.

(b) $(H,2), (H,4), (H,6)$.

(c) $\frac{3}{12} = \frac{1}{4}$. **Answer: 12 outcomes; three favorable outcomes; probability $\frac{1}{4}$**

Problem 2: Complement of an event

The complement is “the marble is blue or green.” There are 20 marbles.

$P(R) = \frac{6}{20} = \frac{3}{10}$, so $P(\text{not } R) = 1 - \frac{3}{10} = \frac{7}{10}$ (check: 14 of 20 marbles). **Answer: $\frac{3}{10}$ and $\frac{7}{10}$**

Problem 3: Union and intersection of events

(a) $A = \{3, 6, 9, 12, 15, 18\}$. Outcomes in both A and B: $\{6, 12, 18\}$.

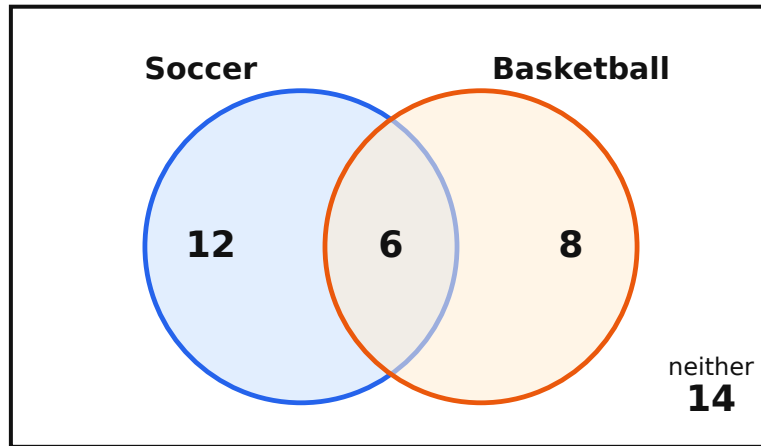
(b) B has 10 even numbers, so A or B has $6 + 10 - 3 = 13$ outcomes.

(c) $P(A \text{ or } B) = \frac{13}{20}$. **Answer: 13 outcomes; probability $\frac{13}{20}$**

Problem 4: Filling in a Venn diagram

(a) Soccer only: $18 - 6 = 12$. Basketball only: $14 - 6 = 8$. Either sport: $12 + 6 + 8 = 26$, so neither: $40 - 26 = 14$.

(b) $\frac{14}{40} = \frac{7}{20}$. **Answer: 12, 8, 14; probability $\frac{7}{20}$**



Completed diagram: $12 + 6 + 8 + 14 = 40$.

Problem 5: Addition rule with decimals

$P(A \text{ or } B) = 0.45 + 0.30 - 0.12 = 0.63$. Neither: $1 - 0.63 = 0.37$. **Answer: 0.63 and 0.37**

Problem 6: Addition rule with cards

13 hearts and 4 kings, with the king of hearts counted twice: $13 + 4 - 1 = 16$ cards.

$\frac{16}{52} = \frac{4}{13}$. **Answer: $\frac{4}{13}$**

Problem 7: Testing independence with numbers

(a) $0.4 \times 0.5 = 0.20$, which equals $P(A \text{ and } B)$: independent.

(b) $0.6 \times 0.3 = 0.18$, which is not 0.15 : not independent. **Answer: (a) independent; (b) not independent**

Problem 8: Independent free throws

(a) $0.7 \times 0.7 = 0.49$. (b) $0.3 \times 0.3 = 0.09$. (c) Make-miss or miss-make: $2 \times 0.7 \times 0.3 = 0.42$.

Check: $0.49 + 0.09 + 0.42 = 1$. **Answer: 0.49, 0.09, 0.42**

Problem 9: Conditional probability formula

(a) $P(A|B) = \frac{0.18}{0.45} = 0.4$.

(b) Yes. $P(A|B) = 0.4 = P(A)$, so knowing B does not change the probability of A. **Answer: 0.4; independent**

Problem 10: Reading a two-way table

(a) $\frac{38}{150} = \frac{19}{75}$.

(b) Among 90 girls, 38 chose pizza: $\frac{38}{90} = \frac{19}{45}$.

(c) Among 72 pizza fans, 38 are girls: $\frac{38}{72} = \frac{19}{36}$.

Answer: $\frac{19}{75}$, $\frac{19}{45}$, $\frac{19}{36}$

Problem 11: Independence in a table

Overall, 50 of 200 walk: $P(\text{walk}) = \frac{1}{4}$. Seniors: $\frac{20}{80} = \frac{1}{4}$. Juniors: $\frac{30}{120} = \frac{1}{4}$.

All three equal, so the events are independent. **Answer: independent**

Problem 12: Comparing conditional probabilities

$P(\text{pizza}) = \frac{72}{150} = 0.48$. Boys: $\frac{34}{60} \approx 0.567$. Girls: $\frac{38}{90} \approx 0.422$.

The values differ, so the events are not independent. **Answer: not independent**

Problem 13: Two aces without replacement

$P(\text{ace first}) = \frac{4}{52}$ and $P(\text{ace second}|\text{ace first}) = \frac{3}{51}$.

$\frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$. **Answer:** $\frac{1}{221}$

Problem 14: Marbles without replacement

(a) $\frac{5}{8} \cdot \frac{3}{7} = \frac{15}{56}$.

(b) Blue then red has the same probability, so $2 \cdot \frac{15}{56} = \frac{15}{28}$. **Answer:** $\frac{15}{56}$ and $\frac{15}{28}$

Problem 15: Tree diagram for a game

(a) $P(\text{win}) = 0.3(0.4) + 0.7(0.7) = 0.12 + 0.49 = 0.61$.

(b) $P(\text{rain}|\text{win}) = \frac{0.12}{0.61} = \frac{12}{61} \approx 0.197$. **Answer: 0.61 and about 0.197**

Problem 16: Awarding medals (permutations)

(a) Order matters: ${}_8P_3 = 8 \cdot 7 \cdot 6 = 336$.

(b) One favorable order out of 336: $\frac{1}{336}$. **Answer: 336 ways; $\frac{1}{336}$**

Problem 17: Choosing a committee (combinations)

(a) Order does not matter: ${}_{10}C_3 = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$.

(b) Maya and Jamal plus one of the other 8 students: 8 committees. $\frac{8}{120} = \frac{1}{15}$. **Answer: 120; $\frac{1}{15}$**

Problem 18: Probability with combinations

Total teams: ${}_{12}C_4 = 495$.

(a) $\frac{{}_7C_4}{{}_495} = \frac{35}{495} = \frac{7}{99}$.

(b) $\frac{{}_7C_2 \cdot {}_5C_2}{{}_495} = \frac{21 \cdot 10}{495} = \frac{210}{495} = \frac{14}{33}$. **Answer: $\frac{7}{99}$ and $\frac{14}{33}$**

Problem 19: Lottery odds

${}_{30}C_5 = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26}{5!} = \frac{17,100,720}{120} = 142,506$.

One winning ticket: $\frac{1}{142,506}$. **Answer: 142,506 tickets; $\frac{1}{142,506}$**

Problem 20: Is the die rule fair?

Rule X: each friend has $\frac{2}{6} = \frac{1}{3}$. Rule Y: Maya $\frac{1}{6}$, Jamal $\frac{2}{6} = \frac{1}{3}$, Sofia $\frac{3}{6} = \frac{1}{2}$.

Only Rule X gives equal chances. **Answer: Rule X is fair; Rule Y is not**

1	2	3	4	5	6
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Maya Maya Jamal Jamal Sofia Sofia

Die result (top) and who is chosen (bottom)

Rule X gives each friend two faces.

Problem 21: A fair dice game

Of 36 equally likely rolls, sums of 8 to 12 occur $5+4+3+2+1=15$ times, and sums of 2 to 6 occur $1+2+3+4+5=15$ times. A sum of 7 occurs 6 times and is rolled again.

$P(\text{Diego}) = \frac{15}{36} = P(\text{Aisha})$, so each wins half of the decided games. **Answer: fair**

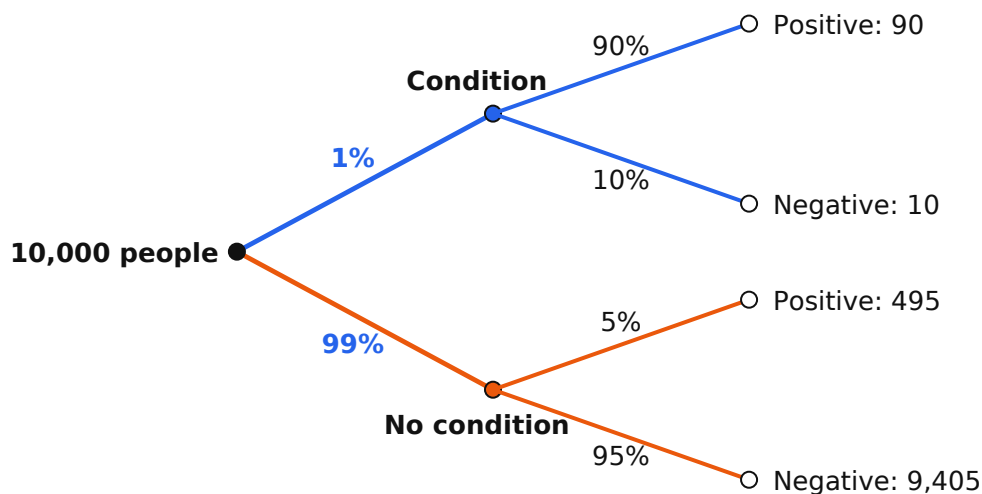
		Second die					
		1	2	3	4	5	6
1st	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Blue: Diego (15). Orange: Aisha (15). Yellow: roll again (6).

Problem 22: Analyzing a medical test

Condition: $10,000 \times 0.01 = 100$ people; 90 test positive. No condition: 9,900 people; $0.05 \times 9,900 = 495$ false positives.

$P(\text{condition}|+) = \frac{90}{90+495} = \frac{90}{585} = \frac{2}{13} \approx 0.154$. **Answer: 90 and 495; about 15.4%**



Completed tree.

Problem 23: Game strategy: stay or switch

The car is behind Door 1, 2 or 3 with probability $\frac{1}{3}$ each. The host never opens the car door or the contestant's door.

Car behind Door 1: staying wins. Car behind Door 2: the host must open Door 3, so switching wins. Car behind Door 3: the host must open Door 2, so switching wins.

$P(\text{stay wins}) = \frac{1}{3}$, $P(\text{switch wins}) = \frac{2}{3}$. If the host opens either goat door at random when he has a choice, $P(\text{car behind 2} | \text{host opens 3}) = \frac{1/3}{1/6 + 1/3} = \frac{2}{3}$. **Answer: switch ($\frac{2}{3}$) beats stay ($\frac{1}{3}$)**

Problem 24: Language electives

(a) Either language: $150 + 90 - 40 = 200$, so neither: 40. $\frac{40}{240} = \frac{1}{6}$.

(b) $P(\text{French} | \text{Spanish}) = \frac{40}{150} = \frac{4}{15}$.

(c) $P(\text{French}) = \frac{90}{240} = \frac{3}{8} \neq \frac{4}{15}$, so not independent. **Answer: $\frac{1}{6}$; $\frac{4}{15}$; not independent**